SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE IN SLOPE STABILITY ANALYSIS

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CERTIFICATE

It is certified that the work contained in the thesis entitled "SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE IN SLOPE STABILITY ANALYSIS" by "GAUTAM BHATTACHARYYA", has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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The thesis pertains to the study of the suitability of nonlinear programming approach using the sequential unconstrained minimization technique in conjunction with limit equilibrium methods of slices proposed by Janbu and Spencer to analyse slope stability problems. Optimization based generalized methods have been developed for :

1. solving the stability equations for a general shear surface to compute the factor of safety associated with a given shear surface using Spencer's method of analysis.
2. searching for the critical shear surface and the corresponding least factor of safety without a priori assumption regarding the shape of the shear surface taking care that the solution satisfies some conditions of acceptability.

The above have generally been formulated as constrained optimization problems. In this study a constrained problem has been converted to an unconstrained one using penalty techniques. As in most cases it is difficult to find an initial feasible design vector, the extended penalty function method suggested by Kavlie has been used. Apart from accepting

infeasible design points the method can also handle equality constraints without any difficulty. For the unconstrained minimization, Powell's conjugate direction method and Davidon-FletcherPowell variable metric method for multidimensional search along with the quadratic interpolation technique of unidirectional search have been used. However, Powell's method being a nongradient method has been adopted in most of the problems. General purpose computer programs for the proposed procedures have been developed to carry out the computations involved in this thesis.

In view of the fact that the efficiencies of these optimization methods are problem oriented, usefulness of the developed procedures have been critically examined by solving a variety of example problems in the following areas of interest :

1. slopes in homogeneous soils with linear and nonlinear strength envelopes,
2. slopes in soils with non-homogeneous and anisotropic undrained shear strength,
3. zoned dams and embankments including those founded on thin shear zone,
4. analysis of failed slopes with well-documented observed failure surfaces,
5. back-analysis of slopes.

From the study of these problems the relative merits of the proposed procedure have been established in :

1. solving stability equations for a given shear surface

over the method of solution suggested by Spencer.

2. finding the critical shear surface and the corresponding factor of safety over other technique like variational calculus, dynamic programming and nonlinear search procedures such as grid search, random search, simplex reflection technique etc.

3. predicting geomechanical parameters over the procedure developed by Yamagami et al.

The performance of the developed computer program with respect to some of the well known currently available computer programs for automated slope stability computations has also been critically examined.

Studies with the failed slopes have established the capability of the developed technique in predicting the failure surface. Danger of a priori assumption regarding the shape of the slip surface without any consideration to the physics of the problem has been highlighted from the case-studies conducted.

Smooth convergence in the proposed numerical scheme has also been established from a study of the variations of the objective function and the composite function with decreasing response factor or the penalty parameter.

From the case-studies it has been observed that the proposed method works quite efficiently. Compared to other existing methods, it has greater potential in analysing complex slope stability problems as it has a greater flexibility in

the form of inserting, deleting and modifying a constraint or a number of constraints as well as modifying the objective function without disturbing the basic flow chart of the scheme.

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Dedicated to

My Father

. . . . I clung to my notion, ill-defined though it was, that a serious study of any important body of human knowledge, or theory, or belief, if undertaken with a critical but not a cruel mind, would in the end yield some secret, some valuable permanent insight, into the nature of life and the true end of man. [Robertson Davies, Fifth Business, Macmillan of Canada, Toronto, 1970][1]

1 By courtesy of Siddall, James N (1982) "Optimal Engineering Design-Principles and Applications", Marcel Dekker, Inc. New York and Basel.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATIONS

b	width of slice in Spencer's Method of analysis
c	cohesion, in general
c_e	c'/F = mobilized cohesion i.e., limit equilibrium parameter in Janbu's method of analysis
c_h	undrained strength in the horizontal direction
c_i	undrained strength of the soil when major principal stress is inclined at an angle i_v with the horizontal
c_o	zero depth intercept of the linear portion of the composite strength curve
$c_t(z)$	undrained strength at a depth z below the ground surface
c_u	cohesion intercept under undrained condition; s_u = undrained shear strength
c_v	undrained strength in the vertical direction
c'	cohesion intercept on the basis of effective stress: effective cohesion
c'_m	c'/F = mobilized cohesion
D	force due to pore water pressure on inter-slice boundary
$\mathbf{D}$	design or decision vector
$\mathbf{D}_k$	design or decision vector after kth cycle of minimization
d_i	ith design or decision variable
E	total horizontal interslice force, with boundary values E_a and E_b in Janbu's method of analysis
ε	tolerance in the extended penalty function formulation
ε_k	$-r_k/\delta_t$ = the tolerance in the kth r-minimization in the extended penalty function method
F	overall factor of safety
F_1, F_2	empirical coefficients signifying the excess strength over the strength predicted by linear distribution
F_{min}	minimum factor of safety
F_v	factor of safety against shear along a vertical interface

'f'	angle between the failure plane and the major principal plane
f(D)	objective function
f_{min}	minimum value of the objective function
∇_f	gradient of the function f
$g_j(D)$	inequality constraint function
H	height of the inter-slice boundary
H_t	height of the embankment
h	mean height of slice
h_t	distance between shear surface and line of thrust in Janbu's method of analysis
i_v	inclination of the major principal stress with the vertical
K	c_v/c_n = degree of anisotropy constant with depth
k_z	c_v/c_n = degree of anisotropy as a function of depth
k	coefficient for slope of inter-slice force
L	height of inter-slice force for total stress above slip surface in Spencer's method of analysis; length in Janbu's method of analysis
L'	height of inter-slice force for effective stress above slip surface
$l_j(D)$	equality constraint function
Δl	length of shear surface along the base of the slice
M	moment, in general
M_n	external stabilizing moment
n	number of slices
n_{dv}	number of design variables
n_{econ}	number of equality constraints
n_{icon}	number of inequality constaints
N	normal force at the base of slice due to total stress in Spencer's method of analysis
N'	normal force at the base of slice due to effective stress in Spencer's method of analysis
N_c	bearing capacity factor
ΔN	total normal force on base of slice in Janbu's method of analysis

P_e	external vertical force on a slice (Spencer's method)
ΔP	external vertical force on a slice (Janbu's method)
Q_e	external horizontal force on a slice (Spencer's method)
ΔQ	external horizontal force on a slice (Janbu's method)
R	radius of the slip curcle
r	penalty parameter or response factor, in general
r_k	penalty parameter or response factor in the kth cycle
r_u	pore water pressure coefficient
S	total shear strength (force) available in Spencer's method of analysis
S_m	mobilized shear force along base of a slice in Spencer's method of analysis
s_u	undrained shear strength
ΔS	mobilized shear force along base of a slice (Janbu's method)
$\mathcal{S}$	direction vector
T	force due to water pressure in tension crack in Spencer's method of analysis; vertical interslice force, with boundary values T_a and T_b (Janbu's method)
t	$\Delta T/\Delta x$ = slope of T - x curve (Janbu's method)
U	force due to water pressure on base of slice
u	pore water pressure
W	weight of slice in Spencer's method of analysis
ΔW	total weight of each slice, including surcharge and external load (Janbu's method)
x	general horizontal distance (abscissa)
x_i	x co-ordinate of the ith interslice boundary
x_L	x co-ordinate of the lower intersection point of the shear surface with the ground surface
x_c	x co-ordinate of the centre of the slip circle
x_T	x co-ordinate of the toe
x_U	x co-ordinate of the upper intersection point of the shear surface with the ground surface
Δx	width of slice in Janbu's method of analysis
y	ordinate of a point on the shear surface, in general
y_c	y co-ordinate of the centre of the slip circle

y_i	y co-ordinate of the intersection of the ith interslice boundary with the shear surface
z	depth in soil, in general
z_i	y co-ordinate of the point of intersection of the ith interslice boundary with the slope boundary (corresponding to x_i)
Z	inter-slice force
z_t	depth of tension crack
z_o	depth of zero active effective stress
Z_n	external stabilizing force in Spencer's method of analysis
Z'	force due to effective stress normal to inter-slice boundary in Spencer's method of analysis
α	slope of base of slice
α_n	horizontal seismic coefficient
α_v	vertical seismic coefficient
α'	nonlinear shear strength parameter
β	slope angle
β'	nonlinear shear strength parameter
$\beta_1, \beta_2, \beta_a$	attenuation rate
γ	bulk density
γ_w	density of water
θ	angle determining slope of inter-slice forces
δ	slope of inter-slice force
δ_t	transition parameter in Extended penalty function method
σ	total stress normal to base of slice
σ'	effective stress normal to base of slice
τ	shear stress
τ_f	shear strength
ϕ_u	angle of shearing resistance under undrained condition
ϕ'	angle of shearing resistance with respect to effective stress
ϕ'_m	mobilized angle of shearing resistance: $\tan \phi'_m = \tan \phi' / F$

$\psi(D, r_k)$	penalty function or composite function
ψ_{min}	minimum value of the penalty of composite function
ψ_1, ψ_2	rate of increase of the linear portions of the composite strength curves (c_v and c_n)
$\tilde{\lambda}^*$	approximate optimal step length
λ^*	optimal step length
λ	step length; parameter introduced by Skempton
λ_o	soil parameter determined empirically

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Stability analysis of earth slopes has always been a topic of immense interest to the geotechnical engineers as well as the geologists not only from the requirement of safety against failures which are often catastrophic in nature, but also from the points of view of economy and other aspects to be considered in designing slopes. Failures of slopes are caused by ground movements. The geologists look upon the soil movement within a slope as a natural process and study the factors governing the origin of such movements, their course and the resulting surface forms. The geotechnical engineers, on the other hand, investigate the stability of slopes on the basis of the principles of soil mechanics, develop methodologies for quantitative assessment of their safety and also recommend the controlling and corrective measures to be adopted for controlling and preventing a slope failure.

There are several types of ground movements causing failure of slopes, which may be broadly divided into five main groups: falls, topples, slides, spreads and flows. Each of the above categories are, again, so varied in real life situations and the factors involved are so difficult to take into account that the stability analysis becomes a very complex problem and may often defy any theoretical treatment. However, among all these the

slide type of failure is the most predominant particularly in man-made slopes and such a mechanism, fortunately, is amenable to theoretical analysis.

There are numerous methods currently available for slope stability analysis namely, the limit equilibrium method, the limit analysis method, the finite element method and the resistance coefficient envelope method. Among these the method which has all along attracted the attention of the researchers and almost monopolised the practising engineers' choice is the limit equilibrium method. Reservations have been raised against the limit equilibrium approach on the ground that factors such as the history of slope formation, initial state of stress are not considered in the analysis. Nevertheless, success in the usage of limit equilibrium method of analysis has been rated as commendable.

The slope stability analysis methods based on limit equilibrium principles have over the years been extensively refined by various investigators. Perhaps the most remarkable refinement has come in the form of development of methods valid for slip surfaces of arbitrary shape whereas in the earlier methods slip surfaces were assumed to be of particular geometry, e.g.,circular.

In the vast majority of the limit equilibrium methods the problem has been treated as a two-dimensional one. Only a few three-dimensional analysis methods have been proposed which, again, need to make an assumption regarding the shape of the slip

surface a priori. Although situations have been cited where three-dimensional analysis is more justified, analytical proof has been presented to establish that the limit equilibrium factor of safety corresponding to a three-dimensional analysis cannot be less than that corresponding to the two-dimensional analysis. An accurate two-dimensional limit equilibrium analysis is therefore of utmost value.

It should, however, be stressed here that the use of appropriate values of shear strength parameters is a prerequisite for any accurate slope stability analysis.

The limit equilibrium slope stability analysis is essentially a problem of optimization namely, the determination of the slip surface that yields the minimum factor of safety. However, the problem was looked upon as one of optimization rather belatedly. Prior to this, efforts were almost entirely devoted to the development of accurate and general safety factor formulations and little attention was paid to the implementation of advanced minimization techniques in locating the critical slip surface. In fact, none of the series of limit equilibrium methods proposed from time to time specifies any minimization procedure or search scheme. The use of computer programs for slope stability analysis, though started as early as 1958, most of the reported computer programs served to provide only automated versions of the existing methods of analysis originally developed for non automated calculations. The high speed computing facility was not made use of in implementing sophisticated autosearch algorithms to find the critical slip surface.

It was only in the mid-seventies that some applications of optimization or mathematical programming techniques to the slope problems appeared in the literature. These early studies, however, were based on a priori assumption that the slip surface is circular. Such assumptions greatly simplifies the computations involved but the restrictions imposed on the slip surface geometry may lead to the bypassing of the true critical slip surface and often result in overestimation of the factor of safety of the slope. Later on, computer programs for non-circular slip surfaces were also reported; but the critical reviews of many such softwares highlighted their inadequacies and limitations.

More recently, applications of mathematical programming techniques e.g., calculus of variation, linear programming, nonlinear programming and dynamic programming to the stability analysis of slopes have been reported. However, the efficiency of these techniques are known to be problem oriented and hence their applicability needs to be critically assessed by solving a variety of problems.

It is therefore evident that the development of methodologies based on accurate limit equilibrium methods valid for general slip surface and augmented by powerful minimization schemes under mathematical programming techniques for effective and efficient determination of the shape and location of the critical slip surface as well as the associated minimum factor of safety will be a useful contribution in the field of slope stability analysis.

1.2 REVIEW OF LITERATURE

1.2.1 General

Over the years works on slope stability have proceeded along many directions covering almost all aspects of the problem. As far as analysis is concerned, these directions can be listed as follows:

1. Type of treatment:
   a) Two dimensional problem
   b) Three dimensional problem
2. Approach of analysis:
   a) Limit equilibrium analysis
   b) Limit analysis
   c) Displacement-type finite element analysis
   d) Resistance coefficient envelope procedure
3. Location of the critical slip surface:
   a) Trial and error procedure
   b) Auto-search methods based on optimization techniques, e.g. linear programming, dynamic programming, nonlinear programming and calculus of variation.
4. Nonlinear strength envelope
5. Earthquake analysis
6. Analysis based on the concept of progressive failure
7. Probabilistic approaches
8. Back analysis of slope failures

Due to the vast scope of the subject concerned it is not desirable to review each aspect. The primary objective of the review of literature is to present the different methods of two dimensional analysis based on limit equilibrium principles and optimization procedures to search for the most critical slip surface in slope stability problems although some of the other aspects of the slope stability analysis are also reviewed. A critical review of the various optimization methods in general and their applications to slope stability problems in particular have been presented. Based on the experiences of various investigators, a qualitative comparison of the promising optimization methods have been drawn so as to be helpful in deciding on the search procedure for determining the critical slip surface and the corresponding factor of safety.

1.2.2 Two-Dimensional Analysis :

The numerous approaches to slope stability analysis can be categorized in two major groups namely limit (plasticity type) formulation and displacement type finite element formulation. By using the limit formulation (Chen, 1975) one can find the range of answers that can be expected from a slope stability analysis. These formulations are referred to as upper and lower bound solutions. The method of characteristics for stresses (Sokolovski, 1960; Reddy, 1980) is an example of lower bound solution. The commonly used limit equilibrium methods are an upper bound type of solution.

One can find in the literature a vast array of expressions for slope stability which vary with the shape of the failure surface, the geometry of the free body or bodies, and the force system acting. In the limit equilibrium method only the concept of statics is applied. Slope stability problem, however is in general statically indeterminate. As a result, some simplifying assumptions are to be made so that unique factor of safety can be evaluated. The differences between various limit equilibrium methods of analysis relate to the assumptions that are made in order to achieve statical determinacy and the particular conditions of equilibrium that are satisfied.

In the stability analysis of slopes and in the back analysis of slides, it is usual to assume that for practical purposes the problem can be considered as two dimensional. Most of the limit equilibrium methods are two dimensional and assume plane strain conditions. Among these, the methods which satisfy overall moment equilibrium are the Fellenius method (1936), the simplified Bishop method (1955), ϕ -circle method (Taylor, 1937, 1948) and the logarithmic spiral method (Taylor, 1937; Huang and Avery, 1976).

Several methods have been proposed which satisfy only the overall vertical and horizontal force equilibrium as well as the force equilibrium in individual slices or blocks. Included in this category are the methods proposed by Seed and Sultan (1967), Department of Navy (1971) and Mendez (1971).

The methods that satisfy both moment and force equilibrium

are the methods developed by Janbu (1954, 1957,1973), Morgenstern and Price (1965), Spencer (1967, 1973), Fredlund and Krahn (1977), Sarma (1973, 1979), Chugh (1982).

Since Taylor (1937) first published his stability charts, various charts have been successively presented by Bishop and Morgenstern (1960), Morgenstern (1963), Spencer (1967), considering the problem to be two dimensional. Application of these charts were reviewed by Hunter and Schuster (1971), Huang and Avery (1976).

Sarma (1979) proposed a new limit equilibrium method for calculating the factor of safety of a slope subjected to a horizontal pseudo-static earthquake force. An attempt has been made to bring all interslice surfaces within the soil mass into a limit state. The slices are not necessarily vertical and the critical inclinations of the slices are found as part of the solution. The analysis is most applicable to actual failed slopes (Chowdhury, 1980). However, Sarma has pointed out that because of the large number of iterations involved in finding the critical set of inclinations of the slices, this method is not suitable for the analysis of sections where large numbers of probable slip surfaces have to be analyzed unless some simplifying assumptions are made.

Spencer (1981) has reported a study along similar lines. It has been shown that when the stability of an earth slope is determined using the limit equilibrium method, a potential slip surface might be intersected by a series of critical shear

planes, as aforementioned, on which the values of the factor of safety would be less than that for the assumed slip surface. If this were true, it could be inferred that the limit equilibrium method overestimates the stability of an earth slope. The risk of such overestimation, it has been shown, could be particularly high for an arbitrarily chosen slip surface of assumed geometry (circular). On the other hand, when the slip circle is a critical one and a tension crack is assumed at its upper end (tension zone) the overestimation, if at all, would be very small.

Chugh (1986a) presented a limit equilibrium method of slice using variable factors of safety along the slip surface in a formulation similar to the Spencer's (1973). His method consists of defining a characteristic that describes the variation in the factor of safety along a slip surface and the analysis procedure seeks to determine a scalar fraction that in combination with the characteristic and the interslice force inclination satisfies the boundary conditions in a slope stability problem. For purely frictional material, it has been shown that the characteristic defined as the ratio of the induced shear stress to effective normal stress is reasonable. However, due to the additional iterations involved in evaluating the factor of safety of a shear surface the method is not likely to be suitable for use in critical surface determination.

Excellent reviews of various methods have been made by several investigators from time to time (Nonveiller, 1965;

Wright, 1969; Chen, 1975; Singh and Sharma, 1976; Fredlund and Krahn, 1977; Chowdhury, 1978; Fang, 1975; Duncan and Wright, 1980; Tavenas et al., 1980; Huang, 1983; Fredlund, 1984; Ramamurthy, 1986; Bromhead, 1986; Clayton and Milititsky, 1986; Graham, 1984; Nash, 1987; Mostyn and Small, 1987). As such, these are not reviewed here.

1.2.3 Three-Dimensional Analysis:

Although there are several methods of slices, only a few three dimensional methods have been proposed to study the end effects which occur in actual slides (Sherrard et al., 1963; Baligh and Azzouz, 1975; Hovland, 1977; Chen and Chameau, 1983; Hungr, 1987; Gens et al., 1988). Critical assessment of these methods have been presented by Fredlund (1984) and Cavounidis (1987).

Some of the three dimensional analyses of slope stability produce three dimensional factors of safety that, in some cases, are smaller than the corresponding two dimensional factors of safety. If the comparison is to be meaningful it should be between minimum factors of safety. By using simple algebra, Cavounidis (1987) demonstrated that the three dimensional minimum factor of safety is always greater than the two dimensional minimum factor of safety.

The present study is based on two-dimensional limit equilibrium methods and hence, no detailed review of the three dimensional slope stability analysis is presented here for reasons of space and brevity.

1.2.4 Analysis with Nonlinear Strength Envelope:

Most of the limit equilibrium methods use linear Mohr-Coulomb relationship in the analysis. There is considerable experimental evidence to show that the Mohr-Coulomb failure envelope exhibits significant curvature for many different types of soils, e.g., stiff clays, dense sands and compacted rock fill.

Maksimovic (1979), Charles and Soares (1984), Zhang and Chen (1987), Yudhbir et. al. (1987) have presented useful contributions in this direction. The difficulty in using nonlinear shear strength envelope arises in the assessment of the normal stress at a particular location on the slip surface. In limit equilibrium methods, the normal stress is generally computed by satisfying equilibrium equation in the vertical or near vertical direction. The shear strength parameters appear in the derived equation. If the shear strength envelope is nonlinear, an iterative approach is to be adopted to compute the corresponding normal stress and shear strength parameters.

Maksimovic (1979) described the treatment of a nonlinear failure envelope with a zero cohesion intercept. The secant angle of friction is represented as an empirical three parameter expression. The limit equilibrium method used to solve for the factor of safety uses similar statics and assumption as that of Morgenstern and Price method (1965). Fredlund (1984) has

critically discussed the method. The developed method is applicable for slip surface of arbitrary shape, nonhomogeneous cross section and nonlinear failure envelope and satisfies all three equilibrium conditions in plane.

Charles and Soares (1984) considered the effect of the curvature of the nonlinear failure envelope on slope stability. Charts based on circular arc stability analysis have been prepared, enabling a rapid assessment of the stability of slopes of simple geometry with curved failure envelope. The effects of the degree of curvature of the failure envelope, the magnitude of the pore pressure ratio within the slope and the depth of the hard stratum have been investigated.

Zhang and Chen (1987) used the upper bound method of limit analysis of perfect plasticity to stability problems of slopes with a general failure criterion and studied the effects of nonlinear failure parameters on the stability. The developed numerical procedure converts the complex system of differential equations to an initial value problem which is solved by Runge-Kutta method.

Yudhbir et al. (1987) developed an autosearch technique for locating the critical shear surface and the corresponding minimum factor of safety without any prior assumption regarding its shape for slopes in soils having nonlinear failure envelope. Penalty function method in conjunction with Janbu's generalized procedure of slices has been used in the analysis. The effects of pore water pressure parameter and the strength parameters on the slope stability have been studied and reported.

1.2.5 Nonhomogeneous and Anisotropic Undrained Strength of Soils:

In most of the references available on the stability of slopes soil has been treated as a homogeneous isotropic material with constant strength throughout the slope under consideration or in the layers into which the slope has been divided arbitrarily. Assuming the validity of $\phi = 0$ analysis, Gibson and Morgenstern (1962) and Kenney (1963) have presented rigorous solutions for the case of a linearly increasing strength with depth. But in nature most of the soils exhibit inherent and induced anisotropy. The importance of studying the effect of nonhomogeneity and anisotropy on the conventional factor of safety in the design of earth slopes and cuts need no emphasis (Wright and Duncan, 1972). The evidence reviewed by Bishop (1966) strongly supports the conclusions that undrained shear strength is not independent of the orientation of the principal stresses. As such, in the present study anisotropy due to the rotation of principal stress directions is considered. Lo (1965), Chen et al. (1975), Ranganathan and Srinivasulu (1979) and Koppula (1984b) presented very useful contribution in this direction. Most of these works assume the validity of strength anisotropy suggested by Casagrande and Carrillo (1944) and linear variation of undrained strength with depth.

Throughout the world there exist areas of surface clays in climates in which there are alternating dry and wet periods. In such climates, the surface clays are subjected to cycles of desiccation and saturation. Capillary forces during drying process subject the clay to preconsolidation pressure which produce overconsolidation to considerable depths resulting in

higher in situ undrained shear strength over this area. Based on published evidence, James et al. (1969) postulated an empirical relationship to account for increased undrained shear strength in the desiccated zone. Considering nonhomogeneous strength distribution and anisotropy as suggested by James et al. (1969) and Casagrande and Carrillo (1944) respectively, Basudhar et al. (1986) studied the stability of slopes in desiccated clays. The reported analysis is general enough to include most of the cases reported in the literature.

1.2.6 Techniques for Solving Factor of Safety Equations:

With the exception of the Ordinary method of slices, all of the limit equilibrium factor of safety equations are nonlinear. The Ordinary method factor of safety therefore does not require any iteration for its solution and is often used to obtain an estimate of the factor of safety for solving other methods.

For methods requiring solution of only one factor of safety equation (e.g., Bishop's simplified method, Janbu's simplified method) it is not necessary to have an elaborate technique to solve for the factor of safety.

Methods which satisfy complete equilibrium require solution of more than one factor of safety equation (e.g., Spencer, Morgenstern-Price, GLE). For such methods it is necessary to use an elaborate technique to solve for the factor of safety.

Spencer (1973) has suggested an iterative scheme. Morgenstern and Price (1967) has suggested a numerical method based on the Newton-Raphson technique. Fredlund (1984) has

presented reviews of a few other solution techniques such as the "best-fit regression" technique (Fredlund, 1974), the "rapid solver technique"(Fredlund, 1981), and the technique used by Maksimovic (1979).

1.2.7 Numerical Difficulties in Solving for the Factor of Safety:

Difficulties which have been reported concerning the solution of nonlinear factor of safety equations may be divided into two kinds; i) the multiplicity of solutions and ii)non-convergence of the solution.

Chugh (1981a, 1981b) reported the existence of more than one numerical solution when solving for the Spencer's factor of safety.

Some convergence difficulties have been reported in solving all the nonlinear factor of safety equations. According to Fredlund (1984), most concern has been expressed over the solution of Janbu's generalized procedure of slices. However, Janbu (1980) has made a few suggestions for obtaining a convergent solution.

Soriano (1976) cited references (Wright, 1969; Sarma, 1973) to state that elaborate iterative schemes such as the one based on Newton-Raphson approach (Morgenstern and Price, 1967; Wright, 1969; Spencer, 1967) are not stable or do not converge to a proper solution. It has been proposed by Bell (1969) that the engineer must overcome these mathematical barriers through the application of his art, ingenuity, experience and judgement. From a mathematical standpoint Soriano (1976) analysed the

nonlinear factor of safety equations to shed some light on the existence and uniqueness of solution. Iterative schemes have been proposed for different methods. It has been also reported that force equilibrium methods are more sensitive to the side force inclination than are the moment equilibrium methods.

According to Ching and Fredlund (1983) most problems associated with non-converging solutions can be attributed to one of the following three conditions:

1) the shape of the assumed slip surface may be unrealistic and produce mathematical instability. This was also pointed out earlier by Whitman and Bailey (1967),
2) high cohesion value may result in a negative normal force and produce instability, and
3) the assumption used to render the analysis determinate may impose unrealistic conditions and prevent convergence.

The suggested remedy is to (i) ensure that the shape of the slip surface is such that the active and passive earth pressure considerations with respect to direction are not violated at the scarp and toe of the slope, respectively, (ii) tension crack is assumed in the analysis and (iii) that assumption regarding the interslice force function should be somewhat consistent with the stresses in the soil mass.

1.2.8 Nature of Slip Surface and Its Effects:

The purpose of stability analysis is to determine the factor of safety of a potential slip surface. The shape of the potential slip surface may be quite irregular depending on the

homogeneity of the slope material. This is particularly true in natural slopes where joints and fractures dictate the locations of the failure surface. If the material is homogeneous and a large circle can be formed, most critical shear surfaces is likely to be cylindrical. If a large circle cannot be developed, such as in the case of an infinite slope with depth much smaller than length, most potential failure surface may be a plane parallel to the slope. If some planes of weakness exist, most critical slip surface will probably be a series of planes passing through the weak strata. In some cases, a combination of plane, cylindrical, and other irregular failure surfaces may also exist (Huang, 1983).

In design, the shape of the unknown slip surface is generally assumed while the location is determined by some trial and error procedure. In most of the analyses the slip surface has been assumed to be of particular geometry, for example, straight line passing through the toe (Culmann, 1866), a circular arc (Fellenius, 1936; Taylor, 1937; Bishop, 1955; Spencer, 1967), a log spiral arc (Frohlich, 1953; Spencer, 1969; Huang and Avery, 1976) an arc of a cycloid (Ellis, 1973). The assumption regarding the shape of the slip surface greatly simplifies the computations involved but the restrictions imposed on the slip surface geometry may lead to the by-passing of the correct critical slip surface (Talesnick and Baker, 1984). Studies in this regard have shown that the smallest value of the factor of safety obtained from the analysis of circular slip surface is

higher than the correct minimum value and the result thus over-estimates the actual safety of the slope. The overestimation may be as high as 15-20 percent in some cases (Celestino and Duncan, 1981).

1.2.9 Back Analysis of Failed Slopes:

The so-called back analysis of earth and rock slopes is the procedure which, instead of conducting tests on rock or soil samples, determines the geomechanical parameters for the materials involved by a reverse calculation using the information regarding the failure surface observed (measured) in the field as input data, so that the parameters obtained are much more pertinent to the real behaviour of the in situ rock or soil. To recommend remedial measures, the forward or conventional stability analysis can be carried out by using the values of the parameters provided by the back analysis. Singh and Ramasamy(1979), Nguyen (1984a,b), Yamagami et al. (1987), Frizzell and Watts (1988), Sauer and Fredlund (1988), have made valuable contributions in this area.

1.2.10 Computer Programs for Slope Stability Analysis:

In all the limit equilibrium methods a number of trial surface are analysed to obtain the critical surface and the corresponding minimum factor of safety, necessitating the use of sophisticated high speed digital computer. Over the years many computer programs have been developed to analyse slope stability problems. An attempt to overview its historical development is made as follows:

Little and Price (1958) were perhaps the first to present a computerized approach of slope stability analysis. The analysis is based on Bishop's adaptation of Swedish method. The authors found the factor of safety for a particular trial circle using Newton-Raphson method. The developed program specifies a set of circles for which the factor of safety are required. Results obtained have shown the presence of local minima in the value of factor of safety which differs from the absolute minimum.

Horn (1960) developed a computer program that can solve a great variety of slope stability problems. Given the property of slopes the computer will automatically search out the minimum factor of safety, solving the whole problem. But the search technique that has been used is highly elementary. It is reported that the technique rarely gets to the bottom of the valley and, as such ,it cannot find the minimum effectively.

Bailey and christian (1969) developed ICES LEASE-I computer program using Fellenius and simplified Bishop methods and assuming circular failure surface. In this program a range of radius for a grid of centres are specified and a search routine is incorporated.

Wright (1969) developed a computer program JANBU using Janbu's generalized procedure of slices and circular failure surface. Grid search technique was used in the program. The program was distributed by GESA (Geotechnical Engineering Software Activity).

Drnevich (1972) developed a computer program based on the

Department of Navy's method (1971) with the exception that the active and passive forces were calculated for more general conditions. He considered the middle block as an active or passive wedge depending on the slope of the failure surface beneath it.

Lefebvre (1971) developed his program STABR which is extensively used for stability analysis of earth slopes. It employs simplified Bishop's method of slices to estimate the safety factor associated with circular slip surfaces. The program can examine particular trial surfaces or it can automatically search for the critical slip circle by means of a built-in pattern search routine.

Morgenstern and Price's method was used in a computer program called MALE (Schiffman, 1972). The program can be used for analysing the stability of earth and rock slopes when failure surface is composed of a series of straight line segments. The slope may be zoned or layered with different materials in an arbitrary manner. It is possible to define an internal piezometric level as well as upstream and downstream external water surfaces. Pore water pressure calculations are based upon piezometric level, the external water surface or pore pressure ratio (r_u).

Adopting Spencer's method a general computer program, SSTAB1 was developed by Wright (1974). The program can be used for both circular and noncircular failure surfaces. For circular failure surfaces an automatic search (lattice method) is available for

locating critical circle corresponding to a minimum factor of safety. The undrained shear strength and the pore pressure may be varied within the slope and interpolations are made to determine their values along the failure surface. The program was modified by the Bureau of Reclamation and renamed SSTAB2 (Chugh, 1981c). Major changes include the use of a new equation-solver to facilitate convergence and the calculation of yield acceleration of a sliding mass.

Jubenville (1975) developed the program BISHOP using simplified Bishop's, method and circular failure surface. It uses regular x,y and radius grid.

Hasselquist and Schiffman (1975) presented their SNOB computer program using Fellenius method and circular failure surface. Here the search is limited to only 20 horizontal and 10 vertical increments.

Features of two dimensional slope stability analysis programs like MALE, SNOB, BISHOP, JANBU, SSTAB1, have been excellently reviewed by Fredlund (1984).

Fredlund (1974, 1975) developed his SLOPE II program containing a total of six methods of analyses all of which can be applied to either circular or noncircular (composite) failure surfaces. The methods of analysis include Fellenius method, Simplified Bishop, Spencer, Corps of Engineers, Janbu's simplified, Janbu's generalized and Morgenstern and Price method. The general features of the program have also been presented by Fredlund (1984) in his review paper.

Janbu's method was also incorporated in a computer program developed by the Purdue University for the Indiana Sate Highway Commission. The original program STABL was written by Siegel (1975) and was later revised by Boutrup (1977) and renamed STABL2. The program can generate circular failure surface, surfaces of sliding block character and more general irregular surfaces of random shapes. It can handle heterogeneous soil systems, anisotropic soil strength properties, excess pore water pressure due to shear, static ground water and surface water, pseudo-static earthquake loading and surcharge boundary loading. The original program was based on Janbu's method but the simplified Bishop's method was later added for circular failure surfaces. STABL2 was further revised and STABL4 and STABL5/PC STABL5 versions were brought out.

Huang (1981) developed the REAME (Rotational Analysis of Multilayered Embankments) computer program to determine the factor of safety of a slope based on cylindrical surface and using simplified Bishop's method. The program has many features which are similar to the ICES-LEASE computer program. The program is available in both FORTRAN and BASIC languages. The method for specifying grid and the procedure for search are the same as those in ICES-LEASE program. The search procedure has been described in detail by Huang (1983).

Huang (1983) also presented his SWASE (Sliding Wedge Analysis of Sidehill Embankments) computer program to determine the factor of safety of a slope when some planes of weakness

exist within the slope. These failure planes may exist at the bottom of an embankment or at any other locations. However, the maximum number of failure planes is limited to three because the program can only handle three blocks. The program has been written in FROTRAN IV and BASIC.

Clayton and Milititsky (1986) reported about a computer program for slope stability analysis using Morgenstern and Price method; however, they did not report the adopted optimization method. The program was developed by Che in 1983.

Bromhead (1986) presented computer programs for slope stability calculations using Bishop's and Janbu's methods. The program is capable of analysing a given trial surface only.

Chen and Chameau (1982) developed the LEMIX computer program for three-dimensional slope stability analysis. They used secant method(Wolfe, 1969) for solving the nonlinear equations to be satisfied for maintaining equilibrium.

A microcomputer program CLARA-3 was reported by Hungr (1987) extending the Bishop's simplified method of slope stability analysis.

Leshchinsky and Baker (1986) made a review of the few analysis procedures which are applicable to three-dimensional failure mechanisms and pointed out some serious shortcomings such as failure to degenerate to known solutions in some cases.

A computer program F3SLOP has been developed by Gens et al. (1988) that calculates the three dimensional factor of safety for a potential slide. The most shear failure surface is found using

an automatic search routine based on optimization techniques to any desired degree of accuracy. The type of failure (slope, toe or base) may be prescribed if desired. In the computations carried out grid spacing for the axes of rotation was taken as 0.02 times the height of the slope. When appropriate, such as in determining the transition points between different modes of failure, the grid spacings were further reduced.

Frizzel and Watts (1988) addresses the use of micro-computer program BACKPACK for an estimation of rock mass strength characteristics by back analysis of plane and wedge failure.

It can be observed from the review of literature presented in the previous paragraphs that most of the programs that were developed mainly pertains to the automated estimation of the factor of safety. Scanty attention was paid to the efficient location of the critical slip surface. As such, in many of the softwares developed for slope stability analysis no mention has been made about the adopted numerical procedure. Maksimovic (1988) noted that the role and significance of slip surface search algorithms has been all but ignored during the past thirty years. The above statement may appear to be a sweeping one but it does give an idea of what was the state of art of the use of the powerful optimization techniques in the slope stability analysis till recent times. It is only to be expected that the advent of high speed digital computers has generated lot of interest among the academic communities to develop computer programs for slope stability analysis using more and more refined

limit equilibrium methods in conjunction with efficient search procedure for a more realistic analysis performed in much reduced computational time. A number of such attempts are reviewed in the following section.

1.2.11 Optimization Techniques in Slope Stability Analysis :

For an efficient computation of extrema of a function it is necessary to cast the problem as mathematical programming problem and use sophisticated methods of optimization.

Although the application of these methods in structural and aerospace engineering had been made in the 60's its application in geotec cal engineering lagged by almost a decade. A brief chronological presentation of the relevant studies in the area of slope stability analysis is made as follows:

Wu and Kraft (1970) performed an optimum cost design of earth dams considering seismic safety against failure.

Krugman and Krizek (1973), using $\phi = 0$ analysis and a direct search optimization procedure, developed a series of charts for evaluating the stability of a system in which a soft layer was underlain by a firm stratum and overlain by a sloping embankment.

Chen et al.(1975) presented charts for stability of slopes in homogeneous and anisotropic soils using the upper bound technique of limit analysis. Optimization method developed by Powell (1964) was used to minimize the function.

Basudhar (1976), Basudhar et al.(1979) used nonlinear programming techniques to slope stability problems. In their studies comparisons of different unconstrained minimization

techniques like Powell's conjugate direction, Fletcher-Reeve's conjugate gradient and Davidon-Fletcher-Powell variable metric methods were reported. However, the studies were based on circular slip surfaces only.

Narayan and Ramamurthy (1980) developed optimization based computer algorithm for slip circle analysis.

When the slip surface is assumed to be circular, the earliest practice of locating the critical shear surface and the corresponding minimum factor of safety was to select a grid of centers and vary the radius at each centre, providing a coverage of all possible conditions. When the slip surface takes a composite shape, it is still possible to use grid search technique (Fredlund, 1981). This technique can also be used to generate slip surfaces of a wedge or sliding block configuration. However, the technique may lead to high C.P.U. time and the cost involved may become prohibitive.

For circular failure surface other efficient methods like conjugate direction, conjugate gradient and variable metric method have also been used by various investigators to reduce the computational time; such references have already been cited. Fredlund (1984) reports that some search routines start at a point and seek the critical centre by moving in a zigzag manner(Wright, 1974) while others use an initially coarse grid of centres which rapidly converges to the critical centre Fredlund 1981).

Celestino and Duncan(1981) highlighted the inadequacy of many

computer programs using the noncircular slip surfaces in regard to the auto-search of the critical slip surface. They proposed a technique involving the minimization of multivariable functions. The method can be used to locate critical slip surface of any shape using a limited number of points (e.g., 4 to 6 points). One point is shifted until its optimum position is located and the same procedure is continued for other points. Two schemes, known as i) the alternating variable method and ii) the quadratic fit have been used for performing the minimization. To illustrate its use they applied the technique in analysing the Birch dam and a few model test cases. For the Birch dam the difference in the factor of safety using circular and non-circular surfaces were not significant. However, the two surfaces were significantly different near the toe. The other study illustrated that the search procedure is capable of locating slip surfaces of unusual shape. However, no comparison of the relative efficiency with respect to other optimization methods were made in this study. The technique developed was not tested for cases having weak shear zones in the foundation. In the analysis no check was made on the acceptability of the critical shear surface with respect to the thrust line and normal stress. As such, its applicability to all types of problems is yet to be established.

Siegel et al. (1981) proposed a random surface generator to locate a critical slip surface of any shape. The procedure uses a random number generator and imposes restriction on the shape

and position of the slip surface to avoid nonsensical results. The slip surface has been approximated by a series of straight lines with the following constraints: i) the starting position on the boundary must be within a logical range, ii) the shape of slip surface must be kinematically acceptable, iii) the base of the slip surface must lie above a lower boundary (i.e. a strong layer) and iv) the terminating position must lie within a logical range on the boundary. Subsequently, Lovell et al. (1984) developed STABL4 and Carpenter (1986) finally came out with the version STABL5/PC STABL5. In the STABL5/PC STABL5 versions for slope stability analysis simplified Janbu, simplified Bishop and Spencer's method can be used.

Dickin (1985) developed a computer program using simplified Bishop method and recommended a search procedure wherein the successive centres of the slip circles are selected down the steepest slope of the three dimensional elliptic valley-shaped surface defined by the co-ordinates of the centre of the circle. He developed the procedure based on his experience with several routines involving both grid search and quadratic fitting techniques. No comparative studies with other optimization algorithms have been made.

Bhowmik (1984), Bhowmik and Basudhar (1989) formulated the problem of finding the critical slip surface as a nonlinear programming problem using the Sequential Unconstrained Minimization Technique (SUMT). They used Janbu's (1973) generalized procedure of slices to find the factor of safety.

Powell's method of conjugate direction and Davidon-Fletcher-Powell variable metric method for multivariable unconstrained minimization in conjunction with quadratic fit and cubic fit respectively for one dimensional minimization were used. They highlighted the effectiveness of the technique for slopes in homogeneous soils only. However, the generalized scheme had a good potential to be extended to nonhomogeneous cases. Bhowmik (1984) also used Spencer's (1973) method for estimating the factor of safety; penalty function method was used to find the critical lip surface of homogeneous slopes.

The sequential unconstrained minimization technique of searching for the critical noncircular slip surface as suggested by Bhowmik (1984) was extended to zoned dams with thin shear zones in the foundation by Satyam Babu (1986), Basudhar et al. (1988); Basudhar and Yudhbir (1989). A computer program SUMSTAB was developed by Satyam Babu (1986); the strength and limitations of the same was further studied by Dhawan (1986). Yudhbir et al. (1987) extended the above technique to incorporate non-linear strength envelope in the analysis.

Nguyen (1985) presented a study involving application of mathematical optimization to classical problem of slope stability. He recommends the use of Simplex reflection technique for searching the minimum. Application of this method has been critically discussed by De Natale (1987), Hachich et al. (1987). Hachich et al. recommends the use of a similar algorithm called simplex plus in which a single initial best guess is required and

which converges to a minimum without user's intervention as safely as and more efficiently than the 'star' method, while retaining the simplicity of the simplex method. In his closure on discussion by De Natale and Hachich et al., Nguyen points out:

"First, the writer is in agreement with De Natale in that the number of function evaluations required to locate a function's minimum is dependent on the closeness of the initial user supplied estimate. In fact, this is the principle behind all optimization algorithms, and to date, there are very few techniques that can claim the capability to differentiate between the convergence to the true or global optimum and those to local optima without users' input or a dramatic rise in the number of iterations. It was found from the simplex runs performed in the study that convergence can be achieved with the number of factor of safety calculations ranging from 8 to around 25 depending on the proximity of the initial estimate to the optimal region.... Hachich et al., are more inclined to advocate for a reduction in the number of initial best guesses. In the writer's opinion, geotechnical practice and geotechnical computer programs will continue to rely on the input of expert judgement, at least until artificial intelligence techniques are ready for reliable, economic and popular implementation in engineering software".

Arai and Tagyo (1985a) proposed a numerical procedure to determine the location of the critical noncircular slip surface giving the minimum factor of safety of a slope in isotropic soil using the simplified Janbu's method of stability analysis in conjunction with mathematical programming technique. The objective function and the decision variables were the factor of safety and the interslice elevations of potential slip surface respectively. Conjugate gradient technique proposed by Fletcher and Reeves (1964) was used for unconstrained minimization.

The same technique was used by Arai and Nakagawa (1986) to study the influence of strength anisotropy on the search for critical noncircular slip surface.

Greco (1987) studied the influence of strength anisotropy on the search for critical noncircular slip surface. In the search procedure the slip surface is represented by a broken line of n vertices whose x and y co-ordinates are the variables in a minimization problem where the objective function is the safety factor. In order to guarantee that the slip surface is geometrically feasible, some constraints have to be imposed on the design variables. He used the pattern search method (Hooke and Jeeves, 1961) for minimization.

Yamagami and Ueta (1987) used optimization technique for back analysis of failed slope. They used the Ordinary method of slices in conjunction with the sequential unconstrained minimization method and flexible tolerance method. The Nelder and Mead's simplex algorithm(Nelder and Mead, 1965) is used to implement unconstrained searches in both the methods.

Arai and Nakagawa (1988) developed a numerical procedure for the slope stability analysis by combining the limit equilibrium method with the lower-bound approach in the limit analysis method. The problem is formulated as an optimization problem for finding a stress field which minimizes the safety factor within the limitations of satisfying the lower-bound conditions. The formulated problem is solved numerically by using the sequential unconstrained minimization technique proposed by Fiacco and McCormick (1968). They introduced a modified objective function wherein a positive constant which adjusts the order of magnitude both of the original objective function and the penalty term. This technique was earlier discussed by Arai and Tagyo (1985b).

the same whereas the DFP and BFGS presented results which are different from the minimum value. In this study no attempt were made to assess the correctness of the methodology developed by comparing it with other known solutions.

De Natale and Abifadel (1988) used Powell's conjugate direction algorithm in order to improve the efficiency of computer-based stability analyses. They observed the technique to be accurate, efficient and highly reliable in analysing slope stability problems. However, in their study they used circular slip surface only. Such studies have been carried out by other investigators in the early seventies. Pattern search technique like Hooke and Jeeves method was used by Krugman and Krizek (1973); Powell's conjugate direction method was used by Basudhar (1976). Maksimovic (1988) developed general slope stability software package for microcomputers to allow any one of the four search techniques in conjunction with an existing limit-equilibrium based stability analysis; the methods are: grid search method, pattern search technique, simplex method and conjugate direction method. Relative merits of the four slip surface search techniques used was earlier established by Gillett (1987). Maksimovic (1988) noted that the simplex and conjugate direction routines are particularly effective when full three dimensional search is required. However, the study pertains to the circular slip surface only.

Yamagami and Ueta (1988b) combined the stresses obtained by finite element analysis with dynamic programming to search for

the critical slip line and associated minimum factor of safety. The greatest advantage of the proposed method is that it can incorporate the effects of the constitutive relations and/or initial stresses in the ground. No assumptions have been made on the shapes of slip lines except that they consist of a chain of line segments. The shape of the critical slip line is thus given as part of the solution. The suggested method is a very promising tool in analysing stability of earthen structures; however, further research are needed in the area for final conclusions.

De Natale and Crennan (1989) have pointed out that variations of the traditional grid-search strategy are built into many modern slope stability programs to identify the critical slip surface and the associated minimum factor of safety. In order to improve the efficiency of computer-aided stability analyses they integrated the simplex procedure of Nelder and Mead (1965) in the STABL5 code of Carpenter (1986). They have observed that the new simplex-based stability program performs effectively and efficiently for a wide range of realistic slope problems.

The movement of the simplex is achieved by using three operations known as reflection, contraction and expansion. If one uses the reflection process for finding the minimum one may encounter certain difficulties like the processes not leading to new simplex or leading to a cyclic process. For better accuracy, the simplex has to be contracted or reduced in size (Rao, 1984).

Apart from the use of nonlinear programming, adoption of calculus of variation in locating the critical slip surface has been advocated by various investigators. Martins (1982), Fredlund (1984), Ramamurthy (1986) have presented excellent reviews of the various studies available in the literature. In this approach, slope stability problems are posed as a minimization problem in the calculus of variation wherein the stress distribution function is determined which minimizes the factor of safety satisfying all equilibrium and boundary conditions and, also, Mohr-Coulomb criterion is not violated anywhere along the slip surface. Minimization of the functional so developed can be obtained by using either indirect or direct method in the calculus of variation. The indirect method was described in detail by Narayan et al. (1976) and direct method by Ramamurthy et al. (1977). Other important contributions in the field of application of this method to slope stability problems have been made by Baker and Garber (1977), Castillo and Revilla (1977). Although a number of solutions have been published to date, the method still requires further research. Fredlund (1984) observes that the variational calculus method still requires a trial and error type of solution for the minimum factor of safety.

Castillo and Revilla (1977) gave solution for a slope in cohesive soils ($\phi = 0$) and used Janbu's (1954) equation as a basis for defining the functional. Comparison with results obtained on the basis of conventional limit equilibrium methods

showed that the factor of safety values are smaller when variational calculus is used. The critical slip surfaces are markedly non-circular in shape and do not pass through the toe as would have been expected from conventional solutions for some range of slope angles.

Baker and Garber (1977) presented a variational calculus method for a general c-ϕ soil. The obtained critical slip surface is shown to have the form of a log-spiral, based on the solution of Euler's equation.

There is lot of controversy about the variational formulation of slope stability analysis. Morgenstern (1977) commented on some difficulties in the application of variational calculus to slope stability problems and cautioned against ignoring these difficulties. He observed that many studies (e.g., Baker and Garber, 1977; Ramamurthy et al., 1977) appeared to ignore shear forces between slices as well as the requirement of moment equilibrium. According to Martins (1982) variational techniques cannot be used for heterogeneous media; however, Baker and Garber (1978) claimed that variational calculus can be used successfully for a general nonhomogeneous , non-isotropic soil with arbitrary distribution of pore pressure and external loads. In his review paper Fredlund (1984) writes:

"In 1980, Luceno and Castillo expressed reservations about the application of the variational calculus methods to slope stability analysis. Their concern was that 'sufficient conditions' may not be satisfied. As a result the computed factor of safety may be neither a relative nor an absolute minimum value. The method proposed by Chen and Snitbhan (1975) and Baker and Garber (1977, 1978) is based on global equilibrium

(i.e., the interslice forces are not considered) and the solution is said to be unbounded and can go to zero. The method proposed by Narayan, Bhatkar and Ramamurthy (1976, 1978) is based on local equilibrium (i.e., an assumption is made regarding the interslice forces) and does not satisfy enough conditions for the functional to be minimum. Narayan and Ramamurthy (1980) disagreed that none of the variational calculus methods satisfy the conditions for functional to be a minimum. They state that for a functional to be a minimum it is not enough if only Euler's condition is satisfied. It has to further satisfy necessary and sufficient conditions to prove the existence of such a minimal is a global minimal. They state that the Legendre's necessary conditions and the Weiesstrass' sufficient condition for a global minimum must also be satisfied".

Ramamurthy in 1986 has made the following observations :

"The variational method suggests that any assumption of internal stress distribution within the potential sliding mass may lead to ill-conditioned functions resulting in misinterpretation of numerical results. The assumed function for internal stress distribution must satisfy all equilibrium and boundary conditions and also the conditions for minimum factor of safety and critical slip surface. The normal stress distribution along the potential sliding surface is related to the critical slip surface......Though the existing methods of analysis yield results which are meaningful by assuming some normal stress distribution, the results themselves do not necessarily refer to the absolute minimum."

The application of dynamic programming to slope stability problem has been advocated by Baker (1980). He has developed an effective procedure based on this principle by which the minimal factor of safety and the corresponding surface are determined simultaneously. The developed computer program SSDP (Slope Stability by Dynamic Programming) couples the minimization scheme with Spencer's method of slope stability analysis. It may be applied to slopes of any geometry, layering, pore pressure and external load distributions. No restriction is placed on the shape of the slip surfaces, and the analysis satisfies all

equilibrium equations.

Baker (1980) pointed out that the dynamic programming approach, by its very nature, yields the absolute minimum only, disregarding any local minima that may exist. He noted that this property of the approach, which is usually considered one of the most attractive features, is actually a disadvantage for the application to the slope stability problems. He argues with an example where the functional and therefore the auxiliary function possesses two local minima. Application of dynamic programming will lead to the lower minimum of the auxiliary functional. This minimum will correspond to one of the minima of the main functional but not necessarily the smallest one, and the slip surface which corresponds to the second minimum of the main functional is not known. One way to overcome this difficulty is to divide the search region into a number of subregions, performing the dynamic programming minimization separately in each subregion in the hope that each of the minima of the main functional is contained in the subregion. In the few cases where such a procedure was utilized in slope stability problems, no evidence of multiple minima was found. This was expressed by the fact that only in one of the subregions a slip surface yielded the smallest value of the functional. Consequently Baker opined that this limitation of the dynamic programming approach was not considered prohibitive.

Talesnick and Baker (1984) used a computer program SSOPT (Baker, 1979) based on Spencer's method and dynamic programming

in reanalysing four documented test embankment failures. The critical slip surfaces generated by SSOPT show good likeness to the observed failure circles. In spite of its having very good potential not much use of the technique has been reported by other investigators.

Munro (1982) and Martins(1982) recommended the use of linear programming in locating the critical slip surface in slope stability problems. However, the same also has not found wide application.

The areas are still in the research stage and need further studies.

1.2.12 A Short Critical Review of the Optimization Procedures :

From the review presented in the earlier sections it is evident that the earliest attempts on the computer-aided slope stability analysis were mainly directed in evaluating the factor of safety satisfying all the conditions of statical equilibrium. Before the advent of high speed digital computers the adopted solution procedures in analysing stability problems were either mostly graphical or the analysis techniques so elementary that enabled to hand compute the function value. However, the appearance of the high speed digital computers on the scene revolutionized the approach and led the researchers to think in terms of automated design and analysis using more and more sophisticated methods. Perhaps, due to the lack of appreciation and exposure of the geotechnical engineering community about the availability of different efficient optimization algorithms that

had been developed by the mathematicians over the years, in the earlier attempts to find the critical slip circle only elementary techniques of optimization were used. As the researchers in the field of geotechnical engineering started getting acquainted with the more refined methods of optimization, the use of various algorithms in the automated slope stability analysis were reported. However, except in a few studies, the knowledge and experience gained by various researchers were not made use of. Even in the seventies when very well known and efficient search techniques and their critical assessment were available, geotechnical research community went on using algorithms which are in general less efficient. The more refined direct search techniques like Powell's conjugate direction, Fletcher-Reeves conjugate gradient and Davidon-Fletcher-Powell variable metric methods were used much later and that too in the simple case of locating the critical slip circle only.

After 1975 determination of the general critical slip surface by using variational calculus generated lot of interest amongst the geotechnical research community. Many papers have been published on the subject matter. However, as has already been mentioned,the method is quite controversial and its application in nonhomogeneous slopes and zoned dams has not been reported. Though some claims have been made about its versatility in tackling slope stability problems in general nonhomogeneous, nonisotropic soils with arbitrary distribution of pore pressure and external loads, its successful application in

general class of problems is yet to be established. Serious doubts have been expressed by various investigators in this respect and about the nature of the solution. It should be borne in mind that if the expressions for the objective function and the constraints are fairly simple in terms of design variables, classical methods of optimization can be used to solve the problem. But if the optimization problem involves objective function and/or constraints which are not explicit functions of the design variables or which are too complicated, one cannot solve it by using the classical analytical methods. In such cases, one has to resort to the numerical methods of optimization for solving the problem.

Pochtman and Kolesnichenko (1972) are perhaps the first to successfully apply dynamic programming technique developed by Bellman (1957) in investigating the stability problem (bearing capacity problem) in geotechnical engineering. However, only in 1980 the technique was used in analysing slope stability problem.

Dynamic programming, when applicable, represents or decomposes a multistage decision problem as a sequence of single stage decision problems which are solved successively. Dynamic programming makes use of the concept of suboptimization and the principle of optimality in solving the problem. Multistage decision problems can also be solved by the direct application of the classical optimization techniques. However, this requires the number of design variables to be small, the function involved to be continuous and continuously differentiable and the optimum

point not to lie at the boundary points. Further, the problem will have to be relatively simple so that the sequence of resulting equations can be solved either analytically or numerically. The nonlinear programming techniques can be used to solve slightly more complicated multistage decision problem. But their application also requires the variables to be continuous and a prior knowledge about the region of the global minimum or maximum. Dynamic programming, on the other hand, can deal with discrete variables, nonconvex, noncontinuous and nondifferentiable functions. The dynamic programming technique, however, suffers from a major drawback, known as the problem of dimensionality or the curse of dimensionality. This problem causes not only an increase in the computational time but also requires a large computer memory. This presents a serious obstacle in solving medium and large size dynamic programming problems. Perhaps due to this reason the method has not found wide application in slope stability problems.

The simplest class of mathematical programming problems is the linear programming problems in which the constraints and objective function are all linear functions of decision variables. The earliest work of applying linear programming in geotechnical engineering was by Lysmer (1970); however, he applied it to stability problems like bearing capacity and earth pressure problems. Only in the early 80's its application in slope stability analysis has been advocated and reported in a few papers. Because the stability analysis of slopes is nonlinear in

nature, the linear programming technique has not been widely adopted by researchers in these areas in spite of the fact that for the cases where the objective function and the constraints are nonlinear the problem can be solved by linearizing the nonlinear equations. This is perhaps due to the fact that the constraints or the objective function have to be linearized repeatedly and solved by linear programming to get a convergent solution due to which the method becomes time consuming and less attractive.

Apart from linear programming and dynamic programming, nonlinear programming has attracted the attention of the researchers in the area of computer aided slope stability analysis. Intuitively or otherwise this approach has most widely been used by the investigators since early 50's. Optimization techniques that are applicable for the solution of constrained problems are presented briefly as these methods are available in any standard text book on mathematical optimization (Fox, 1971; Rao,1984).

The various techniques available for the solution of constrained nonlinear programming problems can be broadly classified into two groups, namely, the direct methods and the indirect methods. In the direct methods the constraints are handled in an explicit manner whereas in most of the indirect methods, the constrained problem is solved as a sequence of unconstrained minimization problems. Direct methods can be further categorized in three groups, namely, heuristic search

methods, constraint approximation methods (the cutting plane method) and methods of feasible directions. Heuristic search methods are mostly intuitive and do not have much theoretical support. In the constraint approximation method, the nonlinear objective function and the constraints are linearized about some point and the approximating linear programming problem is solved by using linear programming techniques. Since in the cutting plane method each of the approximating problems is a linear programming problem, it can be solved quite efficiently. However, the method is applicable only to convex programming problems. Since all the optima of the approximating linear programming problems lie in the infeasible region, specification of a tolerance limit needed for termination might pose problem in many cases.

The methods of feasible directions are those which produce an improving succession of feasible design vectors by moving in succession of usable feasible directions. Each iteration consists of two important steps. The first step consists of finding a usable feasible direction at a specified point and the second step consists of determining a proper step length along the usable feasible direction found in the first step. The Zoutendijk's method of feasible directions and Rosen's gradient projection method can be considered as particular cases of the general methods of feasible directions. In Zoutendijk's method, one has to solve an optimization (linear programming) problem, in each iteration to find a usable feasible direction. Although

this procedure gives the 'best' direction in some sense, it involves much computational work. The gradient projection method of Rosen is an alternative to Zoutendijk method. It does not require the solution of an auxiliary linear optimization problem to find the usable feasible direction. It uses the projection of the negative of the objective function gradient onto the constraints that are currently active. Although the method was originally developed by Rosen for a general nonlinear programming problem, its effectiveness is mainly confined to problems in which the constraints are all linear.

Indirect methods can be classified into two groups, namely, transformation techniques and penalty function methods. If the constraints are explicit functions of the design variables and have some simple forms, it may be possible to make a transformation of the independent variables such that the constraints are automatically satisfied. By such change of variables it is possible to convert a constrained optimization problem into an unconstrained one. However, if it is not possible to eliminate all the constraints by making change of variables it may be better not to use the transformation at all. The partial transformation may, sometimes, produce a distorted objective function which might be more difficult to minimize than the original function. If an optimization problem has m equality constraints, all of them may not be active at the optimum point. If it is known, in advance, which constraints are going to be active at the optimum point, those constraint equations can be

used to eliminate the variables from the problem. Thus if r ($<n$) specific constraints are known to be active at the optimum point, r variables can be eliminated from the problem and a new problem is obtained involving only $(n-r)$ variables and $(m-r)$ constraints. This problem, in general, is much easier to solve compared to the original problem. The major drawback of this elimination of variable technique is that it will be very difficult to know, before hand, which of the constraints are going to be active at the optimum point. It has been reported that even for a very small problem with small number of design variables and constraints the number of new problems to be solved can be very large. Hence even for linear programming problem this approach is not feasible. The simplex method is much more efficient than this technique because the former moves from one basic feasible solution to an improved neighbouring basic feasible solution in a systematic manner.

There are a number of direct approaches to constrained minimization in which the constraints are considered as limiting hypersurfaces and minimization is carried out directly in their presence. In contrast, in the penalty function method the constraints are blended into a composite function and sequential unconstrained minimization of the function is carried out. In some cases, the penalty function method is the most efficient means of solving a particular problem. However, in a number of instances it is preferred mainly because of its convenience and simplicity rather than its efficiency.

The penalty function formulation for inequality constrained problems can be divided into two categories, namely, the interior and the exterior method. In the interior method the sequence of unconstrained minima of the composite function lie in the feasible region and converge to the solution as the penalty parameter is decreased. In the exterior method, the sequence of unconstrained minima of the composite function all lie in the infeasible region and converge to the desired solution from the outside as the penalty parameter is increased.

From the design point of view the interior penalty function method is preferable to the exterior penalty function method (Fox, 1971).

In many complex physical systems, it is extremely difficult to obtain an initial feasible design vector and, as such, the interior penalty function method cannot be used. In such cases the problem has to be solved by using either the exterior penalty function method or obtaining an initial feasible design vector by interior penalty function technique itself following a procedure outlined by Fox (1971) and then using the interior penalty function method. Even when the interior penalty function method is used, owing to the "long step" nature of unconstrained optimization algorithm, the path may be diverted into infeasible regions. In such cases the function is generally set to an arbitrary high value and the minimization procedure is left to correct the situation on its own. Sometimes this approach presents numerical difficulties. This can be avoided by using

the Extended penalty function method enunciated by Kavlie (1971). The method readily accepts infeasible design points and needs no special treatment.

So far a variety of methods for solving optimization problems has been discussed. No attempt has been made to furnish a complete presentation of such techniques. Extensive bibliography on optimization methods are now available in many standard textbooks on mathematical optimization. While discussing the different optimization techniques the focus for discussion falls not only on the different methods available but also on the considerations in picking an approach t solve a problem. A number of factors enter into this decision. Fox (1971) has listed these and discussed them. In hi opinion,

"In general, the unconstrained formulations are the easiest to program and debug. This is particularly true if a minimization algorithm is available in pre-programmed form. Except for the need to have a feasible starting point, the interior penalty function methods are the least complicated. Other penalty function methods follow closely in this respect. Even if the minimization algorithm has to be programmed for the application, the penalty function methods are usually best from the point of view of programming cost".

The running-time cost of optimization programs has received a major share of the attention given to these techniques. In an abstract way, this is indeed the central question because, as mentioned earlier, all optimization problems can be solved by analysing all possible designs and choosing the best. What we seek in developing an optimization program is a practical alternative to the exhaustive search approach. However, most of

the methods that have been discussed, save the most primitive, are such enormous improvements over analysing all possible designs that the point is somewhat academic. The preoccupation with efficiency appears equally academic to many of the people involved in the practical applications of optimization, since any program that solves the problem is usually sufficient, irrespective of running efficiency. In this connection Fox (1971) again notes :

> "Nevertheless, efficiency is important in many cases. For design problems with explicit analysis and a moderate number of variables, the penalty function methods seem best. Of these, the interior penalty function is usually the most effective. It is particularly useful if an approximate analysis is being used. Of course, for linear problems the simplex method is without equal, and for problems with linear constraints and a nonlinear objective function, the gradient projection method is best.
>
> For problems with difficult analysis, much depends on whether the derivatives of constraint functions can be easily calculated; if they can, then some feasible direction method is probably the most efficient. If the only access to derivatives is finite difference, then perhaps penalty functions used with nongradient or finite difference minimizers are the best hope.
>
> ... For the general nonlinear programming problem the interior penalty function appears to be the most reliable unconstrained method, because it provides numerous checks against false convergence and because the sequence of designs is feasible. The method of feasible directions with an optimum solution to the direction-finding problem seems to be the most reliable of the direct methods.
>
> Flexibility of an optimum design program, is of course a desirable feature. By flexibility (as distinguished from generality), we mean the program's capacity to be used in a variety of ways on a particular problem or modifications of a problem ...When flexibility is important, the penalty function methods with nongradient minimizers generally excel because we can easily add or delete constraints, modify the objective function or constraints and interchange the roles of various parts of the problem. This sort of manipulation is more difficult with the direct methods or gradient minimized penalty functions and, of course, any method based on special properties

of the problem (Simplex, gradient projection, etc.) restricts the range of changes that can be made in it".

However, it should be noted that for infeasible initial design points extended penalty function method as proposed by Kavlie (1971) will probably be the best.

If a nonlinear programming problem is to be solved by penalty function methods, a suitable unconstrained optimization method has to be chosen to solve the sequence of unconstrained minimization problems. Thus the efficiency of a penalty function method depends on the efficiency of the unconstrained minimization method used.

A wide variety of unconstrained minimization methods are available. Based on the experiences of various investigators some guidelines are available which may be of great help while choosing the best method for a particular problem and hence the same are presented in the following paragraphs. This aspect of optimization (picking a method) is largely an art. However, the better ones knowledge of the theory the better he is at the art. The discussion will be limited only to the three methods: Powell's (POW), conjugate gradient (CG), and variable metric method (VM). According to Fox (1971) these are the only three methods which should be considered except in special situation. For most of the real life practical problems very often it is not possible to get the derivatives analytically. For problems where exclusive expression for gradient are not available Fox (1971) recommends that

"CG (with finite difference derivatives) is not to be considered except perhaps for very large, well conditioned problems. Its stability is usually too weak to compensate for the use of approximate gradients. The possible exception is that in very large problems (200 variables or more) both POW and VM require excessive storage capacity and hence become impractical.

In small problems (2 to 10 variables) without gradients, it is probably a toss-up between POW and VM with finite difference derivatives; any difference in efficiency for this size problem are strongly dependent on the problem. For the medium size problem (10 to 50 variables), VM with finite difference derivatives is probably preferred for ill-conditioned problems, but otherwise it is still a draw. For large problems (50 to 200 variables), VM is usually more dependable. For very large, ill-conditioned problems (200 variables or more) with no gradient available, one is likely to need divine assistance, but VM is probably the best bet, the storage problem notwithstanding".

Rao (1984) has also recommended similarly.

Moe (1973) presented an excellent state of the art paper on the application of penalty function methods in the analysis and design of engineering problems. He noted

"Although the method is already used with success in practical design work, there are great potentials for further improvements. There is hardly any program working today that includes even the majority of the known features that would have to be incorporated in an optimum version. Much also remains to be done to increase our knowledge concerning optimum choices of search parameters for different types of problems".

The observations made are valid even today. In an excellent review paper on the application of mathematical programming methods Fletcher (1973) explained and critically reviewed what are considered to be important concepts in the design of algorithms for optimization. He also presented a decision-tree for choosing an algorithm which is suitable for solving a particular optimization problem.

1.3 SCOPE OF THE THESIS :

Based on the review of literature and the critical assessment of the optimization schemes presented above, an attempt has been made in this thesis to study the suitability of the nonlinear programming approach in conjunction with two-dimensional limit equilibrium method of slices for automated slope stability analysis without any a priori assumption regarding the nature of the slip surface. Accurate methods of analysis namely, Spencer's method(Spencer, 1973) and Janbu's GPS(Janbu, 1973) have been chosen. The constrained optimization problem has been converted to an unconstrained one using penalty function formulation. In other words, the sequential unconstrained minimization technique(SUMT) has been adopted. Powell's conjugate direction method or Davidon-Fletcher-Powell variable metric method of unconstrained minimization technique for multidimensional search along with the quadratic interpolation technique for unidirectional search have been used. However, the Powell's method being a nongradient method has been adopted in most of the problems. The developed methodology has been applied to different problems to get a wider perspective.General purpose computer programs for the proposed procedures have been developed to carry out the computations involved in this thesis.

The content of the subsequent chapters are as follows:

In Chapter 2 the background and basic principles of the proposed study have been presented. Introduction to the limit equilibrium and the optimization methods used in the thesis have

been presented which may be considered as pre-requisites for the proposed studies. Also, the general way of presentation of the results, assumptions implied and such details as notations, conventions and the units of various quantities are also discussed.

Chapter 3 deals with the general formulation of the proposed technique. For simplicity the general features of the developed procedure have been presented with reference to homogeneous slopes only; necessary modifications of the formulation so developed to analyse slopes in nonhomogeneous soils are presented in subsequent chapters. The chapter also deals with the test of performance of some of the key subroutines for factor of safety computations and its minimization.

Several cases are studied in chapter 4 for slopes in homogeneous soils with linear and nonlinear failure envelopes. Chapter als with the application of the developed technique to slopes in soils with nonhomogeneous and anisotropic undrained shear strength.

Application of the developed procedure to zoned dams through several case studies are presented in chapter 6.

In Chapter 7 the application of the developed methodology to the stability analysis of embankments built on soft ground having irregular undrained shear strength profile has been demonstrated. The case studies considered include some failed slopes with well-documented observed failure surfaces to study the capability of the developed technique in predicting the

failure surface. Two examples are presented to highlight the danger of a priori assumption regarding the shape of the slip surface without any consideration to the physics of the problem.

A study has been initiated in Chapter 8 to explore the effectiveness of the SUMT in the back analysis of failed slopes.

Chapter 9 contains a summary of the work done, general observation, concluding remarks and the scope of future work.

CHAPTER 2

BACKGROUND AND BASIC PRINCIPLES OF THE PROPOSED STUDIES

2.1 INTRODUCTION

In this chapter the necessary background and basic principles of the methodologies adopted in the subsequent chapters have been presented and discussed. Wherever necessary references will be made to the appropriate sections of this chapter. Brief description of the generalised procedure of slices proposed by Janbu (1973) and Spencer (1973) along with their critical appraisal, technique of adopting nonlinear strength envelopes in these procedures originally developed for linear strength envelope, introduction of tension cracks and the adopted unconstrained minimization techniques, in analysing slope stability problems have been presented herein.

2.2 CHOICE OF THE METHOD OF ANALYSIS

Of the various limit equilibrium methods available for the computation of factor of safety associated with a given slip surface, the ones involving the method of slices are most versatile in the sense that they can incorporate a wide variety of field conditions. Of all the available methods of slices, the ones which do not make any a priori assumption regarding the shape of the slip surface and satisfy the equilibrium equations are more accurate. Some of these methods are due to Janbu (1954, 1957, 1973), Morgenstern and Price (1965) and Spencer (1973).

Of the three aforesaid analysis methods, although Janbu's method does not strictly satisfy all the equations of equilibrium, as pointed out by Sarma (1979) and demonstrated by Madej (1971), and it requires an assumption to be made regarding the position of the line of thrust, the method has an appeal because of its computational simplicity. The method can handle a wide variety of forces and field conditions. Because of these advantages, this method has been selected for use.

The method developed by Morgenstern and Price is considered to be the most accurate. It satisfies all the equations of equilibrium and can handle a wide variety of forces and field conditions. But the solution procedure is too rigorous and involves considerable effort on the part of the users.

Spencer's method also satisfies all the equations of equilibrium. The solution procedure, however, is much simpler compared to that of the Morgenstern and Price's method. Unlike Janbu's method, no assumption needs to be made regarding the position of the line of thrust; the line of thrust comes out as a part of the solution. Moreover, this method gives an exact solution whereas Janbu's method makes the system overdetermined. Considering all these, Spencer's method has also been selected for use in most of the studies contained in this thesis.

The purpose of using two methods of analysis when one method is sufficient for locating the critical slip surface is to establish the versatility of the proposed technique in adopting various analysis methods and also to study the influence of the method of analysis on the shape and location of the critical slip surface.

2.3 JANBU'S GENERALISED PROCEDURE OF SLICES

For ready reference, Janbu's Generalised Procedure of Slices (GPS) (keeping the notations intact) has been presented here. In the developed computer program these notations and iteration steps suggested by Janbu have been followed unchanged. But the procedure described herein is more general in the sense that an external moment ΔM has been included in the analysis in addition to ΔP and ΔQ - the external forces in the vertical and the horizontal directions respectively. Figure 2.1(a) shows a cross section of a slope for the general case in which the terrain, the external loads, the boundary conditions, the soil profile, and the shear surface may be irregular.

The shear surface ab in Figure 2.1(a) is only one out of an infinite number of possible shapes and locations of such surfaces.

The soil mass located between the ground surface and the assumed shear surface is divided into a number of slices by vertical lines. A cross section of such a slice is indicated by the dotted area in Figure 2.1(a). This same slice is drawn to an enlarged scale and the system of notation for the forces acting on all four boundaries of the slice is shown in Figure 2.1(b).

The resultant of the total interslice forces are E and T in the horizontal and vertical directions respectively. ΔS and ΔN are the resultants of the shear stress and total normal stress σ over the length Δl along the shear surface. Hence $\Delta N = \sigma \Delta l$ and $\Delta S = \tau\ \Delta l$. In the general case, E, T, ΔN and ΔS are unknowns.

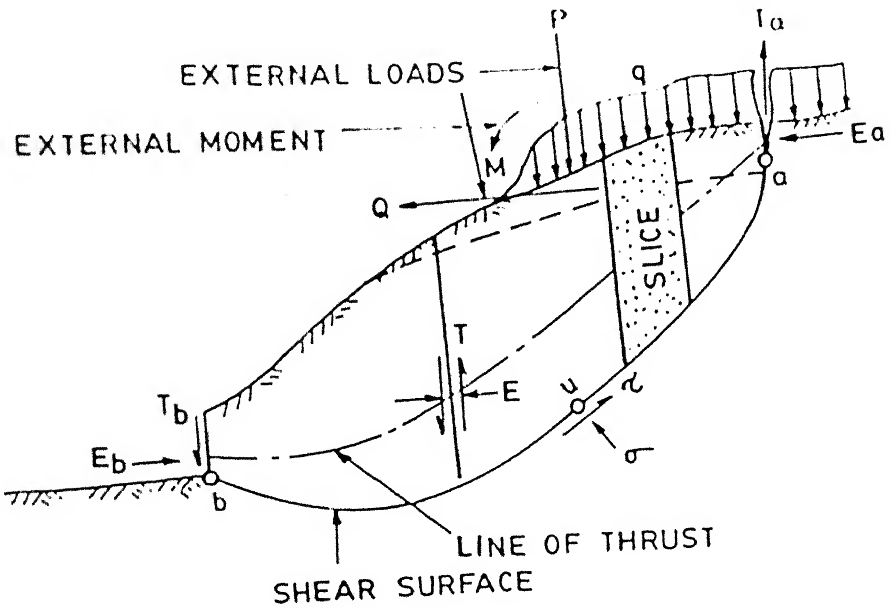


FIG. 2·1(a) DEFINITIONS AND NOTATIONS USED FOR THE JANBU'S METHOD OF ANALYSIS

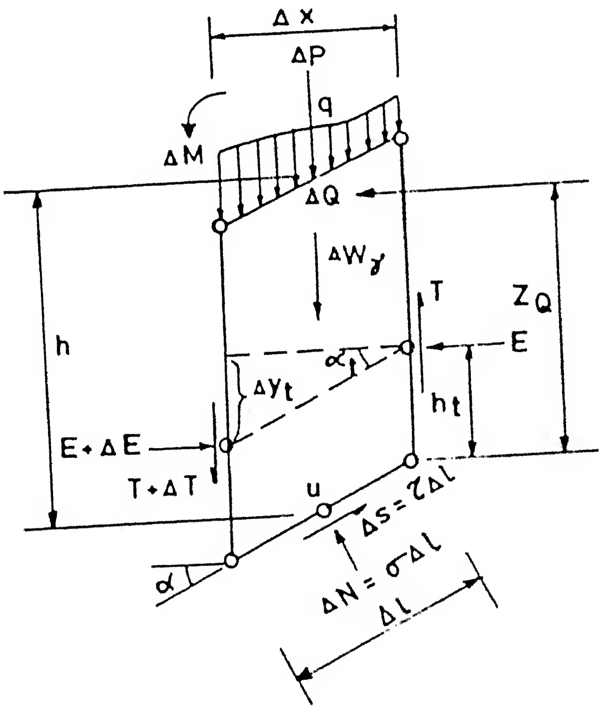


FIG. 2·1(b) FORCES ACTING ON THE BOUNDARIES OF A SINGLE SLICE FOR JANBU'S METHOD

Basic Assumptions : The following basic assumptions are used for the general case:

1. Plain strain conditions apply.
2. The factor of safety F is constant along the entire shear surface
3. The equilibrium shear stress along the shear surface is given by the equation
$$\tau = \frac{\tau_f}{F} \tag{2.1}$$
where τ_f is the shear strength.
4. The total resultant ΔN is assumed to act where $\Delta W = \Delta W_\gamma + q\Delta x + \Delta P$ intersects the base.
5. The position of the line of thrust for the total side force E is assumed to be known.

Known Quantities : The analysis is initiated by selecting a trial surface and subdividing the soil mass into appropriate number of slices. From the shape and location of this surface in Figure 2.1(b), one obtains for each slice :

tan α = slope of shear surface (dimensionless)

Δx = slice width

The average total vertical stress at the base of each slice is computed from
$$p = \frac{\Delta W}{\Delta x} = \gamma z + q + \frac{\Delta P}{\Delta x} \tag{2.2}$$
Further, in Figure 2.1(b),

ΔQ = the magnitude of the horizontal force located either at the ground surface (inclined loads) or in the interior of the soil mass (earthquake force); when horizontal forces act both at surface and at interior, ΔQ is the resultant of them.

z_Q = the distance that ΔQ acts above the assumed shear surface.

When the position of the line of thrust is chosen one measures

h_t = vertical distance between shear surface and line of thrust

$\tan \alpha_t$ = slope of the line of thrust (dimensionless)

Both quantities are measured at each intersection, and not in the middle of the slice (the same is true for z_Q and $\Delta Q/\Delta x$)

ΔM = the magnitude of the external moment acting on a slice in the counterclockwise sense.

Equilibrium Conditions : The requirement for static equilibrium must be satisfied both for individual slices and for the whole body. The complete set of basic equations, which must be satisfied simultaneously is summarized below:

(i) $\tau = c_e + (\sigma - u) \tan \phi_e$ (2.3)

(ii) $\sigma = p + t - \tau \tan \alpha$ (2.4)

(iii) $\Delta E = \Delta Q + (p+t)\Delta x \tan \alpha - \tau\Delta x (1+\tan^2\alpha)$ (2.5)

(iv) $T = -E\tan \alpha_t + h_t \frac{dE}{dx} - z_Q \frac{dQ}{dx} - \frac{dM}{dx}$ (2.6)

where $c_e = \frac{c'}{F}$; $\tan \phi_e = \frac{\tan\phi'}{F}$ and $t = \frac{dT}{dx}$

Equation (2.3) is the defining equation for the state of limit equilibrium. Equation (2.4) is the equation for vertical equilibrium of each slice. Equation (2.5) is the equation for a combination of horizontal and vertical equilibrium for each slice, and Equation (2.6) is the equation for moment equilibrium for a slice of infinitesimal width.

The requirement for overall horizontal equilibrium leads to the following equation (Figures. 2.1(a) and (b)):

$$\sum_a^b \Delta E = E_b - E_a \qquad (2.7)$$

Overall vertical and overall moment equilibrium are automatically satisfied by the foregoing equations when the boundary forces on the end slices are properly handled.

In slope stability analysis, the average factor of safety and four unknowns for each slice, E, T, σ and τ must be determined.

Average Safety Factor: Inserting equation (2.5) into (2.7), one obtains

$$E_b - E_a = \sum_a^b \left[\Delta Q + (p+t)\Delta x \tan \alpha\right] - \sum_a^b \tau \Delta x \ (1+\tan^2\alpha)$$

Introducing $\tau = \tau_f/F$ into the expression above, and considering F = constant (equal to the overall average safety factor along the selected shear surface) and solving the equation for F, one finds:

$$F = \frac{\sum_a^b \tau_f \Delta x \ (1+\tan^2\alpha)}{E_a - E_b + \sum_a^b [\Delta Q + (p+t)\Delta x \tan\alpha]} \qquad (2.8)$$

To derive a formula for the shear strength τ_f, introducing Equation (2.4) into the general shear strength expression, one obtains

$$\tau_f = c' + (p + t - u - \tau \tan \alpha) \tan \phi'$$

Introducing $\tau = \tau_f/F$ and solving for τ_f, one finds

$$\tau_f = \frac{c' + (p + t - u) \tan \phi'}{1 + (1/F) \tan \phi' \tan \alpha} \qquad (2.9)$$

The formula for the average safety factor for the general case is obtained by inserting Equation (2.9) into Equation (2.8).

Using the following abbreviations

$$B = \Delta Q + (p + t)\,\Delta x \tan\alpha \qquad (2.10)$$

$$A = \tau_f\,\Delta x\,(1 + \tan^2\alpha) \qquad (2.11)$$

$$A' = [c' + (p + t - u)\tan\phi']\Delta x$$

$$n_\alpha = \frac{1 + (1/F)\tan\phi'\tan\alpha}{1 + \tan^2\alpha} \qquad (2.12)$$

one obtains

$$A = \frac{A'}{n_\alpha} \qquad (2.13)$$

$$F = \frac{\sum_a^b A}{E_a - E_b + \sum_a^b B}\,. \qquad (2.14)$$

Stresses τ and σ along the shear Surface :

Once the A term for each slice is computed, the shear stress τ is obtained from Equation (2.11) as :

$$\tau = \frac{\tau_f}{F} = \frac{A}{F\,(1 + \tan^2\alpha)\,\Delta x} \qquad (2.15)$$

The total normal stress on the shear surface is now calculated as follows:

$$\sigma = p + t - \tau\tan\alpha \qquad (2.16)$$

Interslice Forces: Introducing Equations. (2.10) and (2.11) into Equation(2.5) and observing that $\tau = \tau_f/F$, one obtains for each slice

$$\Delta E = B - \frac{A}{F} \qquad (2.17)$$

Summing the ΔE values for each slice, starting at point a in Figure 2.1(a), one obtains the total side force E on each interface from the formula

$$E = E_a + \sum \Delta E \qquad (2.18)$$

Along this same interface the vertical shear force T is equal to (Equation 2.6)

$$T = -E \tan \alpha_t + h_t \frac{dE}{dx} - z_Q \frac{dQ}{dx} - \frac{dM}{dx} \qquad (2.19)$$

Derivation of the above expression is given in Appendix A.

After obtaining the T values for all the interfaces one computes ΔT for each slice, and the corresponding values $t = \Delta T/\Delta x$.

Factor of Safety along Interfaces : On the basis of the known, or assumed, pore pressure distribution one can compute the resultant horizontal water force U_h and the horizontal effective resultant $E' = E - U_h$. This resultant produces an average horizontal effective stress, $\sigma'_h = E'/z$, from which the shear strength τ_{vf} on the interface can be computed :

$$\tau_{vf} = c' + \sigma'_h \tan \phi'$$

One can now determine the average factor of safety F_v with respect to shear failure along each interface from the formula

$F_v = \tau_{vf}/\tau_v$, where

τ_v = The average vertical shear stress = T/z

Hence

$$F_v = \frac{c'H + (E - U_h)\tan \phi'}{T} \qquad (2.20)$$

Iteration Procedure :

The iteration procedure to get convergent value of average factor of safety has been discussed in detail by Janbu (1973) and the same has been implemented in the developed computer program. However it is not presented herein for reasons of space.

Critical Comments :

Analysis of the unknowns, available equations and assumptions used in the Janbu's GPS, carried out earlier (Wright,

1969; Bhowmik, 1984), shows that the method is overdetermined as one assumption is made more than required. Therefore, technically it cannot be a rigorous solution. However, as Sarma (1979) points out, "in the solution one of the assumptions i.e., the position of the last normal force, is not used. Had he used it, he would have found that the moment equilibrium of the last slice is not satisfied i.e., $M_n \neq 0$, which he does not test." Moreover, Madej (1971) has demonstrated that Janbu's moment equation for an individual slice is incomplete.

Some convergence difficulties have been reported with regard to Janbu's GPS (Wright, 1969; Siegel, 1975; Fredlund, 1984). It is pertinent to note Janbu's comment on the above:

"--- When using computers blindly one may run into a number of (unnecessary) difficulties. A typical pitfall is to believe that the largest number of slices gives the most accurate results. This is by no means so since all formulae were originally derived for finite differences, neglecting terms of second order. In fact the optimal number of slices is determined by the shape of slices."

Janbu advised that the ratio between slice height and slice width ($H : \Delta x$) should usually be 1 to 2.5 or maximum 3. He also cautioned about another faulty notion that when numerical instability is encountered, the applied procedure must be wrong, while the reason may be one or more of the following :

(i) Statically inadmissible shear surface

(ii) Too large a number of slices

(iii) Wrong line of thrust (to assume crack in tension zone).

(iv) Incorrect input data.

2.4 SPENCER'S METHOD

Figure 2.2(a) shows a cross-section through an embankment with a slip surface of varying curvature. At the upper end of the slip surface there is a vertical tension crack of depth z_t. The force T is due to water pressure in this crack. The external force and moment which need to be applied to the n^{th} slice to stabilize the embankment are Z_n and M_n respectively, which are eliminated in the process of solution.

A typical slice, shown in Figure 2.2(b) is of mean height h and width b. The base of the slice is straight and its slope is α. The width b is assumed to be small.

The slice is subjected to the following forces :

(a) its weight W

(b) the external forces P_e and Q_e in the vertical and the horizontal direction respectively acting at the mid-point of the top of the slice. An external moment M_e also acts in the counterclockwise sense. As a convention they are positive as shown in the figure.

(c) the total force normal to its base N (this has one component N′ due to effective stress and a second component U due to pore water pressure) and thus

$$N = N' + U = (\sigma' + u)b \sec \alpha \qquad (2.21)$$

(d) two inter-slice forces Z_L and Z_R whose slopes are δ_L and δ_R, respectively

(e) the force S_m tangential to the base (this force represents the mobilized shear force) and if S represents the total shear strength available and F the

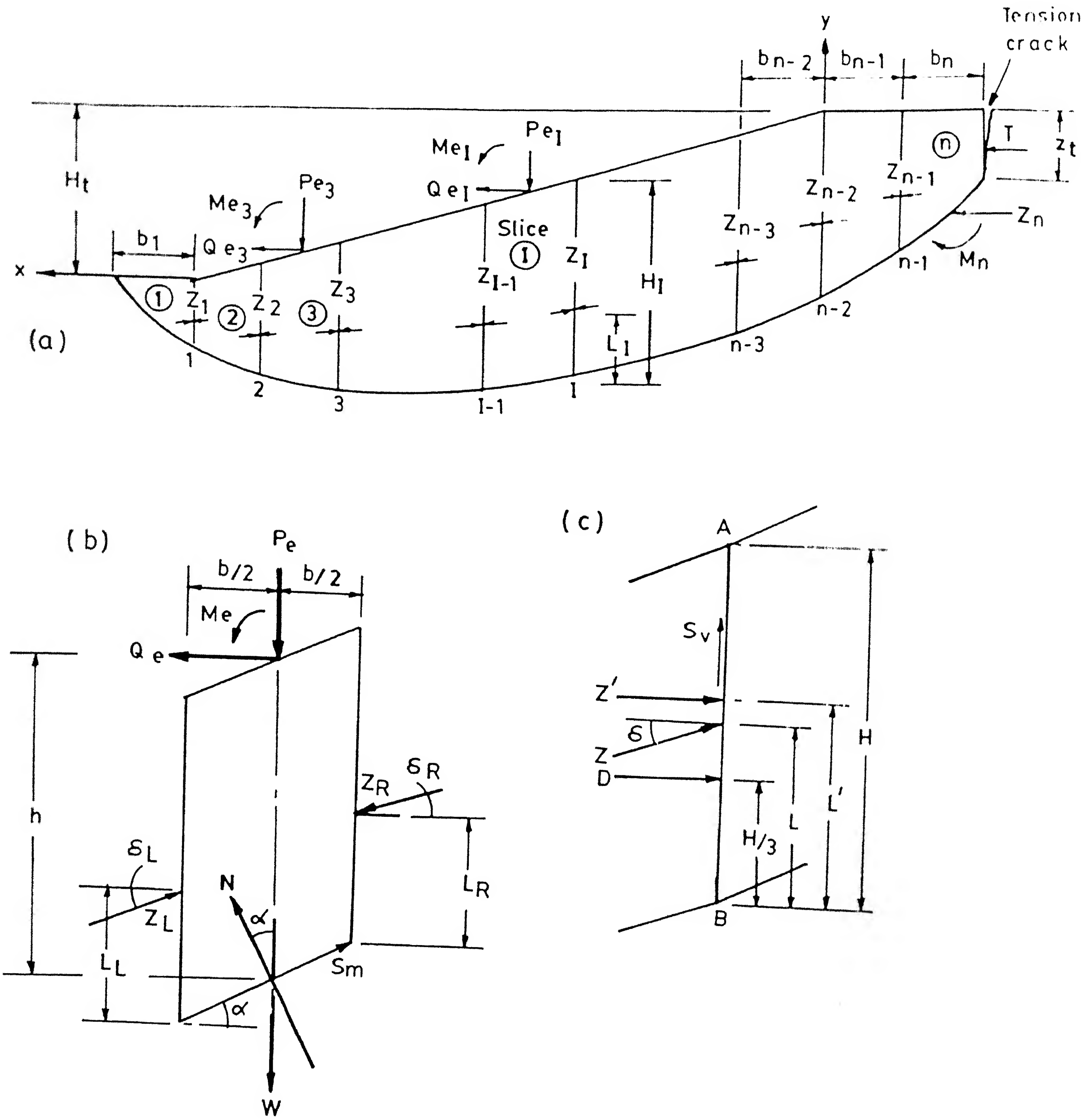


FIG. 2·2 (a) DEFINITIONS AND NOTATIONS USED FOR THE SPENCER'S METHOD OF ANALYSIS
(b) FORCES ON TYPICAL SLICE IN SPENCER'S METHOD OF ANALYSIS
(c) FORCES ON AN INTERSLICE BOUNDARY

overall factor of safety then

$$S_m = \frac{S}{F} = \frac{b \sec \alpha}{F}(c' + \sigma' \tan \phi') \tag{2.22}$$

In Spencer's analysis no external force or moment was considered. The present system of forces is, therefore, more general.

Force Equilibrium :

An expression is obtained for N by resolving the forces on the slice in a direction normal to its base.

$$N = W' \cos \alpha - Q_e \sin \alpha - Z_R \sin(\alpha - \delta_R) + Z_L \sin(\alpha - \delta_L) \quad \ldots(2.23)$$

from which the effective stress σ', at the base of the slice is obtained as

$$\sigma' = \frac{1}{b'}\left[W' \cos \alpha - Q_e \sin \alpha - Z_R \sin(\alpha - \delta_R) + Z_L \sin(\alpha - \delta_L) - ub' \right] \quad \ldots (2.24)$$

Then, resolving parallel to the base and substituting for N, the following expression is obtained for the value, per unit length of embankment, of the inter-slice force on the right hand side of the slice

$$Z_R = \frac{c'_m b' - W' \sin \alpha - Q_e \cos \alpha + \tan \phi'_m \ (W' \cos \alpha - Q_e \sin \alpha - ub')}{\cos(\alpha - \delta_R)[1 + \tan \phi'_m \tan(\alpha - \delta_R)]}$$

$$+ Z_L \left\{ \frac{\cos(\alpha - \delta_L)[1 + \tan \phi'_m \tan(\alpha - \delta_L)]}{\cos(\alpha - \delta_R)[1 + \tan \phi'_m \tan(\alpha - \delta_R)]} \right\} \tag{2.25}$$

In the equations (2.23) through (2.25),

$$c'_m = c'/F$$

$$\tan \phi'_m = \frac{\tan \phi'}{F}$$

$$b' = b \sec \alpha$$

$$W' = W + P_e$$

For the first slice, Z_L is taken as zero and the Equation (2.25) then gives the value of Z_1, the inter-slice force on the first boundary. In the case of the last (n^{th}) slice, it is convenient to take the slope of the external balancing force (Z_n) as zero. Equation (2.25) can then be used to find the value of $Z_n + T$, where T is the force due to water pressure in the tension crack.

Moment Equilibrium :

It is assumed that the slice width b is small and that the four forces N, W, P_e and S_m act through the middle of the base of a slice while Q_e acts through the middle of its top. Commencing at the first slice, moments are taken, about the middle of the base of each slice in turn, of the pair of interslice forces which act on the slice. This process leads to two expressions. The first expression gives the height L of the point of action of an inter-slice force above the base of the boundary on which it acts (Figure 2.2(c)).

$$L_I = \frac{b_I}{2}(\tan \delta_I - \tan \alpha_I) + \frac{1}{Z_I \cos \delta_I} \sum_{i=1}^{I} \left\{ [J] - (Q_{e_i} h_i + M_{e_i}) \right\} \qquad \ldots\ldots\ldots(2.26)$$

The second expression obtained in this process gives the value of the external moment required to stabilize the embankment.

$$M_n = \gamma_w \frac{z_t^2}{2} \left[\frac{b_n}{2} \tan \alpha_n + \frac{z_t}{3} \right] - \sum_{i=1}^{n} \left\{ [J] - (Q_{e_i} h_i + M_{e_i}) \right\} \qquad \ldots\ldots\ldots(2.27)$$

where,

$$J = \tfrac{1}{2} Z_{i-1} \left[\sin \delta_{i-1}(b_i + b_{i-1}) - \cos \delta_{i-1}(b_i \tan \alpha_i + b_{i-1} \tan \alpha_{i-1}) \right] \qquad \ldots\ldots\ldots(2.28)$$

Detailed derivation of the equations are given in Appendix A.

Slope of the inter-slice forces :

With two equilibrium conditions to satisfy ($Z_n = \emptyset$ and $M_n=\emptyset$), two variables are required. The first variable is the value of the overall factor of safety F; the second is the angle θ which together with the coefficient k, determine the slope (δ) of the interslice forces in accordance with the following equation

$$\tan \delta_I = k_I \tan \theta \tag{2.29}$$

Method of Solution Suggested by Spencer :

The method of solution of the force and moment equilibrium equations suggested by Spencer, are as follows :

For particular trial values of the two variables F and θ corresponding values will be required for the external force Z_n and the external moment M_n to stabilize the embankment. The method of solution is a process of successive approximation in which the values of Z_n and M_n are gradually reduced to a negligible level. The process is as follows :

(a) A range of values is chosen for k, one for each inter-slice boundary.

(b) trial values are chosen for F and θ.

(c) The slope δ of each inter-slice force is then computed using Equation (2.29).

(d) Equation (2.25) is then used to determine the value of each inter-slice force in turn until the last slice is reached and the value of Z_n is obtained. The value of F is then adjusted, by successive approximation, until Z_n is negligible.

Assumption Regarding Interslice Forces :

Regarding the selection of inter-slice force function (k-distribution) Spencer (1973) has made the following observations:

"Bearing in mind the various uncertainties involved in analysing the stability of an earth embankment, and in particular the determination of the shear strength parameters and the pore water pressure, there seems little point, so far as the determination of the value of the factor of safety is concerned, in trying many different assumptions regarding the slope of the inter-slice forces. A reasonably reliable value for the factor of safety can be obtained by assuming the inter-slice forces to be parallel. This condition can be included in a computer program so that the data input is reduced.

It is only if there is anxiety about the position of the line of thrust that it may be considered worthwhile obtaining solutions using other assumptions regarding the slope of the inter-slice forces."

In a review paper Fredlund (1984) has reported that Wilson and Fredlund (1983) and Fan (1983) performed a detailed study on interslice force functions computed from finite element analysis. Based on a large number of analyses, a general empirical inter-slice force function was proposed. The function along with charts for various constants used in the function have been presented in detail by Fredlund (1984).

Acceptable Line of Thrust - Introduction of Tension Crack :

In those cases in which the positions of the lines of thrust (obtained as part of the solution) are not satisfactory, Spencer (1973) has made the following recommendations :

(i) The inter-slice forces may be assumed to be parallel, i.e., k = 1, except at the upper end of the slip surface where their slopes should be reduced, i.e., k becoming less than unity.

(ii) The presence should be assumed of a vertical tension crack parallel to the crest of the embankment with water pressure in it. The depth of the tension crack, z_t, can be assumed as the depth of zero active effective stress given by :

$$z_o = \frac{2c'}{\gamma \, F(1-r_u)} \sqrt{\frac{1+\sin \phi'_m}{1-\sin \phi'_m}} \qquad (2.31)$$

The above expression is, however, applicable only to slopes in homogeneous soils in which the pore water pressure increases with depth in direct proportion to the overburden.

Other investigators (Chowdhury, 1978; Janbu, 1980; Ching and Fredlund, 1983) have also recommended the introduction of tension crack in the analysis in order to get rid of occasional convergence difficulties and also to obtain acceptable solutions.

Depth of Tension Crack as Design variable :

Equation (2.31) requires an iterative procedure to solve for the depth of tension crack z_o. Spencer (1973) suggested that the value of F occurring in this expression can be obtained from a preliminary trial taking k = 1 throughout and with no tension crack. In the search for critical slip surfaces the value of F changes from one trial shear surface to another. For slip circle analysis Spencer (1968) has provided a chart to obtain z_o for various homogeneous slopes and soil and pore pressure coefficient values. In the search for critical slip surfaces of general shape, the iteration can be conveniently done by treating z_t as a design variable together with an upper limit for z_t as z_o.

Critical Comments : Study of equations and unknowns in the Spencer method reveals that the method makes exactly the same number of assumptions as are required and hence the obtained solution satisfies the static equilibrium conditions rigorously. According to Wright (1969) ,"the Spencer's method may be the best of those available for analysis of non-circular surfaces, because it combines simplicity with an acceptable degree of accuracy."

2.5 NUMBER OF SLICES TO BE CONSIDERED IN THE ANALYSIS

In the method of slices it is required to decide on the number of slices before any computation is carried out. Based on a comprehensive study on this aspect with respect to circular slip surfaces, Wright (1969) recommended that the number of slices may be taken such that the slice bases have a fixed arc length equal to 10% of the slope height. A series of studies conducted by him revealed that the number of slices corresponding to an arc length equal to 10% of the slope height vary from 10 to 50. Spencer (1967), however, reported that an increase in the number of slices from 16 to 32 amounted to only 0.3% change in the factors of safety (an absolute difference of 0.004) which can be neglected for all practical purposes. This study was also based on circular slip surfaces.

With respect to slip surfaces of general shape, Spencer (1973) considered 14 to 16 slices to demonstrate his method of analysis. However, Janbu (1973) with reference to the GPS procedure, observed that 6 to 9 slices are adequate from practical point of view. As already stated in Section 2.3, Janbu,

from the point of view of convergence, advises not to take too many slices. Bhowmik (1984), based on studies with reference to both Janbu's and Spencer's methods, observed that a number of slices beyond 12 has no significant effect on the factor of safety values in the case of homogeneous slopes.

In the studies contained in this thesis, following Bhowmik's observations the number of slices have been generally kept greater than 12. However, Janbu's recommendations have been given due weightage when using the GPS procedure. In the nonhomogeneous case a study on the effect of number of slices has been carried out, the results of which will be reported at the appropriate place.

2.6 NONLINEAR STRENGTH ENVELOPE

There is considerable experimental evidence to show that the Mohr-Coulomb failure envelope exhibits significant curvature for many different types of soils e.g. stiff clays, dense sands and compacted rock fill. Maksimovic (1979), Charles and Soares (1984) have presented useful contribution in the field of stability analysis of slopes in soils with nonlinear failure envelope.Figure 2.3 shows the point to point variation of strength in a typical nonlinear strength envelope. For example, at points where the normal effective stresses are σ'_l, σ'_i and σ'_h, the corresponding shear strength parameters are (c'_l, ϕ'_l), (c'_i, ϕ'_i) and (c'_h, ϕ'_h) respectively.

De Mello (1977) has suggested the following relationship to represent the curved failure envelope typically exhibited by many

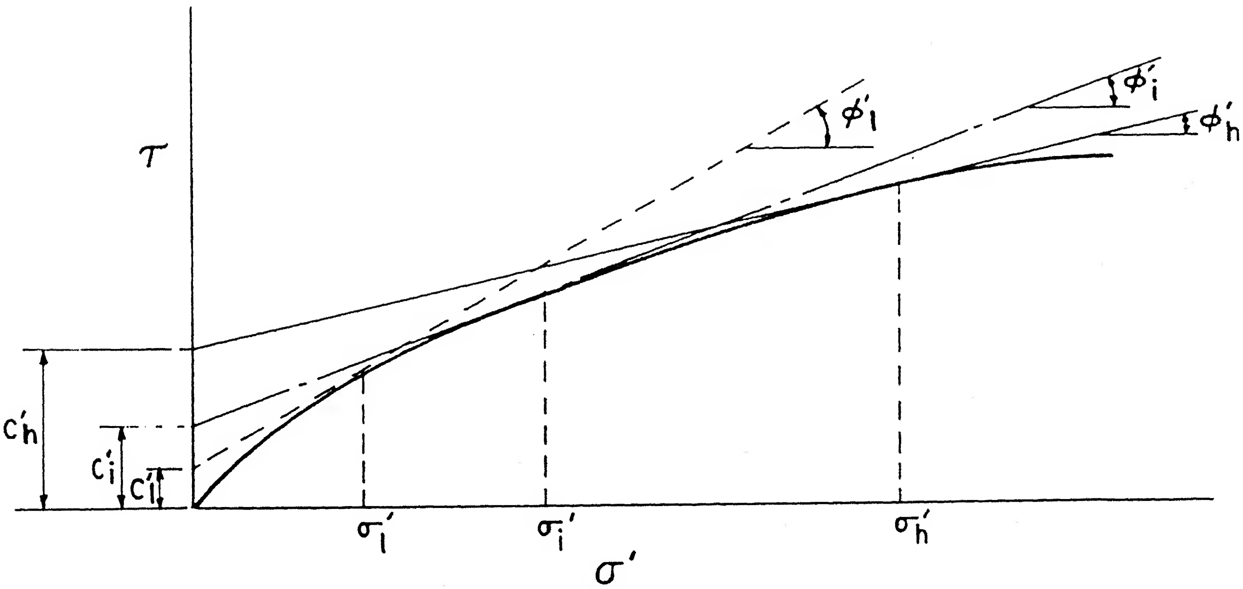


FIG. 2.3 VARIATION OF STRENGTH PARAMETERS IN A TYPICAL NON-LINEAR STRENGTH ENVELOPE

soils over a wide range of normal effective stress :

$$\tau_f = \alpha' (\sigma')^{\beta'} \qquad (2.32)$$

where τ_f is the shear strength and the dimensionless parameter β' defines the degree of curvature of the envelope. The parameter α' has dimension $[\sigma']^{1-\beta'}$ and for any value of σ' the magnitude of τ_f is proportional to this parameter.

The equilibrium shear stress along the shear surface is given by :

$$\tau = \frac{\tau_f}{F} = \frac{\alpha'}{F} (\sigma')^{\beta'} \quad \text{where F is the factor of safety}$$

The relationship in Equation (2.32) was earlier used for factor of safety computations by Charles and Soares (1984) and Yudhbir et al. (1987) in conjunction with simplified Bishop method and Janbu's method respectively.

In the present analysis, an attempt has been made to use De Mello's (1977) model in Spencer method keeping the basic computational scheme developed for linear Mohr-Coulomb envelope undisturbed.

To use the expressions developed by Spencer for linear Mohr-Coulomb envelope, the point to point variation in the effective strength parameters c'_i and ϕ'_i should be used. If σ'_i is the effective normal stress at the base of the i^{th} slice, the effective strength parameters c'_i and ϕ'_i corresponding to σ'_i are obtained as follows :

As shown in Figure 2.3, ϕ'_i is the inclination of the tangent to the curved failure envelope at the effective stress σ'_i

$$\text{Thus, } \tan \phi'_i = \frac{\partial \tau_f}{\partial \sigma'_i} = \frac{\partial}{\partial \sigma'_i} \left\{ \alpha' \ (\sigma'_i)^{\beta'} \right\}$$

$$\text{whence, } \quad \phi'_i = \tan^{-1}\left[\alpha' \ \beta' (\sigma'_i)^{\beta' - 1} \right] \qquad (2.33)$$

Further c'_i represents the intercepts of the aforementioned tangent on the τ_f axis and is given by :

$$c'_i = \alpha' (\sigma'_i)^{\beta'} - \sigma'_i \tan \phi'_i$$

$$= (1 - \beta') \alpha' (\sigma'_i)^{\beta'} \qquad (2.34)$$

The parameters α' and β' for a particular soil are evaluated such that the Equation (2.31) fits the experimental data.

Since σ'_i in Spencer method (Equation (2.24)) itself requires c'_i, ϕ'_i for its evaluation, the solution has to be an iterative one. The iterative scheme adopted for the purpose will be presented in Chapter 3.

2.7 MATHEMATICAL PROGRAMMING FORMULATION

2.7.1. General Statement

An optimization or a mathematical programming problem can be stated as follows :

$$\left.\begin{array}{ll} \text{Find } \mathbf{D} \text{ which minimizes } f(\mathbf{D}) & \\ \text{subject to the constraints} & \\ g_j(\mathbf{D}) \le 0 & j = 1, 2, \ldots, m \\ l_j(\mathbf{D}) = 0 & j = m+1, m+2, \ldots, p \end{array}\right\} \qquad (2.35)$$

where $\mathbf{D} = (d_1, d_2, \ldots d_n)^T$ is the n dimensional vector called the design vector, $f(\mathbf{D})$ is called the objective function and $g_j(\mathbf{D})$ and $l_j(\mathbf{D})$ are, respectively, the inequality and equality constraints. The number of variables n and the number of constraints are unrelated. Thus p could be less than, equal to or greater than n in a given mathematical programming problem. In some problems, the value of p may be zero implying that there

are no constraints on the problem. Such types of problems are called unconstrained optimization problems. If m is not equal to zero the problem is one of constrained optimization.

2.7.2 Penalty Function Method : The Sequential Unconstrained Minimization Technique

2.7.2.1 General

Most of the powerful minimization algorithms are for unconstrained minimization only. To utilize these powerful algorithms a constrained minimization problem is transformed into one of unconstrained minimization by a technique known as the penalty function method. Penalty function methods transform the basic optimization problem into alternative formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. Let the basic optimization problem be of the form given in equation (2.35) and having inequality constraints only. This problem is converted into an unconstrained minimization problem by constructing a function of the form

$$\psi_k = \psi(D, r_k) = f(D) + r_k \sum_{j=1}^{m} G_j [g_j(D)] \qquad (2.36)$$

where G_j is some function of the constraints g_j and r_k is a positive constant known as the penalty parameter or the response factor. The second term on the right hand side of equation (2.36) is called the penalty term. If the unconstrained minimization of the ψ function is repeated for a sequence of values of the penalty parameter r_k ($k = 1,2,\ldots$), the solution may be brought to converge to that of the original problem stated in Equation

(2.35). This is the reason why the penalty function methods are also known as *sequential unconstrained minimization techniques*(SUMT). Penalty function formulations for problems with inequality constraints can be divided into two categories, the interior and the exterior methods. In the interior formulation the decision variables always lie in the feasible region and converge to the minimum from within the feasible region. For this reason, interior penalty function formulation has the advantage that any intermediate sub-optimal decision vector can be taken as an acceptable solution if required.

2.7.2.2 Interior Penalty Function Method

In this approach the objective function is augmented with a penalty term which is small at points away from the constraints in the feasible region but which 'blows up' as the constraints are approached. The most commonly used functions (Fiacco and McCormick) of this sort is

$$\psi\ (D,\ r_k) = f(D) - r_k \sum_{j=1}^{m} \frac{1}{g_j(D)}$$

where f(D) is minimized over all D satisfying $g_j(D) \leq 0$, j = 1, 2,...,m. It should be noted that if r_k, the penalty parameter, is positive its effect is to add a positive penalty to f(D). This is because at an interior point, all the terms in the sum is negative. As the boundary is approached, some $g_j(D)$ will approach zero and the penalty term will 'explode'. Using a reduction factor c,the penalty parameter r_k is made successively smaller in order to obtain the constrained minimum of f(D).

Thus, $r_{k+1} = cr_k$

When both equality and inequality constraints are present Fiacco and McCormick reports some success with the formulation

$$\psi(D, r_k) = f(D) - r_k \sum_{j=1}^{m} \frac{1}{g_j(D)} + \frac{1}{r_k^{1/2}} \sum_{j=m+1}^{p} l_j^2(D)$$

where $\psi(D, r_k)$ is minimized for a decreasing sequence of values of r_k. As r_k is made small the second term allows the minimum to approach the constraints from the inside and the third term successively forces the satisfaction of the equality constraints.

The interior penalty function method has the disadvantage that the algorithm requires a feasible starting point. In complex problems it may be rather time consuming and extremely difficult to find a feasible point by guessing or simple estimation. In these situations the problem has to be solved by using either the exterior penalty function method or obtaining an initial feasible design vector by interior penalty function technique itself following a procedure outlined by Fox (1971) and then using the interior penalty function method. The scheme suggested by Fox (1971) is as follows:

Suppose an engineering assessment of the situation has produced the design D_o which satisfies $g_j(D_o) < \emptyset$, $j = 1, \ldots, m$, but which has $g_j(D_o) > \emptyset$, $j = m+1, \ldots, n$ then that value of the parameter k is found out for which $g_k(D_o)$ is a minimum where $k = m+1, \ldots, n$ and temporarily it is defined as the objective function for the following problem :

Find $\mathbf{D}$ such that

$g_k(\mathbf{D}) \rightarrow \text{Min.}$

subject to

$g_j(\mathbf{D}) \leq \emptyset, \; j = 1, 2, \ldots, m,$

$g_j(\mathbf{D}) - g_j(\mathbf{D}_o) \leq \emptyset, \; j = m+1, \ldots, n.$

This problem is solved by penalty function method. The search is terminated as soon as the objective function becomes negative. The process is repeated until all the constraints are satisfied. However, this method of finding an acceptable design is very time consuming.

Even when an initial feasible design point is available and the interior penalty function method is used, owing to the "long step" nature of unconstrained optimization algorithm, the path may be diverted into infeasible regions. In such cases the function is generally set to an arbitrary high value and the minimization procedure is left to correct the situation on its own. Sometimes this approach presents numerical difficulties.For problems having mixed equality and inequality constraints, the Interior penalty function method is not well established. However, Fox (1971) has suggested some formulations which are not presented here.

2.7.2.3 The Extended Interior Penalty Function Method

A promising alternative approach is to use the extended penalty function method proposed by Kavlie (1971). In this method the infeasible starting points are immediately acceptable to the minimization algorithm. In Kavlie's approach the constrained problem is transformed to an unconstrained one as follows:

$$\psi\ (D,\ r_k) = f(D) + r_k \sum_{j=1}^{p} G\left[g_j(D)\right]$$

where function G is chosen as follows :

$$G\left[g_j(D)\right] = \begin{cases} \dfrac{1}{g_j(D)} & g_j(D) \le \varepsilon \\ \dfrac{\left[2\varepsilon - g_j(D)\right]}{\varepsilon^2} & g_j(D) > \varepsilon \end{cases}$$

where $\varepsilon = - r_k/\delta_t$

and δ_t = a constant that defines the transition between the two types of penalty terms.

Successful application of extended penalty function method to various geotechnical and structural engineering problems having both inequality and equality constraints has been demonstrated by many investigators (Kavlie and Moe, 1971; Basudhar, 1976; Cassis and Schmit 1976; Bhowmik, 1984; Madan Mohan, 1984; Satyam Babu, 1986; Dhawan, 1986; Mondal, 1988).

Initial Values of the Penalty Parameters :

The choice of transition parameter (δ_t) and tolerance (ε) has a great bearing on the effectiveness of the numerical scheme using the Extended penalty function method. Kavlie and Moe (1971) suggested that one may tentatively select δ_t approximately equal to the initial value of the objective function $f(D_1)$. Once a proper choice of the initial penalty parameter or response factor r_1 is made, then ε can be estimated. Regarding the choice of r_1 it has been observed that the composite function ψ is easy to minimize if a substantially large value of r_1 is selected. Unfortunately in that case the minimum of ψ function will be far

away from the desired minimum and hence the number of unconstrained minimization of ψ (r - minimization) increases tremendously and as such, a compromise has to be found. In practice, for a general interior penalty function method, a value of r_1 which gives the value of $\psi(D_1, r_1)$ approximately equal to 1.1 to 2.0 times the value of $f(D_1)$ has been found to be quite satisfactory in achieving quick convergence (Rao, 1984). A factor of reduction (c) between 10 and 50 for the penalty parameter has been found to work quite well and a standard value of 20 has been suggested by Kavlie and Moe (1971).

Basudhar (1976) successfully applied the extended interior penalty function technique to find the optimal lower bound solution of earth pressure and bearing capacity problems with arbitrary choice of δ_t and ε as 10 and -0.1 respectively. However, he cautioned that the values may not be unique for all types of problems and detailed study is needed. He used a reduction factor of 10.

Cassis and Schmit (1976) suggested that the initial value of r may be chosen as

$$r_1 = \frac{(\mu - 1)\ f(D_1)}{\Sigma\ 1/g_j(D_1)}$$

where μ has the most popular value of 2. The tolerance ε can then be estimated from the following relation

$$\varepsilon = \frac{r_1}{\psi_1\ (D_1)}$$

Mondal (1988) observed that convergence is better if the δ_t parameter be chosen at least 100 times the initial penalty parameter r_1.

2.7.2.4 Convergence Criteria for the Penalty Function Methods

Since the unconstrained minimization of $\psi(D, r_k)$ is carried out for a decreasing sequence of values r_k, it is necessary to use appropriate convergence criteria to identify the optimum point and also to avoid an unnecessarily large number of unconstrained minimization. The process is terminated whenever the relative change in function value and/or the change in all the design variables between two consecutive cycles of minimization is less than the desired accuracy, i.e.

$$\left| \frac{f(D_{i+1}) - f(D_i)}{f(D_i)} \right| \le \varepsilon_1$$

$$|D_{i+1} - D_i| \le \varepsilon_2$$

2.7.3 The unconstrained Minimization Techniques

2.7.3.1 General

There are several methods for solving unconstrained minimization problems. These can be classified into two broad categories as the non-gradient or direct search methods and the gradient or descent methods. In view of the non-availability of explicit gradients and the discussion on the relative merits and demerits of the various methods presented in Chapter 1, in the present thesis direct search method of unconstrained minimization has been adopted as it requires evaluation of only objective function and no partial derivatives. There are many direct search techniques available in the literature. From the studies made and reported in the review of literature in Chapter 1 it may be said that out of all the nongradient or direct search techniques Powell's conjugate direction method (1964) is the most

techniques Powell's conjugate direction method (1964) is the most promising and efficient. Of all the gradient based algorithms the Davidon -Fletcher-Powell (DFP) technique is probably the best. Although Powell's method has been mostly used in this thesis, an attempt to solve a problem by the DFP method using derivatives from finite difference has been demonstrated.

For the sake of ready reference the two above mentioned algorithms are presented in the following sections.

2.7.3.2 Powell's Method of Conjugate Direction

Powell's algorithm is an extension of the idea of a pattern move. Fox (1971) describes the method in geometric terms as follows :

> "Given that the function has been minimized once in each of the co-ordinate directions and then in the associated pattern direction, discard one of the co-ordinate directions in favour of the pattern direction for inclusion in the next m minimizations, since this is likely to be a better direction than the discarded co-ordinate direction. After the next cycle of minimizations, generate a new pattern direction and again replace one of the co-ordinate directions".

The basic method has a tendency to choose nearly dependent directions in ill conditioned problems to such an extent that in problems containing many variables, there may be extreme numerical difficulties. One simple remedy is to reset the directions to the original co-ordinate vectors periodically and/or whenever there is an indication that the directions are no longer productive.

The iteration is terminated when the relative change in function value and/or the change in all the design variables in between two consecutive cycles of minimization is less than the desired accuracy.

In most of the problems Powell's original algorithm as
cussed above works successfully. However, for severely ill ditioned problems resetting the search directions to the ginal co-ordinate vectors as suggested may not improve its 'iciency and the algorithm may fail to find the minimum. To rcumvent this Powell suggested an alternative scheme in which e following steps are added to the basic procedure. The scheme known as Powell's modified or rigorous procedure.

gorous Powell Procedure :

The following steps are involved :

(i) Find the minimizing step lengths λ_i^*, $i = 1, 2, \ldots, n$ so that $f(D_i+\lambda_i^* S_i)$ is a minimum along the direction S_i and define

$D_{i+1} = D_i+\lambda_i^* S_i$

(ii) Find the integer m, $1 \le m \le n+1$, so that the quantity $\Delta \equiv f(D_{m-1}) - f(D_m)$ is a maximum

(iii) Calculate $f_3 = f(2D - Z) = f(2D_{n+1}-D_1) \equiv f(D_{n+1} + S_{n+1})$ and define $f_1 = f(Z) = f(D_1)$ and $f_2 = f(D) = f(D_{n+1})$

(iv) If either $f_3 \le f_1$ and/or

$(f_1 - 2f_2 + f_3)(f_1 - f_2 - \Delta)^2 \ge \frac{1}{2} \Delta (f_1 - f_2)^2$

Use the old set of directions $S_1, S_2, \ldots, S_n$ for the next cycle of minimizations and use D_{n+1} for the next D_1. Otherwise, go to the next step.

(v) Define the new search direction $S = (D - Z) = (D_{n+1}-D_1)$ and calculate λ^* so that $f(D_{n+1} + \lambda^* S)$ is a minimum along the direction S. Then start the next cycle of minimizations by taking $D_{n+1} + \lambda^* S$ as the starting

point and $S_1, S_2, \ldots, S_{m-1}, S_{m+1}, S_{m+2}, \ldots S_n$, S as the set of search directions.

In this procedure, step (ii) essentially determines the direction, if any, to be replaced and the steps (iii) and (iv) determine the new direction, if any, to be added.

2.7.3.3 Davidon - Fletcher - Powell Method

The method also known as variable metric method was originally suggested by Davidon (1959) and was further extended and sharpened by Fletcher and Powell (1963). This method is probably the best general purpose unconstrained optimization technique making use of the derivatives that are currently available. The iterative procedure of this method can be stated as follows :

(i) Start with an initial point $\mathbf{D}_1$ and a n x n positive definite symmetric matrix $\mathbf{H}_1$. Usually $\mathbf{H}_1$ is taken as the identity matrix $\mathbf{I}$. Set iteration number as i=1

(ii) Compute the gradient of the function ∇f_i at the point $\mathbf{D}_i$ and set

$$S_i = - \mathbf{H}_i \nabla f_i$$

(iii) Find the optimal step length λ_i^* in the direction S_i an set

$$\mathbf{D}_{i+1} = \mathbf{D}_i + \lambda_i^* S_i$$

(iv) Test the new point $\mathbf{D}_{i+1}$ for optimality. If $\mathbf{D}_{i+1}$ i optimal terminate the iterative process. Otherwise, to step (v)

(v) Update the H matrix as

$$\mathbf{H}_{i+1} = \mathbf{H}_i + \mathbf{M}_i + \mathbf{N}_i$$

where

$$M_i = \lambda_i^* \frac{S_i\ S_i^T}{S_i^T\ Q_i}$$

$$N_i = -\frac{(H_i\ Q_i)(H_i\ Q_i)^T}{Q_i^T\ H_i\ Q_i}$$

and $Q_i = \nabla f\ (D_{i+1}) - \nabla f\ (D_i) = \nabla f_{i+1} - \nabla f_i$

(vi) Set the new iteration number 1 = i+1 and go to step (ii). The basic algorithm is very powerful for a first order method converging quadratically and possessing very good stability

2.7.4 One Dimensional Minimization

2.7.4.1 General

The basic philosophy of most of the numerical methods of optimization is to produce a sequence of improved approximations to the optimum as follows :

(i) Start with an initial trial point D_1.

(ii) Find a suitable direction S_i (i=1 to start with) which points to the general direction of minimum.

(iii) Find an appropriate step length λ_i^* for movement along the direction S_i .

(iv) Obtain the new approximation D_{i+1} as

$$D_{i+1} = D_i + \lambda_i^* S_i$$

(v) Test whether D_{i+1} is optimum. If D_{i+1} is optimum, stop the procedure. Otherwise, set new i=i+1 and repeat step (ii) onwards.

If f(D) is the objective function to be minimized the problem of finding λ_i^* boils down to finding the minimizing step length $\lambda_i = \lambda_i^*$ for fixed values of D_i and S_i. Since f becomes a function of one

variable λ_i only, the method of finding λ_i^* is called one-dimensional minimization.

The numerical methods of one-dimensional minimization can be broadly classified into two categories viz., elimination methods and interpolation methods. Interpolation methods are of two types, namely, methods requiring no derivative and methods requiring derivatives.

The Quadratic interpolation technique belongs to the former and the Cubic interpolation technique belongs to the latter type. Both the methods are described in detail in standard textbooks on optimization techniques; however for the sake of ready reference a brief outline of the two methods are presented below.

2.7.4.2 Quadratic Interpolation Technique

In this technique, the function $f(\lambda)$ is approximated by a quadratic function $h(\lambda)$ which has an easily determinable minimum point. $h(\lambda)$ is expressed as

$$h(\lambda) = a + b\lambda + c\lambda^2,$$

the minimum of which occurs when

$$\frac{dh}{d\lambda} = b + 2c\tilde{\lambda}^* = 0$$

or $$\tilde{\lambda}^* = -\frac{b}{2c}$$

The constants a, b, c of the approximating quadratic can be determined by sampling the function at three different λ values λ_1, λ_2 and λ_3 and solving the equations;

$$f_1 = a + b\lambda_1 + c\lambda_1^2$$

$$f_2 = a + b\lambda_2 + c\lambda_2^2$$

$$f_3 = a + b\lambda_3 + c\lambda_3^2$$

where f_1 denotes $f(\lambda_1)$ and so on.

$c = [(f_3 - f_1)(\lambda_2 - \lambda_1) - (f_2 - f_1)(\lambda_3 - \lambda_1)]/[(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)(\lambda_2 - \lambda_1)]$

$b = (f_2 - f_1)(\lambda_2 - \lambda_1) - c(\lambda_2 + \lambda_1)$

$a = f_1 - b\lambda_1 - c\lambda_1^2$

$\tilde{\lambda}^*$ is considered to be sufficiently good approximation if $f(\tilde{\lambda}^*)$ and $h(\tilde{\lambda}^*)$ differ by a very small amount. For example,

$$\left| h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*) \right| \le \varepsilon_1$$

or,

$$\frac{\left| h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*) \right|}{\left| h(\tilde{\lambda}^*) \right|} \le \varepsilon_2$$

where ϵ_1 and ϵ_2 are small numbers to be specified depending on the accuracy desired. If the convergence criteria stated above are not satisfied, the quadratic polynomial is refitted to get a better approximation to λ^*.

2.7.4.3 Cubic Interpolation Technique

In this technique, the function $f(\lambda)$ is approximated by a cubic polynomial $h(\lambda)$ given by:

$$h(\lambda) = a + b\lambda + c\lambda^2 + d\lambda^3$$

The constants a,b,c and d of the approximating cubic can be determined by sampling the function and its derivatives at two points. As in the case of Quadratic interpolation, the point $\tilde{\lambda}^*$ is considered to be a sufficiently good approximation to λ^* if one or both of the following convergence criteria are satisfied.

$$\left| \frac{h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)}{f(\tilde{\lambda}^*)} \right| \le \varepsilon_1$$

$$\left| \frac{S^T \nabla f}{|S| \; |\nabla f|} \right|_{\tilde{\lambda}^*} \le \varepsilon_2$$

where ε_1 and ε_2 are small numbers whose value depend on the desired accuracy.

If the above mentioned convergence criteria are not satisfied, the polynomial is refitted.

In this thesis, due to the non-availability of explicit gradients, it is the Quadratic interpolation (the Quadratic fit) which has been mostly used. A more detailed discussion on this will be presented in Chapter 4.

2.8 SCALING OF VARIABLES

A technique which improves the convergence of nearly all minimization methods is the scaling of the variables. It is ordinarily considered a good practice in engineering to intelligently nondimensionalize the variables of a problem to improve the numerical behaviour of the solution. Fox (1971) has described procedures for scaling in quadratic and nonquadratic problems. Möe (1973) observes that in general purpose unconstrained minimization routines, it is advantageous to work with scaled variables

$$d'_i = C_i \; d_i \qquad i = 1, \ldots, n_{dv}$$

where C_i, $i = 1, \ldots, n_{dv}$ are the scaling factors. There is n(easy way to find the optimal scaling factors. The theory o: scaling employs the matrix of second derivative which is no generally available. In practice the variables may be scaled s

that they are all of the same order of magnitude. It has been found to be satisfactory to select

$$C_i = \frac{1}{d_i^o}$$

where d_i^o is the selected initial value of d_i. If the initial value is reasonably close to the optimum value, the scaled variables will vary in the neighbourhood of one.

It has also been found to be a good practice to normalize the constraints so that they vary between -1 and Ø as far as possible for better convergence.

If the variables and the constraints are not normalized the problem can still be solved effectively by choosing different step lengths for different variables and different penalty parameters for different constraints (Rao, 1984).

In this thesis, as will be discussed in detail in chapter 3, two minimization schemes namely, the Direct and the Indirect procedures for critical shear surface determination have been developed and used. In the Indirect procedure, the design vector consists of the slip surface co-ordinates only and hence there is not much of scale disparity among the design variables. This allows a common step length of Ø.1 to Ø.ØØ1m for low to medium height (about 3Ø to 4Øm) slopes. For very high slopes with more than 1ØØm height there is likely to be a difference of the order of $1Ø^2$ between the minimum and maximum ordinates of the slip surface. In such cases, generally the design variables have been normalized with respect to either the height H_t of the embankment or x_T the horizontal projection of the slope and a common step length Ø.ØØ1 has been adopted. In the Direct procedure (based on Spencer's method) the design vector consists

of the factor of safety F of the slope and the interslice force angle θ in addition to the slip surface co-ordinates. In the application of this procedure the step lengths may be chosen in two ways :

(i) By choosing a step length of 0.1m or 0.01m for non-normalized variables and 0.001 for normalized variables representing slip surface co-ordinates; a step length of 0.001 may be chosen for F and θ considering the sensitivity of external force (Z_n) and moment (M_n) to these parameters (this aspect will also be discussed in detail in chapter 3).

(ii) Alternatively,by multiplying F and θ by suitable factors so as to raise them to the average order of the variables representing the slip surface co-ordinates and then adopt a common step length.

It will be demonstrated at the appropriate places in the following chapters that normalization of design vectors may not always lead to a better result.

2.9 ON ASSUMPTIONS USED AND PRESENTATION OF RESULTS

1. In the studies involving Spencer's method, reported in this thesis, the interslice forces have been assumed to be parallel (i.e., k = 1 throughout) unless otherwise mentioned.

2. The obtained values of the ordinates of non-circular shea surfaces have been joined by smooth curves unless when the numb of slices are too few (6-10) in which cases they have been join by straight line segments.

3. In presenting the values of design variables at the starting point, in all cases their original values are shown irrespective of whether they are normalized or not. This is done so that from these values the initial trial slip surface can be traced. At the optimal point, however, the values presented are those actually obtained from the program output. In cases they are normalized, the actual values can be obtained by multiplying the presented values by the corresponding values at the starting point.

4. In presenting the values of various constraint functions the figures underlined indicate violated constraints.

5. The calculated responses associated with a computed critical shear surface e.g., positions of lines of thrust, interslice forces and stresses at the base of each slice are presented in the form of tables rather than in plots to show more clearly the actual values obtained as the output of the computer program.

6. Different notations and conventions have been used in dealing with the Janbu's and Spencer's methods. This has been done with a view to keep the original notations and conventions unchanged as far as possible for quick comparison and interpretation of results obtained by using those methods.

7. Although Janbu (1973) defined a potential slip or a sliding surface as a failure surface, in this thesis no distinction has been made in the meaning of 'shear surface' or a 'slip surface'; both have been used to denote 'one out of an infinite number of possible shapes and locations of such surfaces'. The shear

surface that gives the minimum factor of safety is termed the critical shear surface or the critical slip surface.

8. Unless otherwise mentioned, SI units have been followed. According to this, the stresses σ and τ have the units kPa or kN/m^2 whereas the forces E,T or Z, Z', Z_n (as applicable) have the units of kN/m and the unit of moment viz., M_n is kN-m/m. Among the material properties, c' is in kPa, γ is in kN/m^3 ; the unit of length being meter. The interslice force angle θ has been expressed in radian unless otherwise mentioned.

In the following chapters, in all tables presenting the values of design variables and constraints at the starting and optimal points, the respective units of the various quantities are as mentioned above.

CHAPTER 3

GENERAL FORMULATION OF THE PROPOSED TECHNIQUE

3.1 INTRODUCTION

Slope stability analysis is essentially a problem of optimization which, physically, can be stated as follows:

Find the shape and location of that slip surface called the critical slip surface which corresponds to the minimum factor of safety subject to the conditions that the shape of the critical slip surface is physically reasonable and the obtained solution satisfies some acceptability criteria.

The optimization problem stated above can be cast as a mathematical programming problem of the following general form:

Find **D**

such that $f(\mathbf{D}) \longrightarrow \text{Min.}$

subject to : $g_j(\mathbf{D}) \leq 0 \qquad j = 1,2,\ldots\; n_{icon}$

$l_j(\mathbf{D}) = 0 \qquad j = 1,2,\ldots\; n_{econ}$

where $\mathbf{D}$ = the vector of design variables

$= [\, d_1,\ d_2, \ldots\ldots, d_i, \ldots]^T \quad i = 1,\ 2, \ldots, n_{dv}$

$f(\mathbf{D})$ = the objective function

$g_j(\mathbf{D})$ = jth inequality constraint

$l_j(\mathbf{D})$ = jth equality constraint

n_{dv} = the number of design variables

n_{icon} = the number of inequality constraints

n_{econ} = the number of equality constraints

Identification of the objective function, the design variables and design constraints

For the optimization problem under consideration the first step is to identify the objective function, the design variables and the design constraints. In slope stability analysis the above can be identified in a very general way as follows:

For a given site condition, the factor of safety of a slope of given geometry varies with the shape and location of a shear surface. The limit equilibrium methods provide explicit or implicit expressions for factor of safety as a function of a set of co-ordinates with reference to a suitably chosen axis system which completely defines the shape and location of a shear surface. The co-ordinates defining the slip surface, thus, represent the design variables. Since the objective is to minimize the factor of safety expressed as a function of these co-ordinates, it forms the objective function in this case.

To ensure a geometrically reasonable and an acceptable critical slip surface some restrictions or constraints need to be placed on the choice of the design variables. These can be generally stated as follows:

Boundary Constraints: No part of the shear surface is permitted to go beyond either the slope surface or any existing rigid stratum below. In addition, at the two ends of the shear surface its inclinations with the ground surface should not violate the earth pressure conditions - these last mentioned constraints have been referred to as the *End Constraints*.

Curvature Constraints: For a general analysis it is assumed that the potential slip surface is concave upward. In the limiting case the obtained shear surface may be a plane. This restriction is based on the field evidences for majority of slides.

Acceptability Constraints: For the solution to be physically acceptable, the implied state of stress within the sliding mass must be possible. In particular, the failure criterion within the soil mass above the slip surface must not be violated and since it is commonly accepted that soils do not take tension, no state of tension must be implied to exist above the slip surface.

Evaluation of the Objection function and the constraints

Once the design variables, the objective function and the design constraints are identified, the next step is to evaluate the objective function and the constraint functions. The limit equilibrium factor of safety varies both in magnitude and in complexity of computation with the method of analysis - whether it is valid for circular or general or non-circular slip surface etc. The evaluation of the objective function is, thus, method dependent.

The general principle of the proposed technique is such that it can be used for analysing both circular and non-circular or general slip surfaces. For the analysis using non-circular shear surfaces, two discretization models have been considered which are referred to as Model I and Model II. In the Model I, the series of discrete points representing the slip surface are

equally spaced (which means all the slices have the same width) and are allowed to shift both horizontally and vertically during the search for the critical slip surface. In the Model II, unlike Model I, the discrete points can have predetermined spacings not necessarily uniform (which means the slices are not necessarily of the same width) and are allowed to shift only vertically during the search.

For analyses using Spencer's method, an efficient numerical scheme has been proposed to search for and directly arrive at the critical shear surface without going through the process of solving for the factors of safety of a large number of trial surfaces. In this scheme the factor of safety F and the interslice force inclination θ are also treated as design variables in addition to the slip surface co-ordinates. This new procedure of critical surface determination is referred to as the Direct procedure while the conventional procedure is referred to as the Indirect procedure. In addition an efficient numerical method to solve the Spencer's stability equations has also been proposed.

The exact forms of the design variables as well as of the constraint functions also vary depending on the method of slice, the nature of slip surface and the discretization model used in the analysis. Such variations make it necessary to go for methodwise treatment in formulating the problem.

In the following sections, detailed formulations taking care of the above aspects have been presented.

The organization of the present chapter is shown in a flow chart (Figure 3.1).

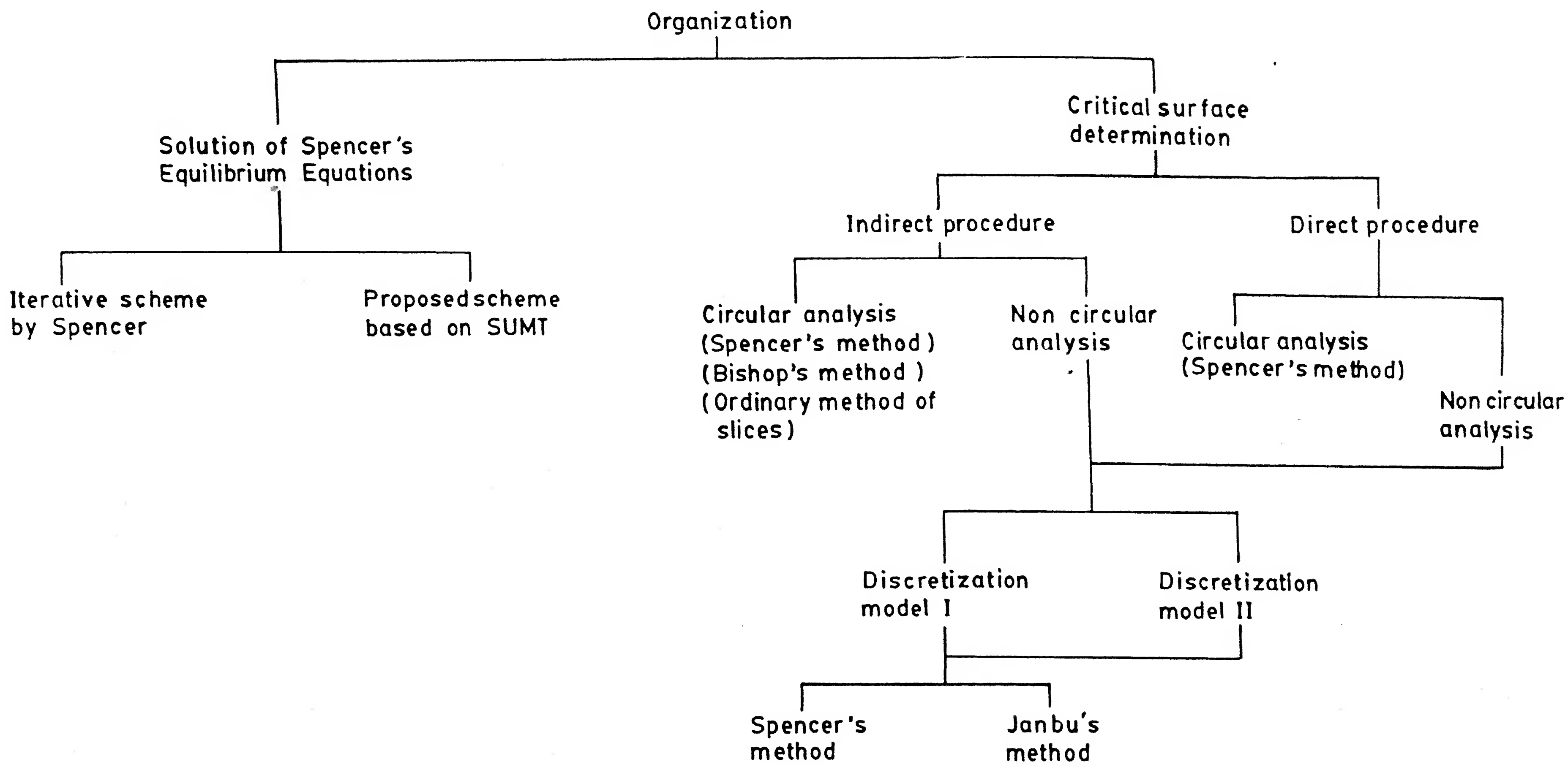


FIG. 3.1 FLOW CHART SHOWING THE ORGANIZATION OF CHAPTER 3

For the sake of simplicity, the general formulation of the developed technique has been presented with reference to homogeneous slopes. The technique, however, is equally applicable to nonhomogeneous soil conditions such as slopes in layered soils, zoned dams etc. Any number of soil layers, pore pressure variations, anisotropic and nonhomogeneous undrained shear strength variations, earthquake effects as well as nonlinear strength envelope can be taken into account. All these aspects will be discussed in detail in subsequent chapters.

3.2. FORMULATION I : THE INDIRECT PROCEDURE

3.2.1 General

In the indirect or the conventional procedure, a large number of trial surfaces are generated by the program and their factors of safety values are evaluated. Among these surfaces the one with the lowest factor of safety is taken as the critical slip surface provided all the relevant criteria are satisfied. The formulation of the design variables, the objective function and the design constraints are presented in the following sections. In all cases the design variables may be normalized as discussed in Chapter 2.

3.2.2 Circular Analysis: Design Variables and Constraints

Referring to Figure 3.2, it is seen that the slip circle is completely specified by x_o, y_o, R and z_t where x_o, y_o and R represent the x and y co-ordinates of the centre and radius of the slip circle respectively and z_t is the depth of the assumed tension

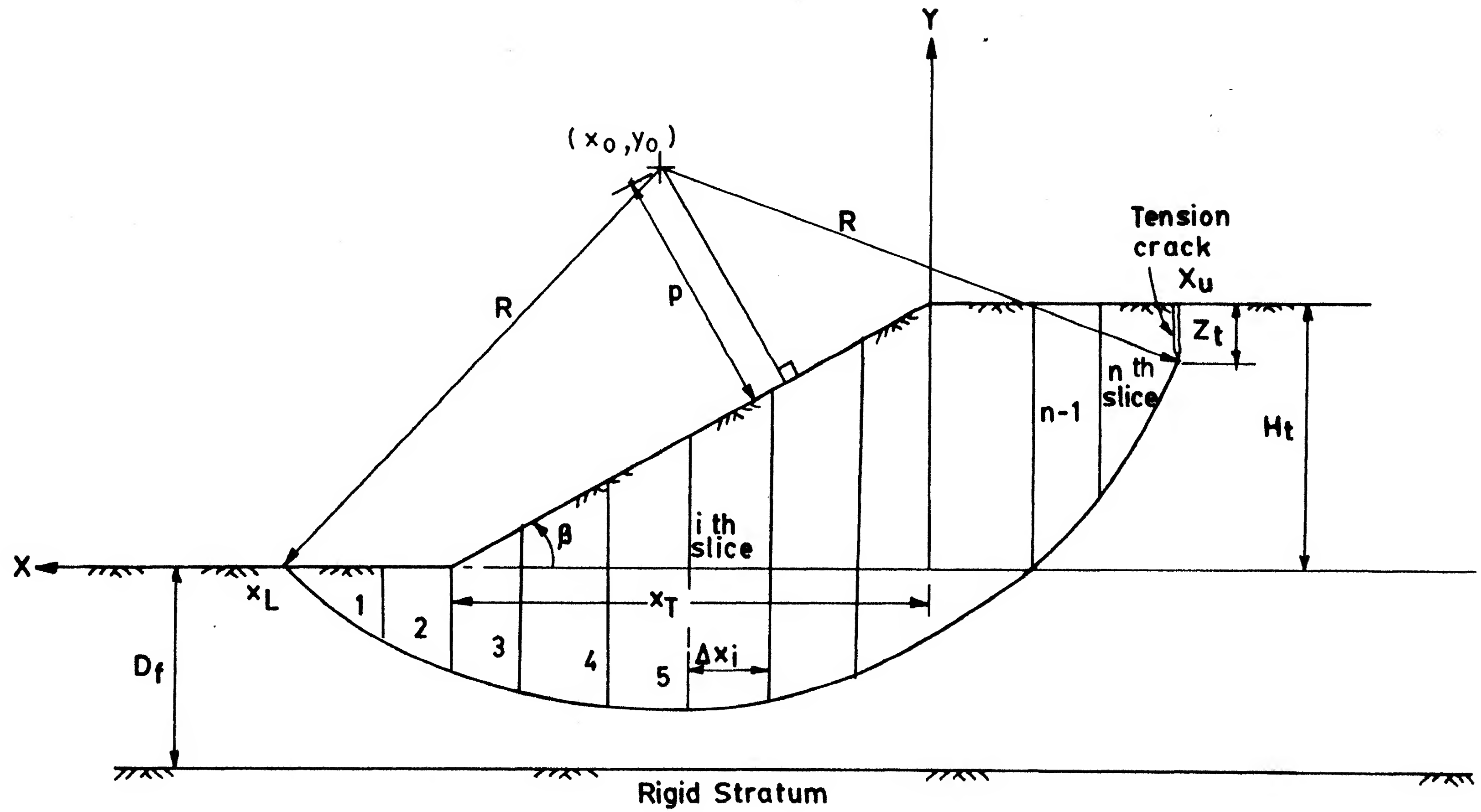


FIG. 3.2 HOMOGENEOUS SLOPE WITH A CIRCULAR SLIP SURFACE

crack. The factor of safety of the slope can be expressed as a function of the above mentioned parameters. The search for the location of the critical slip surface is to find these co-ordinates such that the factor of safety is the minimum.

The design vector in this case is, therefore, as follows:

Case I : When z_t is a design variable

$$D = [\, x_o,\ y_o,\ R,\ z_t \,]^T \qquad \text{and } n_{dv} = 4$$

Case II : When z_t is not a design variable,

$$D = [\, x_o,\ y_o,\ R \,]^T \qquad \text{and } n_{dv} = 3$$

The number of design variables n_{dv} in slip circle analysis is thus independent of the number of slices n. The slices are of uniform width given by

$$\Delta x_i = b = \frac{x_L - x_U}{n} \qquad i = 1,2,\ldots\ldots,n$$

Where x_L and x_U are the x-co-ordinates of the lower and upper intersection points of the slip surface with the ground surface. The expressions for x_L and x_U for the three possible types of failures viz., base failure, toe failure and slope failure are given as follows:

1. For base failure:

$$x_L = x_o + \sqrt{R^2 - y_o^2} \qquad \text{and} \qquad x_U = x_o - \sqrt{R^2-(H_t-y_o)^2}$$

2. For toe failure:

$$x_L = H_t \cot \beta \qquad \text{and} \qquad x_U = x_o - \sqrt{R^2-(H_t-y_o)^2}$$

3. For slope failure:

$$x_L = -\frac{a + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_U = x_o - \sqrt{R^2 - (H_t - y_o)^2}$$

where, $$a = 1 + \frac{H_t^2}{x_T^2}$$

$$b = -2x_o - \frac{2H_t^2}{x_T} + \frac{2H_t y_o}{x_T}$$

$$c = x_o^2 + y_o^2 + H_t^2 - 2H_t y_o - R^2$$

H_t = height of the slope

x_T = x-co-ordinate of the toe

The constraints associated with a slip circle analysis are formulated in this section. As defined in Section 3.1, g_j stands for the jth inequality constraint where the counter j varies from 1 to n_{icon}, the total number of inequality constraints.

Boundary Constraints

1 At least a portion of the slip circle must lie within the slope. This condition will be satisfied if:

(i) The radius of the slip circle is greater than the perpendicular distance of the inclined face of the slope from the centre of the slip circle. Referring to Figure 3.2, this is mathematically expressed as:

$$p - R \leq 0$$

i.e., $$g_j(D) = p - R$$

or, normalizing, $$g_j(D) = \frac{p}{R} - 1$$

where p is the perpendicular distance from the centre of the slip circle to the sloping surface and is given by:

$$p = \frac{(H_t/x_T)x_o + y_o - H_t}{\sqrt{1 + (H_t/x_T)^2}}$$

(ii) The bottom intersection point lies to the left of y-axis This is expressed as,

$$x_L \geq 0$$

i.e., $$g_j(D) = - x_L$$

or, normalizing with respect to H_t,

$$g_j(D) = - \frac{x_L}{H_t}$$

(iii) The x co-ordinate of the centre of the circle also lies to the left of y-axis. This means,

$$x_o \geq 0$$

i.e., $$g_j(D) = - x_o$$

or, normalizing with respect to H_t,

$$g_j(D) = - \frac{x_o}{H_t}$$

2. The slip circle should be prevented from penetrating an existing rigid stratum below. This condition will be satisfied provided,

$$R \leq Y_o + D_f$$

Where D_f is the depth of the rigid stratum (Figure 3.2).

Or, normalizing, $$g_j(D) = \frac{R}{Y_o + D_f} - 1$$

3. End constraints or the maximum obliquity constraints:

In the case of circular slip surface, since the shape is pre-assigned, these constraints are not relevant.

4. Constraint on depth of tension crack:

When z_t is treated as a design variable, it is limited to a maximum depth z_o given by Equation (2.31). This means,

$$z_t \le z_o$$

i.e., $$g_j(D) = z_t - z_o$$

or, normalizing, $$g_j(D) = \frac{z_t}{z_o} - 1$$

Curvature constraints

These constraints are also not relevant here.

Acceptability constraints

The acceptability constraints will be discussed separately at a later section.

Side Constraints

(i) Since a factor of safety value of unity indicates incipient failure condition, there is no need to search for slip surfaces which yield factor of safety values less than unity. This can be effected by applying a constraint as follows:

$$F \ge 0.99$$

$$g_j(D) = -F + 0.99$$

(ii) The interslice slope inclination θ cannot be negative. This means,

$$\theta \ge 0$$

i.e., $$g_j(D) = -\theta$$

or, normalizing with respect to β, where β is the slope angle

$$g_j(D) = -\frac{\theta}{\beta}$$

(iii) Since θ normally does not exceed β (Spencer, 1967), a constraint may be put as follows:

$$\theta \leq \beta$$

i.e.,
$$g_j(D) = \theta - \beta$$

or, normalizing, again, with respect to β

$$g_j(D) = \frac{\theta}{\beta} - 1$$

3.2.3 Non-circular Analysis: Discretization Model I

As stated in Chapter 2, two methods have been considered for the analysis of general or non-circular slip surfaces, namely, the Spencer method (Spencer, 1973) and the Janbu's GPS (Janbu, 1973). For such analysis wherein no a priori assumption regarding the shape of the shear surface is involved, two discretization models, the Model I and the Model II, have been considered and these are described in the following sections. Discretization Model I has been taken up first.

3.2.3.1 Discretization Model I: Design variables and constraints for Spencer's Method of Analysis:

In the discretization Model I, shown in Figure 3.3 (a), the potential sliding mass is divided into n vertical slices of uniform width given by:

$$\Delta x_i = b = \frac{x_L - x_U}{n} \qquad i = 1,2,\ldots,n$$

Referring to Figure 3.3(a), let $y_1, y_2, \ldots y_i \ldots, y_{n+1}$ be the y co-ordinates of the shear surface at the slice boundaries. If

$x_1, x_2, \ldots x_i \ldots, x_{n+1}$ be the corresponding x co-ordinates, then, clearly,

$$y_{n+1} = z_t \quad ; \quad x_1 = x_L \quad ; \quad x_{n+1} = x_U$$

The angle that the base of the ith slice makes with the horizontal is given by

$$\tan \alpha_i = \frac{y_{i+1} - y_i}{b}$$

The shape and location of a shear surface is completely defined by y_2, $y_3, \ldots, y_i, \ldots, y_n$, z_t, x_L, x_U and the factor of safety can be expressed as a function of the above co-ordinates. The search for the critical surface is to find these co-ordinates which minimizes the factor of safety.

The design vector in this case is, therefore, as follows:

Case I : When z_t is a design variables:

$$\mathbf{D} = [\, y_2, y_3, \ldots y_i \ldots, y_n,\ x_L, x_U, z_t \,]^T \qquad \text{and} \quad n_{dv} = n+2$$

Case II : When z_t is not a design variable

$$\mathbf{D} = [\, y_2, y_3, \ldots y_i \ldots, y_n,\ x_L, x_U \,]^T \qquad \text{and} \quad n_{dv} = n+1$$

Clearly in these cases of non-circular shear surface, the number of design variables is directly proportional to the number of slices.

The following are the constraints associated with the analysis of general slip surface using discretization Model I. The formulations essentially refer to Figure 3.3(a).

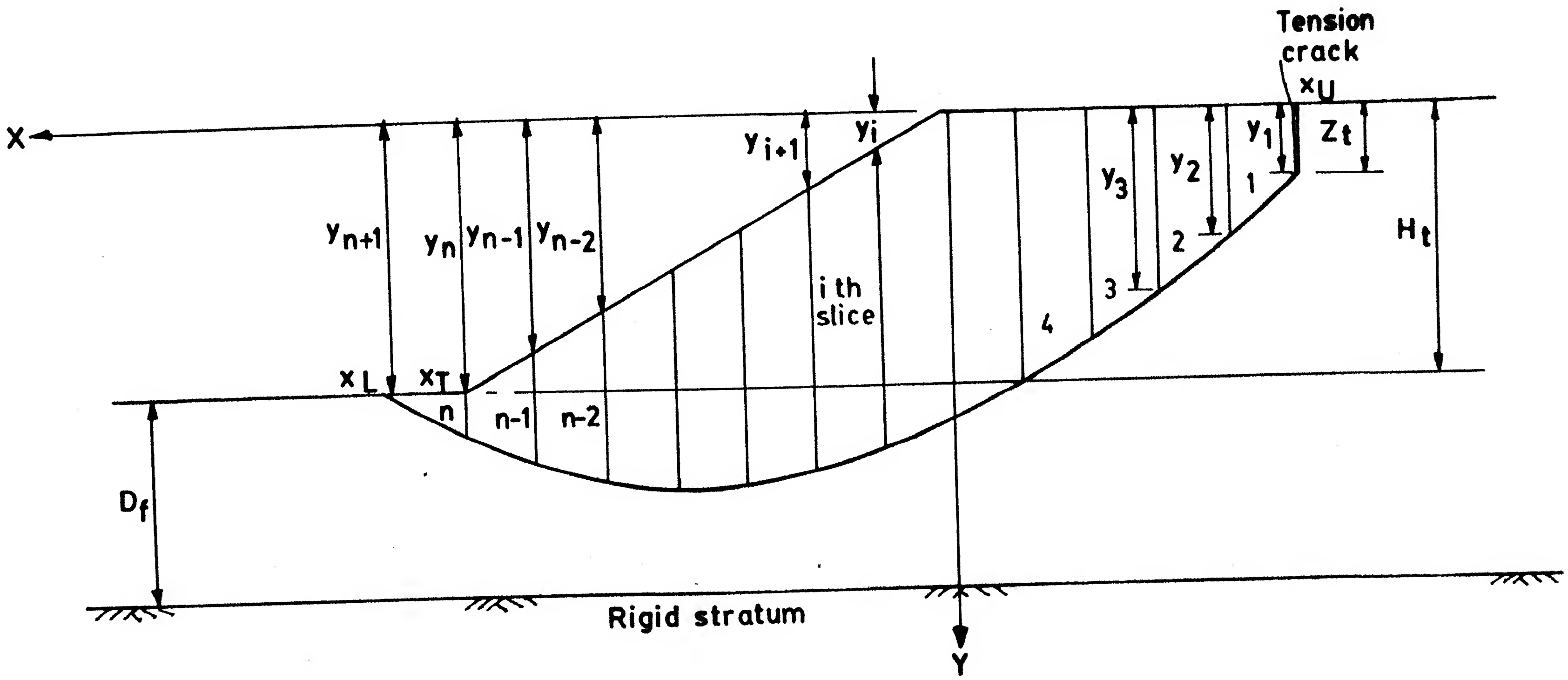


FIG. 3.3(b) DISCRETIZATION MODEL I FOR JANBU'S METHOD IN HOMOGENEOUS SLOPE WITH A NON-CIRCULAR SLIP SURFACE

Boundary Constraints

1. The shear surface must lie within the slope geometry. This will be satisfied if the following are satisfied:

(i) For the portion of the surface lying to the left of the toe, no y co-ordinate should be positive. This means,

$$y_i \leq \emptyset$$

i.e., $$g_j(D) = y_i$$

or, normalizing, $$g_j(D) = \frac{y_i}{H_t} \qquad i = 2,3,\ldots,m_1+1$$

$$j = 1,2,\ldots,m_1$$

where m_1 is the number of interslice boundaries lying to the left of toe.

(ii) For the portion of the shear surface lying within the sloping surface (including the toe), all y co-ordinates must be less than the corresponding y co-ordinates of the sloping surface. This means,

$$y_i \leq \frac{(x_T - x_i)\, H_t}{x_T}$$

i.e., $$g_j(D) = y_i - \frac{(x_T - x_i)\, H_t}{x_T}$$

or, normalizing, $$g_j(D) = \frac{y_i\, x_T}{(x_T - x_i)\, H_t} - 1$$

$$(m_1+2) \leq i \leq (m_1+m_2+1)$$

$$(m_1+1) \leq j \leq (m_1+m_2)$$

Where, m_2 is the number of interslice boundaries within the inclined portion (including the toe) of the slope surface.

(iii) For the portion of the shear surface to the right of y-axis, no y co-ordinates should be greater than the height of the slope. This means,

$$y_i \leq H_t$$

i.e., $$g_j(D) = y_i - H_t$$

or, normalizing, $$g_j(D) = \frac{y_i}{H_t} - 1$$

$$(m_1+m_2+2) \leq i \leq n$$
$$(m_1+m_2+1) \leq j \leq n-1$$

Obviously, the upper boundary constraints in items (i), (ii) and (iii) need actually be applied for all $y_i \geq \emptyset$.

2. The shear surface should be prevented from penetrating a rigid stratum below. Assuming that the rigid stratum boundary is a horizontal one at a depth D_f, as in Figure 3.3(a), the above requires (for all $y_i < \emptyset$):

$$| y_i | \leq D_f$$

i.e., $$g_j(D) = | y_i | - D_f$$

or, normalizing, $$g_j(D) = \frac{| y_i |}{D_f} - 1$$

When the hard stratum boundary is an irregular one, the above requirement may be expressed as :

$$| y_i | \leq | z_i^f |$$

where z_i^f represents the corresponding ordinates of the irregular rigid boundary.

If, however, the lowest point y_M of this boundary is found, then, instead of putting a constraint on all -ve y_i, only one constraint would do. This can be expressed as:

$$|y_M| \leq D_f$$

An alternative to imposing such constraints is to consider a layer with unusually high values of strength properties.

3. When the depth of tension crack is a design variable, as discussed in the case of slip circle analysis,

$$z_t \leq z_o$$

i.e., $$g_j(D) = z_t - z_o$$

or, normalizing, $$g_j(D) = \frac{z_t}{z_o} - 1$$

4. End constraints or the maximum obliquity constraints:

The slopes of the shear surface at its intersections with the ground surface are supposed to satisfy active and passive earth pressure conditions, as is appropriate. This requires the following equalities to be satisfied:

$$|\alpha_1| = \frac{\pi}{4} + \frac{\phi_m'}{2}$$

and $$\alpha_n = \frac{\pi}{4} - \frac{\phi_m'}{2}$$

Where, α_1 and α_n are the base slopes of the first and the last slices respectively and ϕ_m' is the mobilized, angle of shearing resistance given by

$$\tan \phi_m' = \frac{\tan \phi'}{F}$$

The above constraints are based on the assumption that at both ends of the shear surface the ground surface is horizontal

in which case they become planes of principal stresses. If a particular end intersects a slope surface which is not horizontal or passes through the toe, the corresponding constraint is not applicable.

In many cases, however, these two equality constraints may become too stringent to allow a smooth progress of the optimization. Smooth progress of the search scheme can be maintained by considering inequality constraints of the following types.

$$|\alpha_1| \leq \left(\frac{\pi}{4} + \frac{\phi_m'}{2}\right)$$

i.e.,
$$g_j(D) = |\alpha_1| - \left(\frac{\pi}{4} + \frac{\phi_m'}{2}\right)$$

and
$$\alpha_n \leq \left(\frac{\pi}{4} - \frac{\phi_m'}{2}\right)$$

i.e.,
$$g_j(D) = \alpha_n - \left(\frac{\pi}{4} - \frac{\phi_m'}{2}\right)$$

Curvature Constraints

For the shear surface to be concave upward the followin relationship must be satisfied.

$$y_i \leq \frac{y_{i-1} + y_{i+1}}{2}$$

or
$$-(y_{i-1} - 2y_i + y_{i+1}) \leq \emptyset \qquad i = 2, 3, \ldots, n$$

i.e.,
$$g_j(D) = -(y_{i-1} - 2y_i + y_{i+1}) \qquad j = 1, 2, \ldots, n$$

These may also be normalized with respect to H_t as:

$$g_j(D) = -\frac{y_{i-1} - 2y_i + y_{i+1}}{H_t}$$

Field evidence suggests that in most of the case imposition of the above restriction is reasonable (Bell, 196 However, the shear surface sometimes may be a composite cur

surface governed by special geologic features (Janbu, 1973); in such cases problem formulation will be different in generating the curvature constraints.

Side Constraints

As discussed earlier in connection with the slip circle analysis, some side constraints on F and θ may be imposed.

Acceptability Constraints

These will be separately dealt with in a later section.

3.2.3.2 Discretization Model I : Design Variables and Constraints for Janbu's Method of Analysis

For analyses using Janbu's method as shown in Figure 3.3(b), a different convention has been followed regarding the choice of axis system and the numbering of the slices. This has been done to implement Janbu's original analysis as it is. Due to this difference, there will be minor modifications in the general formulations as already presented with reference to the Spencer's method and they have been incorporated in the computer program developed for the purpose. However, for the sake of brevity and to avoid repetition the modified versions of the formulations have not been presented here.

3.2.4 Non-circular Analysis: Discretization Model II

In this section the discretization Model II is explained first and then the detailed formulations of the design variables and constraints are presented with reference to the Model II. As before, analyses using Spencer's and Janbu's methods have been dealt with individually.

3.2.4.1 Discretization Model II: Design Variables and Constraints for Spencer's Method of Analysis

In the discretization Model II shown in Figure 3.4(a) an estimated search region is divided into n vertical slices. In contrast to the Model I, here the (n+1) vertical slice boundaries are suitably chosen and their positions are kept unchanged throughout the search. The widths of the n slices, therefore, also remain unchanged and need not be uniform.

Referring to Figure 3.4(a), let $y_1, y_2, \ldots, y_{n+1}$ be the ordinates of the shear surface at the fixed slice boundaries marked 1,2...,(n+1) respectively. If there is no tension crack, the shape and location of the shear surface marked $S_1, S_2 \ldots, S_{n+1}$ is completely defined by $y_1, y_2, \ldots, y_{n+1}$. But the points, S_1, S_2 on the left and S_n, S_{n+1} on the right fall outside the slope boundary and hence they have no physical significance. The actual intersection points A and B of the shear surface with the ground surface can be obtained by linear interpolation as follows:

Lower Intersection Point x_L :

By inspection, the slice number 'i' which contains the lower intersection point, A, is found out such that

$$y_i \geq \emptyset \geq y_{i+1}$$

and then, x_L is obtained as

$$x_L = x_i - \frac{\Delta x_i}{y_i - y_{i+1}} \cdot y_i$$

where, Δx_i represents the width of the i^{th} slice.

Upper Intersection Point x_U :

Again, by inspection, the slice number 'j' containing the upper intersection point, B, is found out such that

$$y_j \leq H_t \leq y_{j+1}$$

and then x_U is obtained as:

$$x_U = x_{j+1} + \frac{\Delta x_j}{y_{j+1} - y_j} (y_{j+1} - H_t)$$

From Figure 3.4(a) it is clear that the shape and location of a shear surface is completely defined by the ordinates of the shear surface at its intersection with the fixed slice boundaries. The design vector is, therefore, as follows:

$$D = [\, y_1\ y_2, y_3, \ldots y_i \ldots, y_{n+1}]^T \qquad \text{and} \qquad n_{dv} = n+1$$

When tension crack is assumed in the analysis, the formulation is modified as follows:

Case I : When z_t is a design variable

Referring to Figure 3.4(b), the shape and location of the shear surface $S_1, S_2, \ldots, S_{n-1}, B'$ is completely defined by y_1, y_2, y_3, $\ldots y_i \ldots$, y_k, x_U and z_t.

Therefore,

$$D = [\, y_1, y_2, \ldots y_k, x_U, z_t]^T \qquad \text{and} \qquad n_{dv} = k+2$$

where, k is such that,

$$x_{k+1} \leq x_U \leq x_k$$

For example in Figure 3.4(b), it is clear that $k = n-1$. So, in this particular case, $n_{dv} = n+1$. For some other position of the shear surface, n_{dv} might be different. Now, considering that the total number of design variables, n_{dv}, cannot be allowed to vary during the search when the slip surface takes various positions,

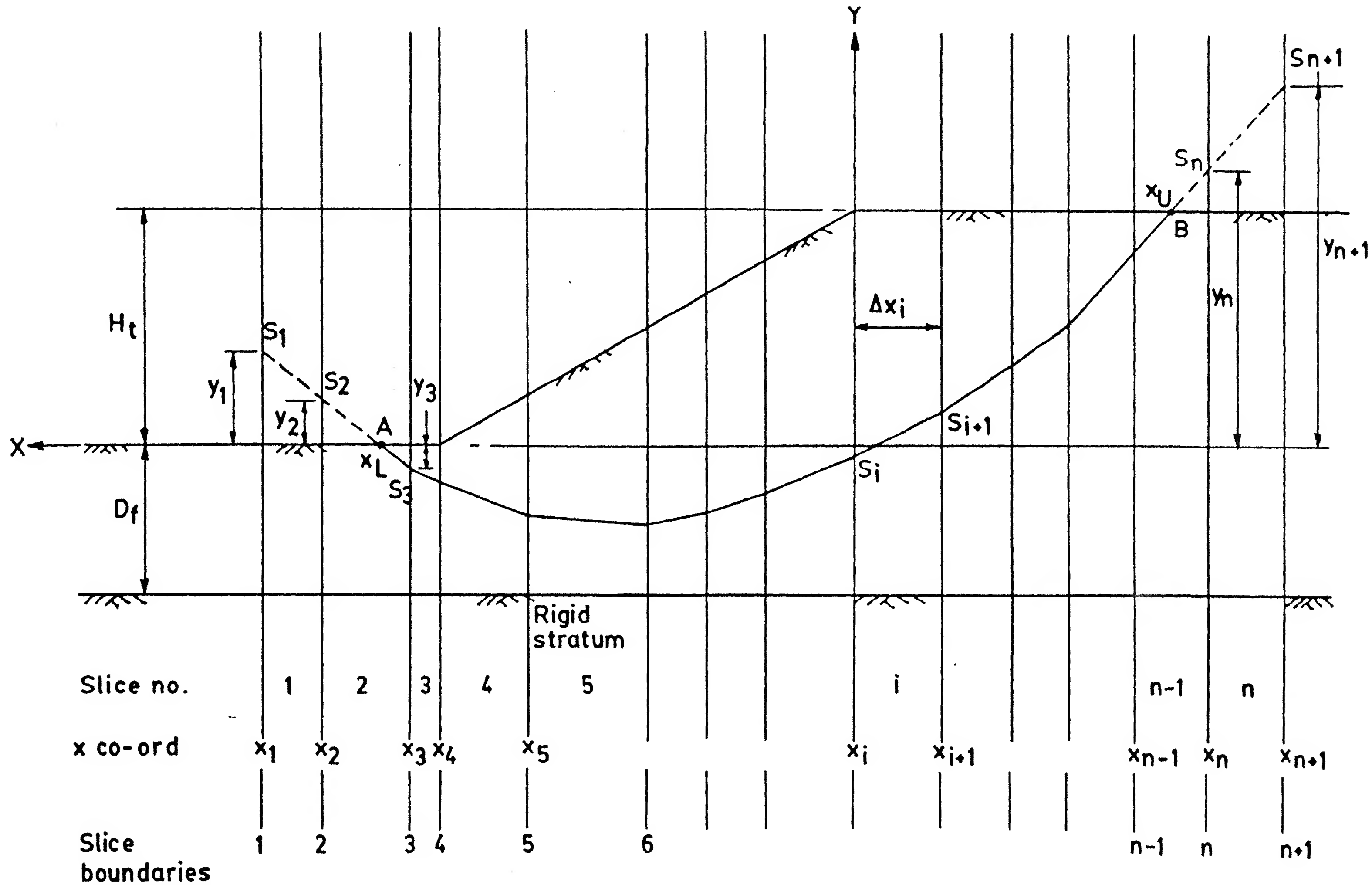


FIG. 3.4 (a) DISCRETIZATION MODEL II FOR SPENCER METHOD IN HOMOGENEOUS SLOPE WITH A NON-CIRCULAR SLIP SURFACE (No tension crack)

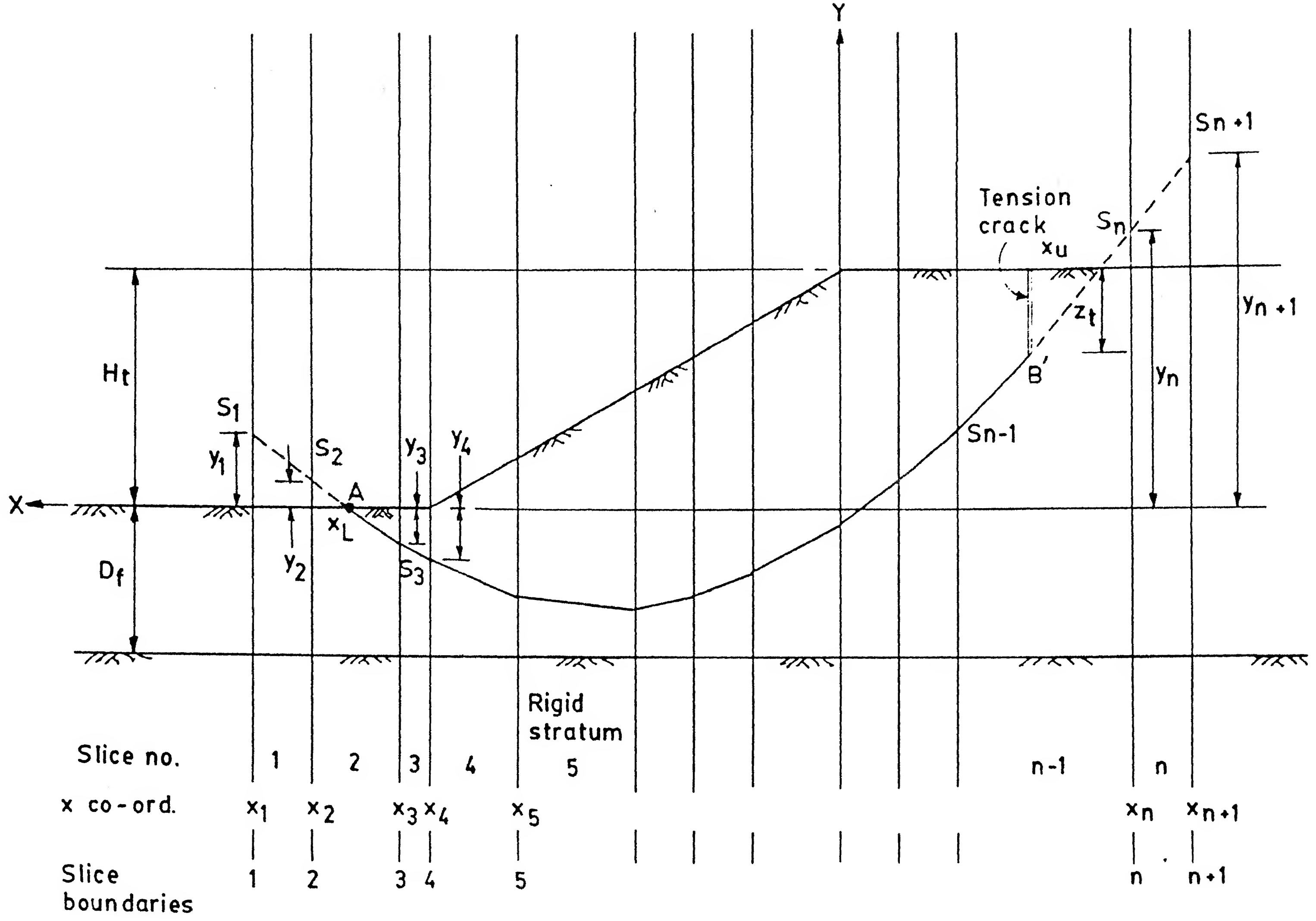


FIG. 3.4 (b) DISCRETIZATION MODEL II FOR SPENCER METHOD IN HOMOGENEOUS SLOPE WITH A NON-CIRCULAR SLIP SURFACE (Tension crack)

one needs to make an intuitive guess regarding the most likely position of the upper intersection point, i.e., the domain of x_U. The unrestricted nature of the search can still be retained by taking a large enough width for the most probable slice containing the upper intersection point.

Case II : When z_t is not a design variable

Unlike case I, in this case no such guess needs to be made. The design vector is the same as in the cases where no tension crack is assumed to be present. However, here x_U is obtained in a slightly different manner. After obtaining k such that

$$y_k \leq (H_t - z_t) \leq y_{k+1}$$

x_U is obtained from :

$$x_U = x_{k+1} + \frac{\Delta x_k}{y_{k+1} - y_k} (y_{k+1} - H_t + z_t)$$

The constraints associated with the analysis are explained and formulated as follows:

Boundary Constraints

1. The points of intersection of the shear surface with the tw extreme slice boundaries must be above the slope geometry or th ground surface. This requires:

(i) $y_1 \geq 0$

i.e., $g_j(D) = -y_1$

or, normalizing, $g_j(D) = -\frac{y_1}{H_t}$

and, (ii) $y_{n+1} \geq H_t$

i.e., $g_j(D) = -y_{n+1} + H_t$

or, normalizing, $g_j(D) = -\frac{y_{n+1}}{H_t} + 1$

2. The other kinds of boundary constraints including the maximum obliquity constraints being the same as presented in the case of Model I are not presented here to avoid repetition.

Curvature Constraints

Because of non-uniform widths of slices in Model II, the form of these constraints are somewhat more general than those in Model I. Here, the shear surface will be planar or concave upward if it satisfies the following conditions:

$$y_i \leq \frac{y_{i-1} \cdot \Delta x_i + y_{i+1} \cdot \Delta x_{i-1}}{\Delta x_{i-1} + \Delta x_i}$$

i.i., $$g_j(D) = - \left[y_{i-1} - (1+f) y_i + f \cdot y_{i+1} \right]$$

where, $$f = \frac{\Delta x_{i-1}}{\Delta x_i} \qquad i = 2,3,\ldots,n$$

Acceptability Constraints

These will be discussed separately in a later section.

Side Constraints

These are as presented and discussed earlier.

3.2.4.2 Discretization Model II : Design Variables and Constraints for Janbu's Method of Analysis

As discussed in connection with Model I, the formulation for Janbu's method is somewhat different because of different convention followed in the choice of the axis system and the numbering of the slices. However, as before, the modified formulation, though properly implemented in the computer program, is not presented here to avoid repetition.

3.2.5 Critical Comments on the Two Discretization Models

The essential difference between the two models presented above is that whereas in Model I the discrete points may shift both horizontally and vertically as the search progresses, only vertical movement is allowed in Model II excepting, however, the situation where z_t is included in the design vector in which case the upper intersection point may also shift along the ground surface. One advantage of Model II over Model I is the adoptability of unequal slice-widths. Moreover, Model II may be found more suitable and convenient in heterogeneous soil conditions such as zoned dams wherein one can use smaller slice-widths in some zones to represent more accurately the sharp variations in soil properties and steep phreatic surfaces if any.

Bell (1969), Bhowmik (1984), Dhawan (1986), Yudhbir and Basudhar (1986), Basudhar et. al. (1988) and others have reported successful use of the principle of Model I without tension crack. Celestino and Duncan (1981) have observed that the Model I works more effectively than the Model II; however, they have not substantiated their observations with data. Arai and Tagy (1985a) have reported that "reasonable solutions were hardl obtained" through Model I while Model II worked well. Here agai no comparative study has been reported and no tension crack wa incorporated in the model.

In the present thesis limited studies have been carried o using Model II in analysing the Okete Dam where there are ve abrupt changes in properties from one zone to another. The re of the studies, have been carried out using only the Model I.

3.2.6 Indirect Procedure : Formulation of the Objective Function

3.2.6.1 General

In formulating the slope stability problem as one of optimization the objective function is, as discussed earlier, the safety functional called the factor of safety which, for the given geometry of the slope and the soil and pore pressure conditions, becomes a function of the co-ordinates of a set of discrete points on the shear surface which form the design variables. The evaluation of the objective function corresponding to a given design vector, therefore, involves computation of the factor of safety associated with a shear surface, which will be discussed in the following sections.

3.2.6.2 Solution of the Factor of Safety Equations

With the exception of the Ordinary method of slices, the computation of limit equilibrium factor of safety involves solution of an equation or a pair of equations which are nonlinear in nature. The simplified Bishop Method, for instance, needs one nonlinear equation to be solved. This can be done with the help of a simple iterative scheme starting with a trial value of factor of safety equal to that given by the Ordinary method of slices which is obtained straightaway. For Janbu's GPS, the iterative scheme proposed by Janbu is generally adopted in which the starting value is generated by the iterative scheme itself.

Spencer's method (Spencer,1973) like the Morgenstern and Price method (1965) and unlike the above mentioned methods is one method which requires solution of a pair of nonlinear equations.

The solution procedures for such methods are much more involved than those for the methods requiring solution of only one factor of safety equation. Spencer (1973) has provided broad outlines of a numerical scheme to solve the force and moment equilibrium equations associated with the proposed method. The original version of the scheme has already been presented in Chapter 2 and hence not repeated here.

Referring to Section 2.4, it is noted that steps (d) and (e) as originally suggested by Spencer imply the use of an iterative method to 'adjust' F and θ until Z_n and M_n respectively are 'negligible'. Spencer, however, has not specified or recommended any particular method for the same. The well known Bisection method being simple can be used for the purpose. The method, however, is known to suffer from slow convergence. The computation time can be reduced by using the Modified Regula Falsi Technique, another good method, which has a faster convergence.

Morgenstern and Price (1967) have proposed a method based o two-variable Newton-Raphson technique for the solution of th stability equations in the Morgenstern-Price method.] principle, the same can also be applied to solve the Spencer' equations. However, the technique requires a number intermediate controls which is likely to be inconvenient a time-consuming when a large number of trial shear surfaces are be analysed in the search for critical shear surface Furthermore, reservations has been expressed regarding the use the Newton-Raphson Technique (Hamming, 1962) in finding zeroes of a function.

Apart from the time considerations, it has been reported (Soriano, 1976) that for some cases the iterative schemes proposed by Morgenstern and Price (1967), Wright (1969) and Spencer (1973) are not stable or do not converge to a proper solution.

3.2.6.3 Solution of Spencer's Equations: The Proposed Method

In view of the shortcomings of the iterative schemes used in solving the complete equilibrium methods like the Spencer's method, a new numerical solution technique is proposed herein. The nonlinear programming technique has been used to formulate the problem as follows:

Find $D = [F, \theta]^T$

such that $f(D) = Z_n^2 + M_n^2 \longrightarrow$ Min.

Subject to the non-negativity constraints

(i) $F \geq 0$

(ii) $\theta \geq 0$

Further, an upper limit on θ may also be placed as discussed earlier. All the symbols have their usual meanings.

The logic behind taking $Z_n^2 + M_n^2$ as the objective function are the following:

1. The minimum of a sum of squares is zero.
2. The sum of the squares of some terms will be zero only when each term is zero.

Thus, by minimizing $Z_n^2 + M_n^2$ the dual purpose of satisfying both the force and the moment equilibrium conditions i.e.,

$$Z_n = \emptyset$$

and $$M_n = \emptyset$$

are indirectly achieved and the optimal design vector will give the factor of safety and the interslice force angle associated with the given shear surface.

Scale Factor:

Because of the difference in the order of magnitude of Z_n and M_n the objective function $(Z_n^2 + M_n^2)$ becomes considerably eccentric and as such it takes large number of iterations to converge. In such cases it is generally advisable to introduce a scale factor to reduce the eccentricity (Fox, 1971). Therefore, a quantity S_f has been introduced such that in the modified objective function defined as,

$$f(D) = Z_n^2 + S_f . M_n^2$$

the contributions of the terms Z_n^2 and $S_f . M_n^2$ are of the same order of magnitude. Following a pertinent suggestion by Morgenstern and Price (1967), in the present analysis S_f has been adopted as:

$$S_f = \frac{\left[\dfrac{\partial Z_n}{\partial F}\right]^2 + \left[\dfrac{\partial Z_n}{\partial \theta}\right]^2}{\left[\dfrac{\partial M_n}{\partial F}\right]^2 + \left[\dfrac{\partial M_n}{\partial \theta}\right]^2}$$

Further, the objective function as a whole may be normalized a:

$$f(D) = \frac{Z_n^2 + S_f\, M_n^2}{\left[\gamma\, b_{av}\, H_t\right]^2}$$

where, b_{av} = an average width of the slices.

γ = a representative unit weight of the soil.

H_t = the height of the slope.

Expressions for Z_n, M_n and their Derivatives:

The expressions for Z_n, M_n, $\dfrac{\partial Z_n}{\partial F}$, $\dfrac{\partial Z_n}{\partial \theta}$, $\dfrac{\partial M_n}{\partial F}$ and $\dfrac{\partial M_n}{\partial \theta}$

have been derived and presented in Appendix A.

Method of Solution:

The above problem is solved by using the principle of the sequential unconstrained minimization technique. The Interior penalty function method may be used in conjunction with the Powell's method and the Quadratic fit for the multidimensional and unidimensional unconstrained searches respectively. The Interior penalty technique is applicable here because a feasible starting point is readily available.

3.3 FORMULATION II: THE DIRECT PROCEDURE

3.3.1 Principle of the Direct Procedure

The procedure of determination of critical shear surfaces for which formulation has been presented in the preceding sections has been referred to as the Indirect procedure as it involves evaluations of a large number of trial slip surfaces in the process of arriving at the critical surface. When the

analysis is carried out by methods like the simplified Bishop and Janbu's GPS the Indirect procedure is quite effective and the entire computation gets completed in a short time. The reason is that for such methods the iterative solution schemes are relatively simple and rapidly convergent. However, when it is intended to use a more sophisticated and complete equilibrium method in particular the Spencer's method (Spencer, 1973), considerable computational difficulty as well as time is involved in the process of evaluation of factor of safety of a trial slip surface, and hence in the determination of the critical slip surface. This inefficiency becomes all the more pronounced in the case of critical surface determination requiring analysis of a large number of trial surfaces.

It has, therefore, been felt that if the slope stability problem is formulated in a manner such that the critical slip surface is determined directly without going through the numerous evaluations of trial surfaces it would be very useful as far as analysis by the Spencer's method is concerned. This is likely to be achieved by treating F and θ also as design variables in addition to the slip surface co-ordinates while putting force and moment equilibrium requirements as equality constraints. The optimal design vector would now give not only the shape and location of the critical shear surface but also the factor of safety, F, and the interslice force angle, θ, associated with the critical shear surface. Since the objective in this new formulation is the same as in the Indirect Procedure discussed earlier namely, to minimize the factor of safety, the objective function is, again, the factor of safety of the slope. In th

formulation, thus, the objective function also appears as a design variable. The procedure of critical slip surface determination based on this new formulation will henceforth be referred to as the Direct procedure. The basic problem may be stated as follows:

Find the shear surface as well as the corresponding factor of safety, F, and the interslice force angle, θ, such that the factor of safety of the slope is minimized subject to the conditions that,

(i) the force equilibrium condition is satisfied i.e., $Z_n = 0$

(ii) the moment equilibrium condition is satisfied i.e., $M_n = 0$

(iii) the shape of the critical surface is geometrically reasonable and that the obtained solution satisfies some acceptability criteria.

3.3.2 Direct Procedure: Design Variables and Constraints

From the principle of the Direct procedure discussed above, it is clear that the design vector in this case is comprised of the shear surface co-ordinates, which also constitute the design variables in the Indirect procedure, plus two variables in the form of the factor of safety, F and the slope of the interslice forces, θ. There is no loss of generality if the design vector in the Direct procedure is expressed in terms of that in the Indirect procedure as follows:

$$D_{dir} = \left[D_{ind}, F, \theta \right]^T$$

and
$$n_{dv_{dir}} = n_{dv_{ind}} + 2$$

where,

D_{dir} = Design vector in the Direct procedure

D_{ind} = Design vector in the Indirect procedure

$n_{dv_{dir}}$ = No. of design variables in the Direct procedure

$n_{dv_{ind}}$ = No. of design variables in the Indirect Procedure

The above relationships hold good irrespective of whether circular or non-circular shear surfaces are analysed and in the latter case whether discretization Model I or II is used and also whether tension crack is a design variable or not. The design variable may be normalized as discussed earlier.

The complete set of constraints associated with the Direct formulation includes all the inequality constraints associated with the Indirect formulation. In addition, two equality constraints are also present in the form of the two equilibrium conditions ($Z_n = 0$; $M_n = 0$) which are, however, inherent in the formulation itself. The inequality constraints have been discussed in detail in connection with the Indirect procedure and hence are not presented here.

Equality Constraints

The two equilibrium equations may be expressed in the form of equality constraints as follows:

$$l_j(\mathbf{D}) = Z_n^2$$

and

$$l_j(\mathbf{D}) = M_n^2$$

where, l_j stands for the jth equality constraint function, as defined earlier.

The two equality constraints, however, may be combined to form only one constraint as,

$$l_j(\mathbf{D}) = Z_n^2 + M_n^2$$

As discussed in Section 3.2.6.3, it is desirable to introduce a scale factor S_f to improve the numerical behaviour of the minimization scheme. With the introduction of S_f, the equality constraint takes the form:

$$l_j(\mathbf{D}) = Z_n^2 + S_f.M_n^2$$

or, normalizing, as before,

$$l_j(\mathbf{D}) = \frac{Z_n^2 + S_f\ M_n^2}{\left(\gamma\ b_{av}\ H_t\right)^2}$$

where the symbols have their meanings as defined earlier.

Expression for S_f:

In view of the fact that in the search for critical slip surface, Z_n and M_n are functions of not only F and θ but also of the changing slip surface co-ordinates, it seems logical to use an expression of the form:

$$S_f = \frac{\sum_{i=1}^{n_{dv_{dir}}} \left(\frac{\partial Z_n}{\partial d_i} \right)^2}{\sum_{i=1}^{n_{dv_{dir}}} \left(\frac{\partial M_n}{\partial d_i} \right)^2}$$

where d_i = the i^{th} design variable, as an extension to the earlier expression presented in section 3.2.6.3 is used unchanged, the efficiency of the numerical scheme is not likely to be affected by a great degree. The rather tedious process of finding a series of partial derivatives of Z_n and M_n can be avoided in that case.

Acceptability Constraints

There is no difference in the forms of such constraints as applied in the Direct and the Indirect procedures. These constraints are dealt with separately in the next section.

3.4 ACCEPTABILITY CONSTRAINTS

3.4.1 General

For obtaining a physically acceptable solution it is essential not only to satisfy the equilibrium and boundary conditions and failure criterion along the shear surface but also to satisfy some conditions of acceptability or admissibility criteria such that the implied state of stress within the soil mass is feasible. Morgenstern and Price (1965), Whitman and Bailey (1967), Hamel (1968), Spencer (1973), Janbu (1973)

Chowdhury (1978), Sarma (1979), Chugh (1986b) and others have discussed and presented their views on this aspect in detail and hence are not presented here. Due considerations to their suggestions and recommendations about the forms of acceptability constraints have been given in the present formulations (both in the Direct and the Indirect). These constraints may be of several types as follows:

(i) No state of tension must be implied to exist above the slip surface. In the iterative process it is normally ensured by imposing a restriction on the position of the line of thrust and/or on the non-negativity of the normal stress acting on the base of each slice.

(ii) The forces obtained from the solution do not violate the Mohr-Coulomb failure criterion anywhere within the sliding body. It is normally ensured by checking that shear failures do not occur along the vertical interslice boundaries. However, this condition while necessary is not really sufficient, since the most critical condition generally occurs on a plane which is inclined to the vertical (Seed and Sultan, 1967; Sarma, 1979; Spencer, 1981).

(iii) The direction of forces should be kinematically admissible.

The constraints to be imposed in an actual case may be one or more of the above, depending upon the method of computation of factor of safety adopted in the analysis. These are discussed in the following sections. Deviations, from them, if any, will be discussed at the appropriate places.

3.4.2 Acceptability Constraints Associated with the Janbu's Method of Analysis

With a view of ensure that the obtained solution does not violate the admissibility criteria as discussed above, the following constraints may be imposed in the Direct and the Indirect procedures using Janbu's method of analysis:

1. The shear stresses acting on any interslice boundary must be less than those required for critical equilibrium. In other words, the average factor of safety, F_v, with respect to shear failure along each vertical interface should be greater than unity, i.e.,

$$F_v \geq 1.0$$

However, even if this condition is satisfied, the values of the factor of safety on the vertical interfaces could be less than that for the slip surface. If this is true, it could be inferred that the stability of the slope is overestimated. Possibly it is from this point of view that Janbu (1973) has stated that in a theoretically correct solution,

$$F_v \geq F$$

Accordingly, the latter requirement has been considered in the present analysis. The constraint function may be expressed as:

$$g_j(D) = F - F_{v_i}$$

or, normalizing,

$$g_j(D) = \frac{F}{F_{v_i}} - 1, \qquad i = 1, 2, \ldots, n$$

For homogeneous slopes, the average F_v values are easily calculated. However, as Janbu has point out, for layered soils the computation of average F_v values becomes much more complicated. Moreover, as stated already, the vertical interfaces are in general not likely to be the critical shear planes. Considering these the above mentioned constraints are not incorporated in the case of analysis of heterogeneous slope sections.

2. No tension is allowed within the sliding body.

In Janbu's method, a reasonable line of thrust is assumed a priori. This itself is likely to guard against development of tension within the sliding mass. In spite of that, tension may develop near the crest and may be manifested in the form of negative normal stresses on the base of a slice. In such situations, Janbu (1973) has recommended the introduction of tension crack in the analysis. As already stated, the developed computer program has the provision to include depth of tension crack either as constant or as a design variable. The tension crack may be assumed to be dry or water-filled.

3. The direction of forces obtained from the solution should be kinematically admissible

Although no such constraint has been directly imposed, in all cases the obtained results have been scrutinized to examine whether the sign of the forces are kinematically admissible or not, consistent with the method of analysis used.

3.4.3 Acceptability Constraints Associated With the Spencer's Method of Analysis

The following acceptability constraints have been generally imposed on the proposed procedures for critical surface determination using Spencer's method of analysis. Judicious choice of the constraints is needed as it is not desirable to burden the numerical scheme with too many constraints if not unavoidable so that the progress of the minimization is not unduly affected.

1. In Spencer's method, to avoid development of tension, control is effected on the position of the line of thrust. The line of thrust obtained as part of the solution should not fall outside of the sliding mass. This involves two requirements:

(i) $$L_i \geq 0$$

i.e., $$g_j(D) = -L_i$$

or, normalizing, $$g_j(D) = -\frac{L_i}{H_i} \qquad i = 1,2,\ldots,n-1$$

(ii) $$L_i \leq H_i$$

i.e., $$g_j(D) = L_i - H_i$$

or, normalizing, $$g_j(D) = \frac{L_i}{H_i} - 1$$

where, L_i is the height of the point of application of the interslice forces from the slip surface and H_i is height of the i^{th} interslice boundary.

Situations may arise where it would be hard to satisfy all of the above constraints particularly those in locations near the crest, implying development of tension. In such situations

introduction of tension crack and/or other assumptions regarding interslice force function is likely to be helpful in satisfying these constraints.

2. The constraints against shear failure along the vertical interslice boundaries, as explained in connection with the Janbu's method of analysis in the previous section, may be applied in the analysis by Spencer's method also. However, as already stated, unless absolutely essential, too many constraints should not be imposed. In view of this, the constraints on F_v need not be applied if it is seen that when the constraints on the line of thrust are satisfied the obtained F_v values are all greater than the value of F. It is relevant to note here that Spencer (1973) has laid great emphasis on obtaining a satisfactory line of thrust towards obtaining an acceptable solution. After finding the critical shear planes for homogeneous slopes, Spencer (1981) also demonstrated that in those cases in which the obtained line of thrust is satisfactory in the sense that no state of tension is present within the sliding mass, the solutions generally show good agreement between the average factor of safety against shearing on the slip surface and the factors of safety on the critical shear planes (which are not necessarily vertical).

3. Regarding the directions of the forces obtained from the detailed computer output, the signs of the forces particularly the resultant effective interslice forces are scrutinized, following the discussion presented in Section 3.4.2.

3.4.4 On Inclusion of Acceptability Constraints in the Proposed Technique for Solving Spencer's Equations

In Section 3.2.6.3, formulation has been presented for the proposed numerical scheme for solving the Spencer's stability equations for a general slip surface. The constraints used in the formulation are the non-negativity constraints on the design variables F and θ. Now, for an acceptable solution the line of thrust which comes out as part of the solution should be reasonable. In view of this, it may seem logical to include, in the above mentioned formulation, the constraints on the line of thrust as used in the Direct or the Indirect formulations for critical surface determination present in the preceding sections. Although in that case a feasible starting point may not be easily available, the solution can be obtained by using the Extended penalty function method which does not need a feasible starting point, and, unlike the Exterior penalty function method, is capable in most cases of locating the optimum feasible solution. However, in those cases in which the line of thrust obtained as a part of the solution is already reasonable, it is quite likely that the imposition of such constraints would make no difference to the solution obtained by using analysis without these constraints. In other cases in which the obtained line of thrust is unreasonable, the imposition of such constraints are likely to make a difference in the solution in the sense that by virtue of the property of the Extended penalty function method the optimum solution might be a feasible one. This means that the constraints on the line of thrust would be marginally satisfied

But this would, in all probability, result in a poorer convergence for $Z_n = 0$ and/or $M_n = 0$ i.e., either force or moment equilibrium or both would remain unsatisfied. On the other hand when such constraints are not applied, both these conditions are satisfied to the desired convergence.

From a physical standpoint also the inclusion of such constraints appears to be superfluous. This may be explained as follows:

For a given slope section with a given shear surface and the interslice force function (k-distribution), equilibrium conditions can be satisfied for a unique combination of F and θ. If they do not result in a reasonable line of thrust they cannot be forced to do so without violating the equilibrium conditions.

To improve the line of thrust, however, there are methods such as (i).introduction of tension crack and (ii).use of various k-distributions, as discussed in detail by Spencer (1973). When such trials fail it could well be the case that for the concerned shear surface a solution in the feasible region does not exist.

3.5 ANALYSIS USING NONLINEAR STRENGTH ENVELOPE

3.5.1 General

The use of nonlinear failure envelope and the nonlinearity model proposed by De Mello (1977) have been discussed in Chapter 2. It has been mentioned with reference to De Mello's model that an iterative solution is required to obtain the point to point variation of the strength parameters c'_i and ϕ'_i ($i = 1,2,\ldots,n$) so

that slope stability computations can be carried out keeping the basic formulations of the Spencer method originally developed for linear strength envelope unchanged. Such an iteration is necessary once for each slice for a given shear surface and will henceforth be referred to as the inner iteration loop.

Both the Direct and the Indirect procedures of critical shear surface determination formulated in the preceding sections with reference to linear strength envelope are capable of handling nonlinear strength envelopes. This is effectively done by accommodating the inner iteration loop mentioned above within the main iterative or numerical scheme which may be called the outer iteration loop. The steps involved are discussed as follows.

3.5.2 Direct Procedure using Nonlinear Strength Envelope

The following are the steps involved in the direct determination of critical slip surface using the Direct procedure based on Spencer's method and De Mello's model of nonlinear strength envelope:

Let j=1 correspond to the beginning of the optimization started with an initial guess for the design vector.

Step 1. An initial guess for the set of effective normal stress values, σ'_i (i=1,2,....,n) can be chosen in two ways:

(i) Strength parameters (c'_o, ϕ'_o) obtained from a linear idealisation of the given nonlinear τ - σ' curve may be used in the calculations such that,

$$\left.\begin{array}{l} c'_i = c'_o \\ \phi'_i = \phi'_o \end{array}\right\} \text{ for } i = 1,2,\ldots,n$$

and then σ'_i may be obtained from the Equation (2.24).

(ii) σ'_i values may be computed straightway using the Ordinary method of slices.

Step 2. To calculate c'_i and ϕ'_i from the expressions given respectively by Equations (2.34) and (2.33) derived in Chapter 2.

$$c'_i = (1-\beta')\,\alpha'\,(\sigma'_i)^{\beta'}$$

$$\phi'_i = \tan^{-1}[\,\alpha'\beta'(\sigma'_i)^{\beta'-1}\,]$$

Step 3. To use the above values of c'_i and ϕ'_i to calculate Z_n and M_n from the expression derived in Appendix A

Step 4. To calculate σ'_i values from Equation (2.24).

Step 5. To calculate the objective function f (equal to F in the Direct procedure) and the composite function ψ (explained in a later section) as usual.

Step 6. To go to the optimization subroutines to change the design vector. At this stage it is checked whether the optimum is reached or not. In case it is the optimal solution, the iteration is stopped. Else, to make: $j = j+1$ and to go to step 2.

Remarks: Finally, at the optimum point, all the obtained c'_i and ϕ'_i values correspond to the obtained σ'_i values for the critical

surface searched out. It is noticeable that no inner iteration loop is needed to adjust σ'_i and c'_i, ϕ'_i values, thereby saving a lot of computation.

3.5.3 Indirect Procedure Using Nonlinear Strength Envelope

In the Indirect procedure each and every trial shear surface is analysed separately. When the proposed method for the solution of Spencer's equilibrium equations is used, the evaluation of factor of safety of a trial shear surface taking nonlinearity of strength envelope into account can be done following the same steps as outlined in the previous section. However, as is the case with the analysis using linear Mohr-Coulomb envelope, the time of computation is likely to be much more when the Indirect procedure is used.

3.6 METHOD OF SOLUTION : THE SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE (SUMT)

3.6.1 General

In the preceding sections the determination of the critical slip surface as well as the solution of the equilibrium equations in the Spencer method have been formulated as constrained minimization problems. Among the number of approaches for solving the constrained optimization problems the penalty function method has been chosen as the method of solution in view of its several advantageous features as discussed in Chapters 1 and 2. In this indirect approach the constrained minimization problem is transformed into a sequence of unconstrained

minimization problems. The Fiacco-McCormick (Fox, 1971; Rao, 1984) interior penalty function for inequality constraints has the following form:

$$\psi\ (\mathbf{D},\ r_k) = f(\mathbf{D}) - r_k \sum_{j=1}^{n_{icon}} \frac{1}{g_j(\mathbf{D})} \qquad \text{for } g_j(\mathbf{D}) \le \emptyset \qquad (3.1)$$

where r_k is a positive factor called penalty parameter which is decreased after each stage (k) in the sequence of unconstrained minimization of the composite function ψ and n_{icon} is the number of inequality constraints.

In the Extended interior penalty function due to Kavlie (1971) the individual constraint contributions to the penalty term of the compound interior-extended penalty function have been defined as follows:

$$G_j(\mathbf{D}) = \begin{cases} \dfrac{1}{g_j(\mathbf{D})} & \text{for } g_j(\mathbf{D}) \le \varepsilon \\[2ex] \dfrac{2\varepsilon - g_j(\mathbf{D})}{\varepsilon^2} & \text{for } g_j(\mathbf{D}) > \varepsilon \end{cases} \qquad (3.2)$$

where $\varepsilon = \dfrac{r_k}{\delta_t}$

and δ_t = a constant that defines the transition between the two types of penalty terms.

With the introduction of the Extended interior penalty function concept, the penalty function formulation of Equation (3.1) is replaced by

$$\psi\ (\mathbf{D},\ r_k) = f(\mathbf{D}) - r_k \sum_{j=1}^{n_{con}} G_j(\mathbf{D}) \qquad (3.3)$$

where, n_{con} = the total number of constraints.

Due to the non-availability of a feasible starting point in the majority of cases and based on other points discussed in Chapters 1 and 2, in the present study the Extended penalty function formulation has been adopted. Powell's method and quadratic interpolation method have been chosen for the multidimensional and unidimensional searches respectively in carrying out the sequential unconstrained minimizations of the composite function ψ. The suitability or otherwise of the Davidon-Fletcher- Powell method of multidimensional search has also been studied.

3.6.2 Handling the Equality Constraint in the Direct Formulation:

The Interior penalty function method is not very well established for equality constraints. Studying the Extended interior penalty function formulation it has been intuitively felt that it may be a very promising alternative scheme in handling the equality constraints involved in the Direct formulation presented in this chapter. In that case, however, it is likely to require a special consideration with regard to the termination criteria as discussed below:

As already discussed in Section 2.7.2.4, normally, the penalty function operations are terminated when the relative change in the objective function values and/or the change in the design variables in between two successive r-minimization cycles falls below a desired accuracy.

This can be expressed in terms of relative difference as,

$$\left| \frac{f(D_{i+1}) - f(D_i)}{f(D_i)} \right| \leq \varepsilon_1 \tag{3.4a}$$

$$\left| D_{i+1} - D_i \right| \leq \varepsilon_2 \tag{3.4b}$$

However, when an equality constraint is present, it may so happen that after a few cycles the process stops as the termination criteria as in Equation(3.4) are duly satisfied, and yet the equality constraint function has not diminished to the desired degree. To safeguard against any such premature termination, additional conditions may be imposed on the values of Z_n and M_n separately or on the equality constraint function itself as follows:

$$\left.\begin{array}{l} |Z_n| \leq \varepsilon_{Z_n} \qquad \text{and} \qquad |M_n| \leq \varepsilon_{M_n} \\ \text{or, combining the two,} \quad l_j(D) \leq \varepsilon_1 \end{array}\right\} \tag{3.5}$$

Where ε_{Z_n} and ε_{M_n} are pre-assigned positive quantities beyond which Z_n and M_n may respectively be considered negligible. ε_1 is the corresponding quantity for the equality constraint function $l_j(D)$ defined earlier. An exercise to determine how small ε_{Z_n} and ε_{M_n} should be from practical considerations has been carried out and is reported in the subsequent sections.

3.6.3 Use of Interior Penalty Function Method

In finding the critical slip surfaces using the Indirect formulations involving only inequality constraints the Interior penalty function method may also be used provided a feasible starting point is available.

This is one of the most commonly used test functions and is known as the Rosenbrock's banana function (Rosenbrock, 1960). The optimal solution of this problem (Rao, 1984) is as follows:

$$\mathbf{X}^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \quad \text{and} \quad f_{min} = f(\mathbf{X}^*) = 0.0$$

Solution

Minimization has been carried out using two models. Model I, called POW, consists of Powell's method of multidimensional search in conjunction with the Quadratic fit for unidimensional search. Model II, called DFP, consists of the Davidon-Fletcher-Powel method and the Cubic fit for multidimensional and unidimensional searches respectively. The adjustable parameters, e.g., the step length and convergence criteria used are as given in Table 3.1. In order to obtain the most accurate results, such strict convergence criteria have been set. It is possible that the same accuracy would have been obtained with less strict convergence criteria also. Results in the form of optimal design vector and the minimum function values are presented in Table 3.2. It can be observed from Table 3.2 that the obtained results is very close to the known solution of the problem.

Test Function 2

$$f(\mathbf{X}) = -\left\{\frac{1}{1 + (x - y)^2} + \sin\left(\frac{1}{2}\pi yz\right) + \exp\left[-\left(\frac{x+z}{y} - 2\right)^2\right]\right\}$$

using the starting point $\mathbf{X}_1 = \begin{bmatrix} 0.0 \\ 1.0 \\ 2.0 \end{bmatrix}$

and initial function value $f(\mathbf{X}_1) = -1.5$

TABLE 3.1

Values of the Adjustable Parameters used in Test Function 1

Method	Step-length	Convergence Criteria
Quadratic fit	Ø.1	$\left\| \dfrac{h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)}{f(\tilde{\lambda}^*)} \right\| \le \varepsilon_1 = 1.0\text{x}10^{-8}$
Cubic fit	Ø.1	$\left\| \dfrac{S^T . \nabla_f}{\|S\| \|\nabla_f\|} \right\|_{\tilde{\lambda}^*} \le \varepsilon_2 = 1.0\text{x}10^{-3}$
POW	——	$\| D_{i+1} - D_i \| \le \varepsilon_3 = 1.0\text{x}10^{-8}$
DFP	——	$\|\nabla_f\| \le \varepsilon_4 = 1.0\text{x}10^{-3}$

Note : The symbols have the same meanings as given in Chapter 2

TABLE 3.2 Optimal Solution of Test Function 1

Model	No. of iterations required	x_1	x_2	f_{min}	cpu time (secs)
POW	1Ø	Ø.9999	Ø.9997	$2.016\text{x}10^{-8}$	Ø.19
DFP	3Ø	Ø.9998	Ø.9996	$3.581\text{x}10^{-8}$	Ø.21

TABLE 3.3 Optimal Solution of Test Function 2

Model	No. of iterations required	x	y	z	f_{min}
POW	8	Ø.3885	Ø.4574	2.Ø84Ø	-1.993Ø
RIGPOW	8	1.ØØØØ	1.ØØØØ	Ø.9999	-3.ØØØØ
Rigorous Powell (reported by Powell, 1964)	6	Ø.9272	Ø.9641	1.Ø424	-2.9968

This function has minima at:

$$x = y = z = (4n + 1)^{1/2}, \text{ n is an integer (Fox, 1971)}$$

This function was chosen by Powell (1964) to illustrate the application of Rigorous or Modified Powell procedure as the ordinary Powell's method failed to converge to the proper solution. As such, in the present study also, the working of the subroutine written for the Rigorous Powell method is checked with respect to this function.

Solution

The minimization model, called RIGPOW, combines Rigorous Powell method with Quadratic fit. In this case the values of the adjustable parameters are set at : $\varepsilon_1 = 1.0 \times 10^{-5}$ and $\varepsilon_3 = 1.0 \times 10^{-5}$ The results obtained with an initial step length of 0.1 are fairly close to the actual solution; however, a step length of 0.05 gave the best results.

This shows that for such complicated functions, the results may vary depending upon the step-length used in the search. As shown in Table 3.3, the optimal solution obtained both in the present study as well as that reported by Powell (1964) are close to the actual solution, the present result being closer to the actual solution. This demonstrates the satisfactory working of the subroutine developed for the Rigorous Powell method. It is also seen from the Table 3.3 that the ordinary Powell method has not converged to the actual solution.

3.7.3 Testing the Performances of the Subroutines for Function Evaluation

As it is intended in the proposed study to use two methods of analysis, namely, the Janbu's GPS and the Spencer's method (Thrust line criterion - Spencer, 1973) two subroutines have been developed for the function evaluations. Test of performance of each of the subroutines is discussed in the following sections.

3.7.3.1 Testing the Subroutine for Janbu's Method : Test Problem 1

In order to ensure that the Janbu's GPS has been correctly programmed, one solved example from Janbu's paper (Janbu, 1973) has been selected. Marked as Test Problem 1, the slope with the given slip surface, soil and pore pressure properties and the assumed line of thrust has been shown in Figure 3.5.

A comparison of the results obtained by using the developed program with those reported in Janbu's paper is presented in Table 3.4. The table shows that except for the values of F_v, the factors of safety along the vertical interfaces, present results are in excellent agreement with the reported values. Apparent disagreement of some of the F_v-values (max. difference is about 17%) could be due to the following:

(i) F_v-values are highly sensitive to the respective heights of the interslice boundaries. Any small error introduced in scaling and reading them from the figure is likely to result in considerable errors in the calculated F_v- values.

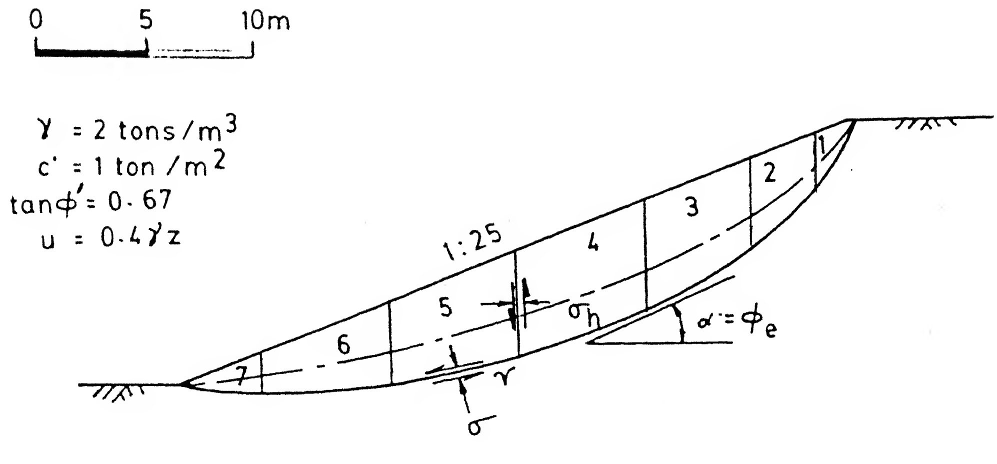


FIG. 3.5 SLOPE SECTION IN TEST PROBLEM 1 (Janbu, 1973)

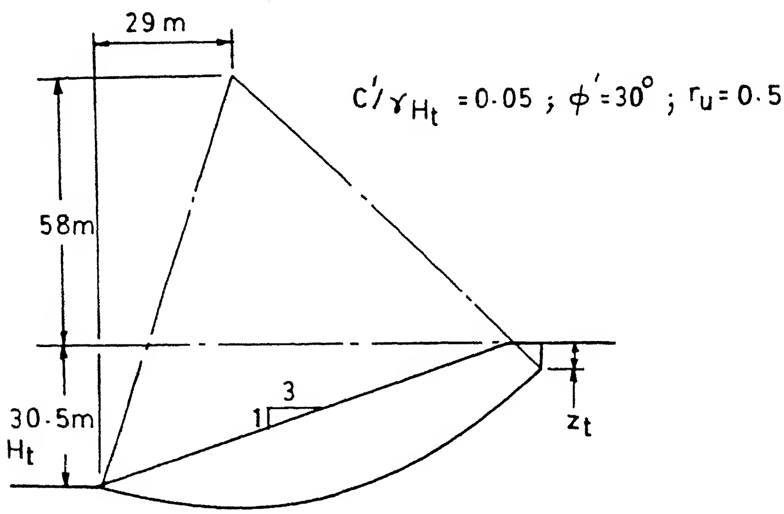


FIG. 3.6 SLOPE SECTION IN TEST PROBLEM 2
(Spencer, 1973)

(ii) Values of F_v shown as Janbu's solution in Table 3.4 are actually read from a graph and hence may not be accurate.

3.7.3.2 Testing the Subroutine for Spencer Method: Test Problem 2

As stated earlier, there are provisions for the computation of the Spencer's factor of safety by using two different techniques of solution of the equilibrium equations :

(i) the iterative scheme suggested by Spencer (1973) and

(ii) the technique based on SUMT as proposed in this thesis.

To test the programs developed for both these schemes, one solved example from Spencer's paper (Spencer, 1973) has been taken up. Marked as Test Problem 2, the slope along with the soil and pore pressure properties is as shown in Figure 3.6. The given slip surface terminates at the bottom of a vertical tension crack running parallel to the crest with water pressure acting in it. This example has been chosen to check the effectiveness of the developed computer program in handling tension cracks. Spencer (1973) has reported results for various depths of tension crack as well as k-distributions. In this study, however, only the case with $z_t = 0.3H_t$ and parallel interslice forces has been considered.

Solution using Spencer's scheme:

Spencer's method of solution as presented in Chapter 2 has the following prerequisites:

TABLE 3.4

Comparison with Janbu's Solution (Test Prob. 1)

Slice No.	E	T	τ	σ'	F_v
	0.000 (0.000)	0.000 (0.000)			
1			0.801 (0.801)	0.272 (0.268)	
	2.750 (2.753)	-0.684 (-0.691)			3.86 (3.86)
2			1.503 (1.503)	1.815 (1.821)	
	11.536 (11.512)	-3.737 (-3.724)			1.90 (1.70)
3			2.336 (2.336)	3.647 (3.647)	
	22.927 (22.915)	-7.777 (-7.768)			1.70 (1.57)
4			3.012 (3.012)	5.136 (5.136)	
	25.658 (25.656)	-8.433 (-8.429)			1.81 (1.70)
5			3.040 (3.040)	5.196 (5.196)	
	21.444 (21.453)	-6.712 (-6.712)			2.07 (1.76)
6			2.673 (2.673)	4.389 (4.389)	
	7.484 (7.499)	-2.617 (-2.616)			2.27 (2.05)
7			1.614 (1.614)	2.059 (2.059)	
	0.000 (0.000)	0.000 (0.000)			

Factor of Safety : F_o = 1.38 (1.38)
F = 1.474 (1.474)

Note : Figures within parentheses are the ones reported by Janbu
E and T are in tons/m and τ and σ' are in tons/m^2.

A. Initial guess for F and θ

Some trial values of F and θ are necessary to start the computation. The following guidelines may be used:

$$F = F_{oms} \qquad \text{(a)}$$

and

$$\theta = \beta/2 \qquad \text{(b)}$$

where, F_{oms} is the factor of safety given by the Ordinary method of slice and β is the slope angle.

B. An algorithm to 'adjust' F and θ by way of successive approximations, until Z_n and M_n respectively are negligible. As already stated, two algorithms, namely, the Method of bisection and the Modified regula falsi technique have been considered in this thesis.

C.(i)To ascertain the lower limits ε_{Z_n} and ε_{M_n} of Z_n and M_n respectively beyond which they may be considered 'negligible' - these are the requirements of steps (d) and (e) in the Spencer's iterative scheme (Chapter 2).

(ii) To ascertain the convergence limits ε_F and ε_θ for F and θ respectively with regard to the step (f) in the same scheme such that, for convergence, $|(F_1-F)/F_1| \le \varepsilon_F$ and $|(\theta_1 - \theta)/\theta_1| \le \varepsilon_\theta$. However, the latter two limits are related to the former two and step (f) may be expressed, in other words, as "the process is then repeated, from (c), until Z_n and M_n both become negligible". A sensitivity analysis pertaining to this will be reported in a later section.

Results:

The results of analysis of the given slip surface is presented in Table 3.5. Following the guidelines in equations (a) and (b) of this section the starting values of F and θ have been taken as 1.30 and 0.15 respectively, the values of F_{oms} and β in this case being 1.34 and 0.32 respectively. For trapping the initial interval of uncertainty, fixed step size of 0.2 for F and 0.05 for θ have been adopted. The following values have been assumed:

ε_{Z_n} = 0.001, ε_{M_n} = 0.001 and the step (f) convergence requirements has also been applied in terms of values of Z_n and M_n.

Results have been obtained using the Method of bisection as well as the Modified regula falsi method. By the Method of bisection, the final results are : F = 1.45487; θ = 0.257289 rad. the step-wise obtained results have also been presented in Table 3.5. The final results using the Modified regula falsi technique has been obtained as F = 1.45499 and θ = 0.257290 rad.and it required a total of nine steps as against fourteen required in the earlier case. However, the detailed results are not presented here for reasons of space and brevity.

Solution using the proposed scheme based on SUMT :

In the solution by the proposed numerical scheme, the design variables F and θ have been normalized by dividing by their respective initial values and a step-length of 0.1 has been

TABLE 3.5

Typical Stepwise Results using Spencer's Iterative Scheme (Method of Bisection)(Test Prob.2)

Step No.	F	θ (rad)	Z_n (kN/m)	M_n (kN-m/m)	Remarks
	1.30	0.15	0.5765×10^{3}	0.2989×10^{5}	Initial Guess
1	1.39670	do	0.4959×10^{-4}	0.2364×10^{5}	
2	do	0.26587	0.3659×10^{3}	0.1526×10^{-3}	
3	1.45979	do	0.4997×10^{-3}	-0.1933×10^{4}	
4	do	0.25655	-0.2966×10^{2}	0.8545×10^{-3}	
5	1.45443	do	-0.1411×10^{-3}	0.1697×10^{3}	
6	do	0.257354	0.2605×10^{-3}	-0.8545×10^{-3}	
7	1.45490	do	-0.7439×10^{-3}	-0.1481×10^{2}	
8	do	0.257284	-0.2281×10^{0}	-0.3357×10^{-3}	
9	1.45486	do	0.8354×10^{-3}	0.1301×10^{1}	
10	do	0.257290	0.2082×10^{-1}	0.3052×10^{-3}	
11	1.45487	do	-0.4845×10^{-3}	-0.1211×10^{0}	
12	do	0.257289	-0.2361×10^{-2}	0.5493×10^{-3}	
13	1.45487	do	-0.2251×10^{-3}	0.1266×10^{-1}	
14	do	0.257289	-0.7248×10^{-4}	0.4272×10^{-3}	Convergence

Note : Rounding off to two-decimal places, at convergence F=1.45 and θ = 0.26 rad.

adopted. Interior penalty function method in conjunction with Powell's method and Quadratic fit has been used with the following convergence criteria.

Method	Convergence Criteria
Quadratic fit	$\left\lvert \dfrac{h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)}{f(\tilde{\lambda}^*)} \right\rvert \le 0.001$
Powell	$\lvert D_{i+1} - D_i \rvert \le 0.001$
Interior Penalty	$\lvert Z_n \rvert \le 0.001$ and $\lvert M_n \rvert \le 0.001$

Results

Initial guess: $F = 1.30$, $\theta = 0.15$

Initial normalized design vector: 1.0, 1.0

Constraints (only side constraints): $g_j = -1.30, -0.15, -0.17$

First two constraints are the nonnegativity constraints on F and θ; the last one represent the upper limit on θ.

Initial function values : $Z_n = 0.5765 \times 10^3$, $M_n = 0.2989 \times 10^5$

Scale factor : $S_f = 0.373 \times 10^{-2}$,

$$\text{Obj function : } f = \frac{Z_n^2 + S_f . M_n^2}{(\gamma b H_t)^2} = 0.2886$$

Initial value of penalty parameter, $r_1 = 1.0 \times 10^{-3}$

$$\text{The Composite function } \psi = f - r \sum_{j=1}^{3} \frac{1}{g_j(D)} = 0.3091$$

Optimal Solution :

$F = 1.45491$, $\theta = 0.257302$

Normalized Design Vector: 1.1192, 1.7153

Constraints g_j : -1.4549, -0.2573, -0.0827

No. of r-minimization required = 2

Final Values of $Z_n = -0.6982 \times 10^{-4}$, $M_n = 0.4761 \times 10^{-3}$

Scale factor $S_f = 0.3956 \times 10^{-2}$

Obj function $f = 0.454 \times 10^{-15}$

$\psi = 1.6664 \times 10^{-5}$

Rounded off to two decimal places, F = 1.45 and θ = 0.26 which are identical with the results obtained using Spencer's iterative scheme. These results are in very close agreement with those reported by Spencer (1973). Convergence was also achieved keeping the scale factor $S_f = 1.0$. However, in that case the c.p.u. time required was more, namely, 2.55 seconds as against 1.60 seconds required in the other case.

Position of the Line of Thrust

Table 3.6 shows the positions of the line of thrust and the normalized values of the normal component of resultant interslice forces for effective stress (Z') obtained by using the proposed technique and also a comparison of the corresponding values obtained by Spencer (1973). The Z' values presented in the Table are normalized as $Z'/\gamma b H_t$. It can be observed that the line of thrust for total stress (L/H) is identical with that obtained by Spencer; the line of thrust for effective stress (L'/H) differs at the 13th, 14th and 15th interslice boundaries

Table 3.6
Comparison with Spencer's Solution

Line		Interslice Boundaries														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	L/H	0.44	0.38	0.36	0.35	0.34	0.33	0.33	0.33	0.32	0.32	0.32	0.31	0.31	0.31	0.33
2		0.44	0.38	0.36	0.35	0.34	0.33	0.33	0.33	0.32	0.32	0.32	0.31	0.31	0.31	0.33
3	L'/H	0.47	0.40	0.37	0.35	0.34	0.33	0.33	0.32	0.31	0.30	0.29	0.28	0.27	0.25	0.27
4		0.47	0.40	0.37	0.35	0.34	0.33	0.33	0.32	0.31	0.30	0.29	0.28	0.26	0.24	0.30
5	Z'	0.08	0.17	0.27	0.36	0.43	0.48	0.51	0.51	0.49	0.45	0.38	0.30	0.21	0.12	0.03
6		0.08	0.17	0.27	0.36	0.43	0.48	0.51	0.51	0.49	0.45	0.38	0.30	0.21	0.12	0.03

by a small margin (3-4%). The normalized Z′ values are, however, identical with those reported by Spencer.

3.7.4 Sensitivity Analysis (with reference to Test problem 2)

In order to ascertain the order of values of Z_n and M_n which may be considered negligible, the following sensitivity studies have been carried out:

(i) The sensitivity of Z_n to F (θ = const.)

(ii) The sensitivity of M_n to θ (F = const.)

From Table 3.5, it is seen that keeping θ = const. at 0.15, F has been adjusted, using the method of bisection, to a value of 1.39670737 (correct upto 8 places of decimal) such that $|Z_n| \leq 0.001$.

Then, in the next step, keeping F = const. at 1.39670737, θ has been adjusted to a value of 0.2656868.

Truncating the correct values at various decimal places, sensitivity analyses have been carried out as reported in Tables 3.7 and 3.8.

Table 3.7 shows that one needs to consider values of $Z_n/(\gamma b H_t)$ smaller than 1×10^{-5} only when F is desired to be correct upto at least four places of decimals. Since the accuracy in the value of factor of safety beyond second decimal place is of no physical significance, the values of factor of safety beyond second decimal place is of no physical significance, the values of $Z_n/(\gamma b H_t)$ beyond 1×10^{-2} need not be considered.

TABLE 3.7

Sensitivity of Z_n to F (Test problem 2)

θ = const.

F	θ	$Z_n/(\gamma b H_t)$	$M_n/(\gamma b H_t^2)$
1.39	0.15	1.0523×10^{-2}	2.2089×10^{-1}
1.396	0.15	1.1053×10^{-3}	2.1756×10^{-1}
1.3967	0.15	1.1503×10^{-5}	2.1717×10^{-1}
1.396707	0.15	5.8689×10^{-7}	2.1716×10^{-1}
1.3967073	0.15	1.0583×10^{-7}	2.1716×10^{-1}
1.39670737	0.15	1.3897×10^{-8}	2.1716×10^{-1}

TABLE 3.8

Sensitivity of M_n to θ (Test problem 2)

F = const.

θ	F	$M_n/(\gamma b H_t^2)$	$Z_n/(\gamma b H_t)$
0.26	1.39670737	1.1958×10^{-2}	9.6906×10^{-2}
0.265	1.39670737	1.4522×10^{-3}	1.0186×10^{-1}
0.2656	1.39670737	1.8364×10^{-4}	1.0246×10^{-1}
0.26568	1.39670737	1.4367×10^{-5}	1.0254×10^{-1}
0.265686	1.39670737	1.6686×10^{-6}	1.0254×10^{-1}
0.2656868	1.39670737	1.4021×10^{-9}	1.0254×10^{-1}

Table 3.8 demonstrates that only when the desired accuracy in the values of 'θ' is beyond four places of decimals, values of $M_n/(\gamma b H_t^2)$ smaller than 1×10^{-4} are of significance. Keeping the above in mind, the limiting values of Z_n and M_n have been decided for various cases and mentioned at the appropriate places.

From Tables 3.7 and 3.8 it can also be observed that for a fixed θ, Z_n is more sensitive to F whereas, for a fixed F, M_n is more sensitive to θ.

Time Considerations (with reference to Test problem 2)

The above mentioned limits on the values of Z_n and M_n have considerable effect on the computation time. The relative efficiencies of the various method for solving Spencer's equations in terms of c.p.u. time have been presented in Table 3.9. It is seen that the solution scheme using Modified regula falsi technique is much faster than that using the Method of bisection. But the proposed scheme is the fastest. Studies in this regard for varying accuracies have also been carried out and presented in the same Table. It is seen that the c.p.u. time increases considerably as the degree of accuracy increases.

3.7.5 CONCLUSIONS

Results presented for the test problems have demonstrated:

1. that the developed program subroutines are correct.

2. that one need not consider an unreasonable accuracy in satisfying the force and moment equilibrium equations in computing the factor of safety of a given shear surface.

TABLE 3.9

CPU Time Required For One Function Evaluation By Spencer's Method (Test Problem 2)

Run No.	ϵ_{Zn} step (d)/(e)	ϵ_{Mn} step (f)	CPU time in seconds Spencer's iterative scheme using: Bisection Method	Modified Regula Falsi Technique	Proposed method based on SUMT: Without S_f	With S_f
1	0.01	0.01	1.85	1.09	–	–
2	0.01	0.001	2.37	1.33	–	–
3	0.001	0.001	2.72	1.40	–	–
4	0.001	0.001*	4.36	2.40	2.55	1.60

* In this particular excercise, convergence requirement in step (f) of Spencer's original scheme has been applied on Zn and Mn instead of F and θ (as discussed in Section 3.7.3.2) as follows:
$|Zn| \leq 0.001$ and $|Mn| \leq 0.001$

CHAPTER 4

APPLICATION OF THE PROPOSED PROCEDURE TO SLOPES IN HOMOGENEOUS SOIL CONDITIONS

4.1 INTRODUCTION

Stability of slopes in homogeneous soils is one of the most widely studied topics in geotechnical engineering. Nevertheless, it is still drawing attention of the research workers in the field. The reasons may be stated as follows :

In developing any new procedure of locating the critical shear surface for slopes in general heterogeneous soil conditions (with complex geometry, layering, pore pressure etc.), problems may be encountered either due to an inherent flaw in the principle itself or due to some numerical difficulties. For this reason it is essential to apply the procedure first to simple slopes in homogeneous soils. Only after sufficient experience and insight have been gained in solving these simple cases, the method can be extended to more general and complicated case with confidence. Apart from this, in order to compare the relative efficiency of the proposed scheme with respect to the existing ones, it is required to study some example problems the solutions of which are available in the literature. And such examples mostly belong to the homogeneous slopes.

With the above in view, the proposed technique has been applied to a few slopes in homogeneous soils. These are:

Example Problem 4.1: This example problem deals with a comparative study of the Direct and the Indirect procedures proposed in this thesis.

Example Problem 4.2 : The basic objective of taking up this example problem is to present a comparative study of the proposed Direct procedure with the variational calculus and the dynamic programming techniques.

Example Problem 4.3 : This example problem has been considered to demonstrate the effectiveness of the proposed technique to a steep slope. The results are also compared with those obtained by using other techniques mentioned above.

Example Problem 4.4 : In this illustrative example an attempt has been made to obtain an acceptable solution using an interslice force function proposed by Wilson et al.(1983).

Example Problem 4.5 : Application of the proposed Direct procedure to slopes in soils having nonlinear strength envelope has been demonstrated through this example. Results have been compared with those reported in the literature.

4.2 EXAMPLE PROBLEM 4.1

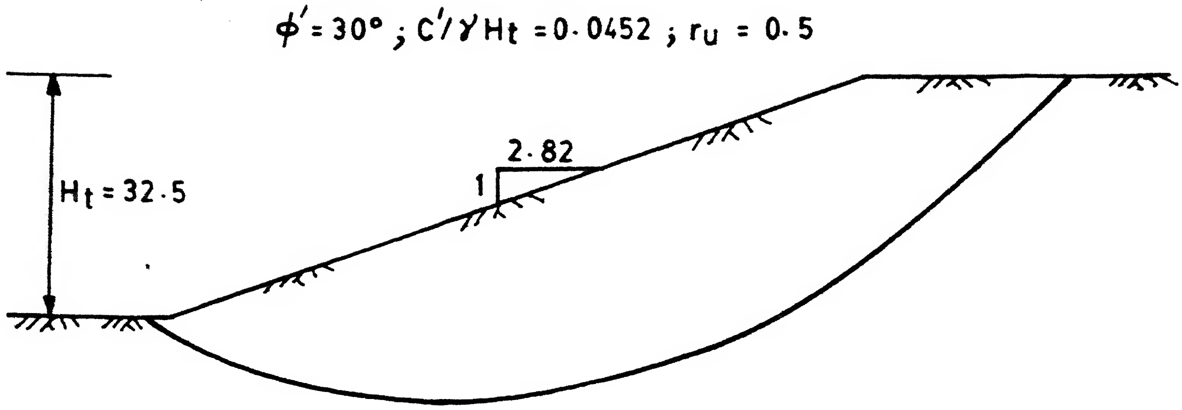


Figure 4.1(a) SLOPE SECTION IN EXAMPLE PROBLEM 4.1 (Spencer, 1973)

4.2.1 General

The slope shown in Figure 4.1(a) refers to one of the two homogeneous slopes considered by Spencer (1973) to illustrate the "Thrust line criterion". The soil and pore pressure characteristics are as indicated in the figure. Actual values of γ and c' taken for computation are 20.0 kN/m^3 and 29.38 kPa respectively. This slope has been analysed in the present studies to find its critical slip surface and the associated minimum factor of safety using Spencer's method, adopting both the proposed procedures to study the relative merits of the Direct procedure to the Indirect procedure. This is a non-circular analysis using discretization Model I.

4.2.2 Solution by Spencer's Method

General

The slip surface marked 3 in Figure 4(a) of Spencer's paper (Spencer, 1973)has been chosen as the first trial shear surface. 14 slices have been considered to keep the same number of slices as was used by Spencer. The interslice forces are assumed to be parallel i.e. k=1 throughout and no tension crack has been assumed. For minimization, in both Indirect and Direct procedures, the Extended interior penalty function technique in conjunction with Powell's method and Quadratic fit has been used.

Results and Discussion

A. Indirect Procedure : In the present case, the iterative scheme suggested by Spencer along with the Method of bisection has been used for the purpose of function evaluation. The factor of safety, F, of the initial shear surface, which is the value of the objective function at the starting point, has been calculated as 1.42297 as against 1.42 reported by Spencer. The corresponding interslice force angle, θ, is Ø.25ØØ76 rad as against Ø.25 rad reported by Spencer. It is observed that the obtained values of F and θ rounded off to two decimal places are identical with Spencer's values.

The design variables, 15 in number, are normalized by dividing by their initial values and an initial step length of Ø.Ø1 has been taken. A value of transition parameter δ_t = Ø.Ø1 and an initial value of tolerance, ε_o = -Ø.1 have been adopted. This makes $\psi \approx (1-1.5)f$,where f and ψ are the objective function

and the composite function respectively.A factor of reduction equal to 10 has been applied on the penalty parameter, r; the same value has been used throughout the thesis.

The minimum factor of safety (F_{min}) obtained in this case is 1.36235 and the corresponding θ is obtained as 0.266554 rad. The associated line of thrust, interslice forces and the stresses at slice bases are presented in Table 4.1. The design vector and the constraints at the starting and optimal points are given in Table 4.2. The obtained shear surface having minimum factor of safety is shown in Figure 4.1(b)

B. **Direct Procedure :** Including the two additional design variables F and θ, the total number of design variables in this case is 17. For F and θ initial values of 1.50 and 0.25 respectively have been taken. As in the Indirect procedure, the design variables are normalized; initial step length is 0.01 and the Extended penalty parameters δ_t = 0.01 and ε_o = -0.1.

The minimum factor of safety (F_{min}) and θ obtained here are 1.35793 and 0.265368 rad. Thus the minimum factor of safety obtained using the Direct procedure is marginally smaller than that obtained using the Indirect procedure. The shear surface corresponding to the F_{min} is shown in Figure 4.1(b) along with the surface obtained by using the Indirect procedure reported above. As is expected, the two surfaces are almost coincident. The associated line of thrust and other internal variables are presented in Table 4.3. It is seen both in Tables 4.1 and 4.3 that the lines of thrust obtained using the Indirect as well as the Direct procedure are unacceptable as the corresponding

TABLE 4.1

Calculated Responses Associated with the Solution to Example prob. 4.2 by Indirect procedure

Slice No.	σ' kPa	τ kPa	L/H	L′/H	$\frac{Z'}{\gamma b H_t}$	F_v
1	49.44	38.50				
			0.63	0.67	0.07	2.71
2	83.62	57.10				
			0.44	0.47	0.15	2.35
3	128.57	75.71				
			0.38	0.41	0.25	2.08
4	150.23	84.10				
			0.36	0.38	0.32	1.92
5	164.86	90.20				
			0.35	0.36	0.36	1.81
6	219.37	92.07				
			0.34	0.34	0.38	1.72
7	198.11	95.69				
			0.33	0.33	0.39	1.64
8	180.86	95.98				
			0.32	0.31	0.37	1.58
9	168.67	89.73				
			0.32	0.30	0.32	1.53
10	139.86	78.21				
			0.31	0.28	0.23	1.51
11	119.56	70.70				
			0.31	0.25	0.14	1.51
12	93.75	59.71				
			0.31	0.20	0.05	1.56
13	53.94	42.90				
			0.40	0.11	-0.03	2.04
14	5.53	23.19				

Table 4.2

Design Vector and Constraints in the Indirect Procedure

Example Problem 4.1

Starting Point

F = 1.42297 θ = 0.250076

Design Variables (Not normalized)

-1.4444	-2.5278	-3.6111	-3.8999	-4.1888	-4.1888	-2.7444	
-1.1555	1.0111	3.9865	6.9865	13.4333	20.9440	91.6500	-15.6000

Constraints(all inequality) :

-1.5317	-1.4653	-1.4431	-1.3589	-1.3084	-1.2570	-1.1443	-1.0532
-0.9586	-0.8533	-0.7662	-0.5867	-0.3557	-0.2499	-0.39E-04	-0.2200
<u>0.26E-04</u>	-0.69E-01	-0.3448	-0.53E-01	-0.4999	-0.7999	-0.62E-02	-0.4934
-0.79E-01	-0.1931	-0.4088	-0.3465	-0.3237	-0.3127	-0.3041	-0.2979
-0.2920	-0.2865	-0.2807	-0.2744	-0.2711	-0.2657	-0.4153	-0.5912
-0.6535	-0.6763	-0.6873	-0.6959	-0.7021	-0.7080	-0.7135	-0.7193
-0.7256	-0.7289	-0.7343	-0.5847	-0.4330	-0.2500	-0.91E-01	

δ_t = 0.01 ε_o = -0.1 r_o = 0.1E-02

f = 1.42297 ψ = 1.65696 Z_n = 0.3799E-01 M_n = -0.2014E-02

Optimal Point

F = 1.36235 θ = 0.266554

Design Variables (Normalized)

-1.3158	-1.2875	-1.1775	-1.1365	-0.9715	-0.7399	-0.6886	-0.2409
2.1252	1.4404	1.4115	1.1101	1.0523	1.0428	-0.4714	

Constraints(all inequality)

-2.5633	-1.8513	-1.6613	-1.4904	-1.3495	-1.2175	-1.1121	-1.0143
-0.9026	-0.7673	-0.6386	-0.5011	-0.3218	-0.2876	-0.1095	-0.1923
-0.1224	-0.1493	-0.77E-01	-0.2128	-2.9305	-0.5428	-0.92E-01	-0.95E-01
-0.1392	-0.1512	-0.6270	-0.4389	-0.3831	-0.3606	-0.3469	-0.3374
-0.3295	-0.3232	-0.3179	-0.3115	-0.3065	-0.3090	-0.4040	-0.3780
-0.5611	-0.6169	-0.6394	-0.6531	-0.6626	-0.6705	-0.6768	-0.6821
-0.6885	-0.6935	-0.6910	-0.5960	-0.3724	-0.2665	-0.74E-01	

No. of r-minimization required = 2

f = 1.36235 ψ = 1.56897 Z_n = -0.9632E-02 M_n= -0.3448E-02

Note: Out of a total of 55 constraints, the first 13 are bounday constraints, next 13 are curvature constraints, next 26 are constraints on the line of thrust, and last 3 are side constraints respectively.

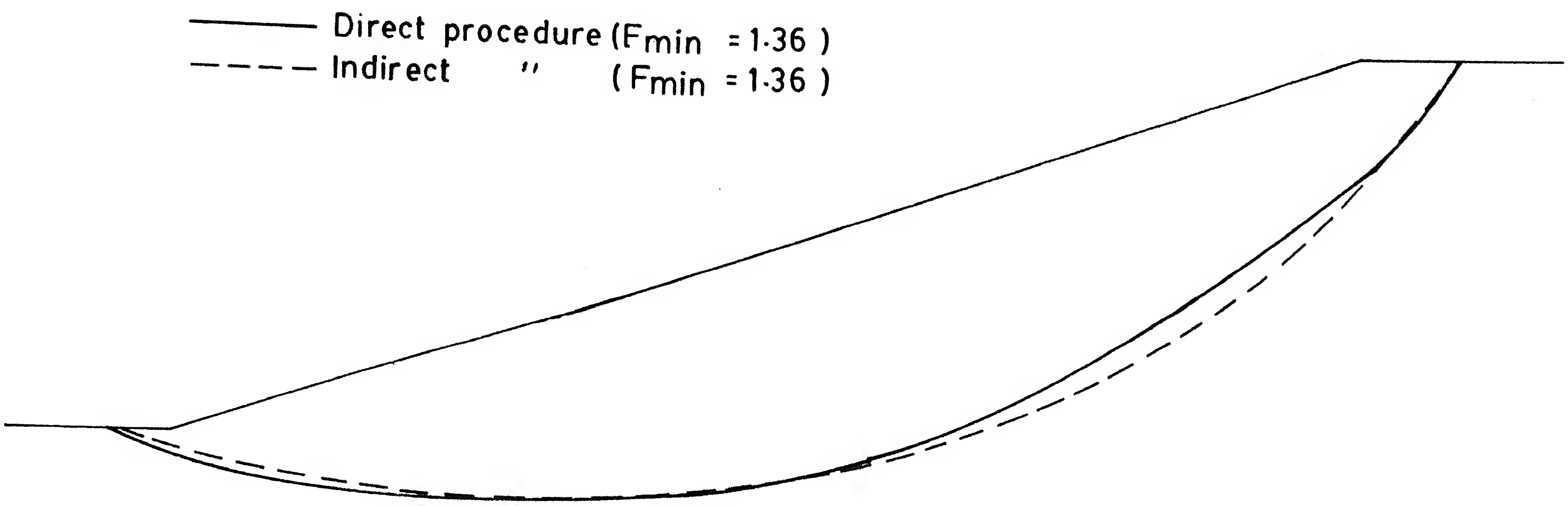


FIG. 4·1(b) COMPARISON OF THE CRITICAL SHEAR SURFACES FOR THE EXAMPLE PROBLEM 4·1

TABLE 4.3

Calculated Responses associated with the Solution to Example prob. 4.1 by Direct procedure

Slice No.	σ' kPa	τ kPa	L/H	L′/H	$\frac{Z'}{\gamma b H_t}$	F_v
1	43.07	40.14				
			0.72	0.77	0.08	2.65
2	86.47	58.42				
			0.47	0.51	0.17	2.33
3	120.52	72.62				
			0.40	0.44	0.25	2.10
4	142.57	82.00				
			0.37	0.40	0.31	1.95
5	155.69	87.84				
			0.36	0.37	0.35	1.83
6	163.47	91.56				
			0.34	0.35	0.37	1.74
7	168.49	93.28				
			0.33	0.33	0.37	1.67
8	162.80	90.87				
			0.33	0.32	0.35	1.61
9	150.17	85.50				
			0.32	0.30	0.29	1.57
10	139.46	80.93				
			0.31	0.28	0.22	1.53
11	113.76	70.00				
			0.31	0.25	0.14	1.54
12	88.30	59.17				
			0.31	0.19	0.05	1.59
13	49.40	42.55				
			0.40	0.14	-0.03	2.14
14	4.64	23.63				

effective interslice forces at the last interslice boundary are found to be negative. By introducing tension crack and/or assuming other k-distributions, it may be possible to obtain acceptable lines of thrust. However, since the main objective of the current example is to draw a comparison between the results obtained using the Direct and the Indirect procedures, no such effort has been made. However, from these tables, it is also seen that all the F_v values are greater than the corresponding values of F and that the total normal stress and shear stress values are all positive as they should be. The design vector and constraints at the beginning and end of minimization are given in Table 4.4.

Tables 4.2 and 4.4 demonstrate that the Extended penalty function technique can successfully handle the infeasible design points as well as equality constraints.

Influence of the scale factor S_f on the numerical scheme has been studied as follows:

Initially, an attempt was made to use the Direct procedure without introducing the scale factor S_f. The data and the initial conditions were the same as those used in the analysis in which the scale factor was incorporated and results of which have been presented already. The only change is in the evaluation of the equality constraint

$l_j = \dfrac{Z_n^2 + S_f . M_n^2}{(\gamma b H_t)^2}$, where, now $S_f = 1.0$ in place of the expression for the same in terms of partial derivatives of Z_n and M_n. The final results are :

TABLE 4.4

Design Vector and Constraints in the Direct procedure
For the Example problem 4.1

Starting Point

F = 1.5000 θ = 0.2500

Design Variables (Not normalized)

-1.4444	-2.5278	-3.6111	-3.8999	-4.1888	-4.1888	-2.7444	-1.1555
1.0111	3.9865	6.9865	13.4333	20.9440	91.6500	-15.6000	1.50, 0.25

Constraints(inequality) :

-1.5317	-1.4653	-1.4431	-1.3589	-1.3084	-1.2570	-1.1443	-1.0532
-0.9586	-0.8533	-0.7662	-0.5866	-0.3556	-0.2499	-0.39E-04	-0.2200
0.26E-04	-0.69E-01	-0.3448	-0.53E-01	-0.4999	-0.7999	-0.62E-02	-0.4934
-0.79E-01	-0.1931	-0.4086	-0.3458	-0.3229	-0.3126	-0.3042	-0.2982
-0.2943	-0.2909	-0.2886	-0.2899	-0.2982	-0.3953	-6.2506	-0.5914
-0.6542	-0.6770	-0.6874	-0.6959	-0.7018	-0.7057	-0.7091	-0.7114
-0.7100	-0.7018	-0.6047	5.2506	-0.5100	-0.2500	-0.91E-01	

Constraint(equality) : 0.1459E-01

δ_t = 0.01 ε_o = -0.1 r_o = 0.1E-02

f = 1.5000 ψ = 2.2746 Z_n = -0.5673E+03 M_n = -0.5940E+04

Optimal Point

F = 1.3580 θ = 0.2654

Design Variables (Normalized)

-1.6122	-1.5229	-1.2516	-1.1714	-0.9730	-0.7482	-0.6455	0.1703
2.8303	1.4973	1.4521	1.1349	1.0691	1.0600	-0.4795	0.9053
1.1061							

Constraints(inequality) :

-4.3270	-2.1491	-1.7532	-1.5281	-1.3607	-1.2246	-1.1067	-0.9898
-0.8693	-0.7569	-0.6270	-0.4893	-0.3110	-0.3468	-0.2211	-0.1374
-0.1185	-0.1103	-0.1342	-0.3420	-3.5407	-0.1545	-0.1792	-0.91E-01
-0.1342	-0.1323	-0.7198	-0.4683	-0.4037	-0.3740	-0.3563	-0.3439
-0.3344	-0.3266	-0.3195	-0.3133	-0.3072	-0.3078	-0.3995	-0.2802
-0.5317	-0.5963	-0.6260	-0.6437	-0.6561	-0.6656	-0.6734	-0.6805
-0.6867	-0.6928	-0.6922	-0.6005	-0.3680	-0.2654	-0.75E-01	

Constraints(equality) : 0.5878E-12

No. of r-minimization required = 5

f = 1.35793 ψ = 1.37793 Z_n = -0.2613E-03 M_n = 0.7385E-01

Note: Out of a total of 55 inequality constraints, the first 13 are curvature constraints, next 26 are constraints on the line of thrust, and the last 3 are sideconstraints on F and θ respectively.

F_{min} = 1.36012, θ = 0.269029 rad. ψ_{min} = 7.5087 x 10^3. More detailed results are not presented herein for the sake of brevity. It is seen that the value of F_{min} is quite close to 1.35793 obtained in the solution with scaling. However, the equality constraint is rather poorly satisfied in this case. Whereas in the solution with S_f the constraint reduced to a final value of the order of 10^{-13}, in this case its value reduced to only of the order of 10^{-5} (actual value = 0.8067 x 10^{-5}). This is due to the fact that whereas the values of Z_n and M_n at the final point were of the order of 10^{-3} and 10^{-2} respectively in the analysis with scaling, they are of the order of 10^2 (Z_n=4.0474x10^2) and 10^0 (M_n = 4.1686) in the case without scaling. The closeness of the minimum factor of safety in the two solutions of this problem seems only incidental and may not be the case with other problems.

C. Time Considerations in the Indirect and the Direct procedures:

Table 4.5 presents, with reference to the current example problem, the cpu time required in the computations using the two procedures. From the table it is seen that true to the expectations, the Direct procedure needs cpu time only about one-eighth of that required by the Indirect procedure. However, it may seem contradictory that the Indirect procedure needs about one-fourth the total number of function evaluations required in the Direct procedure. This can be explained by noting the difference in meaning of the term 'function evaluation' in the two cases. In the case of Direct procedure, the function evaluation means evaluation of the ψ function when the slip surface and F

and θ are known. No iteration is involved in finding F and θ for the trial surface concerned and hence Z_n and M_n are calculated only once per surface. The function evaluation in the case of Indirect procedure, on the other hand, means iteration over F and θ to satisfy the conditions of equilibrium, $Z_n=0$ and $M_n=0$ for a given trial surface and this involves several evaluations of Z_n and M_n. The time for one function evaluation in the Indirect procedure varies with the adopted iterative procedure. In the present analysis, as already mentioned, Spencer's iterative scheme using the Method of bisection has been adopted. If, either the Spencer's scheme uses Modified regula falsi technique or the proposed equation-solver based on SUMT instead of Spencer's scheme, it is expected that the CPU time required by the Indirect procedure will be appreciably less than what is presented in Table 4.5 (page 183)

D. Choice of δ_t and ε_o

With reference to the same problem, herein a study has been carried out to investigate the effect of various choices of the Extended penalty parameters δ_t and ε on the results obtained by using the Direct procedure.

As already demonstrated, by choosing δ_t = 0.01 and ε = -0.1, the obtained F_{min} = 1.35793. This choice makes $\psi \approx 2f$.

In another trial, δ_t and ε_o have been chosen as 0.001 and ε_o = -0.1 such that $\psi \approx 1.1$ f. The obtained minimum value now is F_{min} = 1.34377, the required CPU time being more or less the same. Thus it is seen that by choosing δ_t and ε_o such that ψ is quite close to f, the result is theoretically improved (whatever

may be the physical significance); this has been generally observed in case of other problems also; however, those results are not presented here. This is contrary to the suggestions (Cassis et al. 1976; Rao, 1984) that if ψ is about twice the value of f, more effective minimization results. This could be because of the presence of equality constraint in the Direct procedure.

Based on the above findings, in the case studies contained in this thesis, the Extended penalty parameters have been generally so chosen that $\psi \approx (1 - 1.1)f$.

E. Effect of Step Length and Scaling of Design Variables :

With respect to the Direct procedure, the effects of initial step length and normalization of design variables have been studied separately as follows:

To study the effect of initial step length, at first the design variables have been normalized by dividing them by their initial values. With these normalized variables, analyses have been carried out using three different initial step lengths of 0.1, 0.01 and 0.001 (out of which results obtained using 0.01 have already been reported). It has been observed that while the final design vectors in the three cases are marginally different, the F_{min} values remain almost unchanged. However, the time of computation increases substantially when smaller step lengths are used.

To study the effect of normalization of the design variables, if any, analyses have been carried out for the following two cases:

Case I: design variables are normalized as in above.

Case II: design variables are not normalized but adjusted as discussed in Chapter 2.

Once again, the difference in the results obtained in the two cases are not significant and as such these results are not presented here.

F. Progress of Minimization

Figures 4.2 and 4.3 show the variation of the f and ψ functions with the penalty parameter (r) as well as with the number of function evaluations in case of Direct and Indirect procedures respectively . It is seen that while in the Indirect procedure the decrease in the function values is monotonous, it is not so in the case of Direct procedure. On inspection of the detailed results it has been observed in the latter case that in the first few cycles the function values (f and ψ) decrease continually till they reach some minimum values and the scheme shows a tendency to keep all the inequality constraints satisfied. However, at these stages the equality constraint remains violated i.e., its value does not sufficiently reduce. Afterwards, the minimization procedure attempts to effectively satisfy the equality constraint by re-adjusting two design variables in particular, namely, F and θ. This eventually results in subsequent increase in the value of F till convergence is achieved.

G. Solution by Direct Procedure using DFP method

An attempt has been made herein to solve the same problem with the same initial conditions by the Direct procedure using

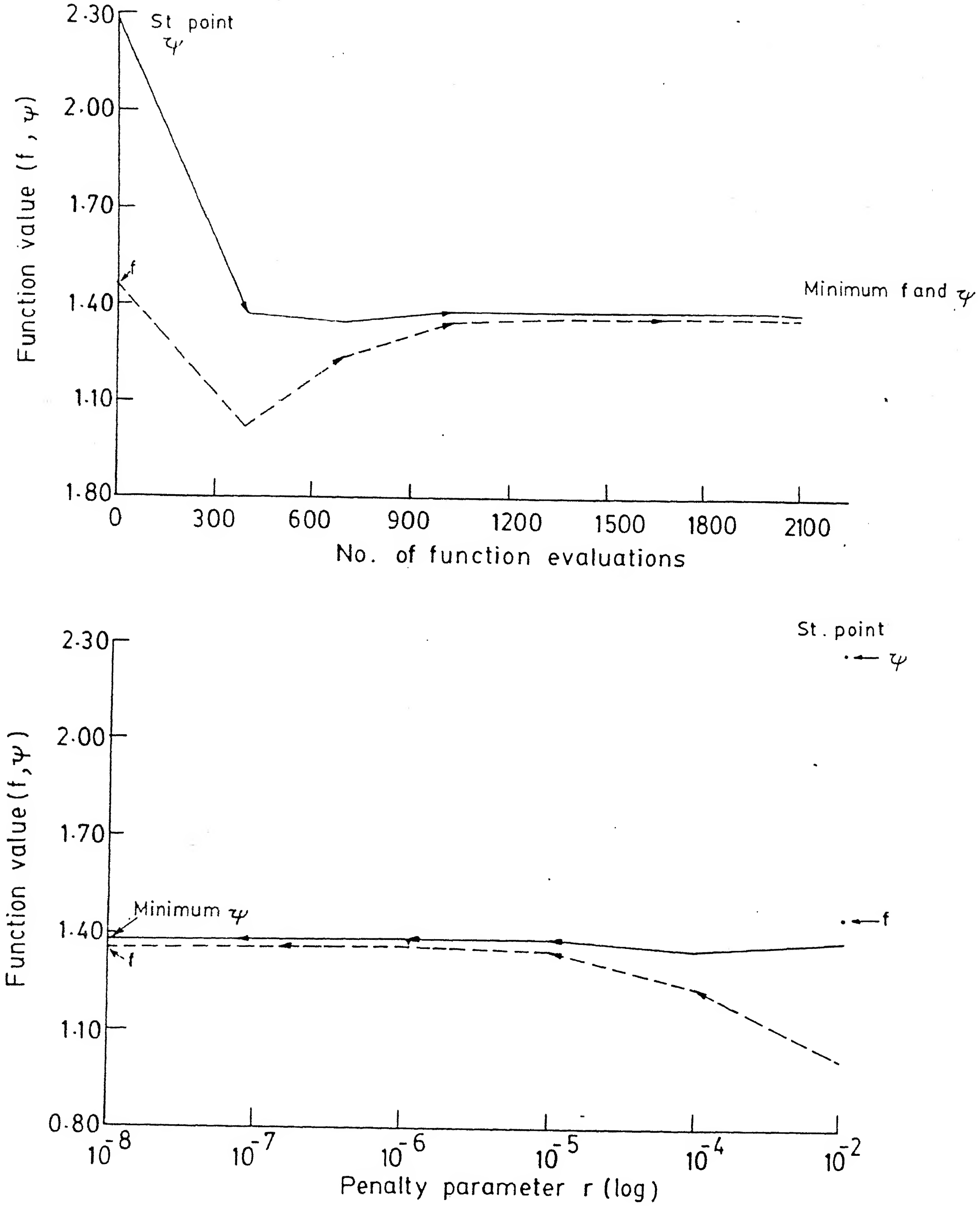


FIG. 4·2 PROGRESS OF MINIMIZATION (Direct procedure)

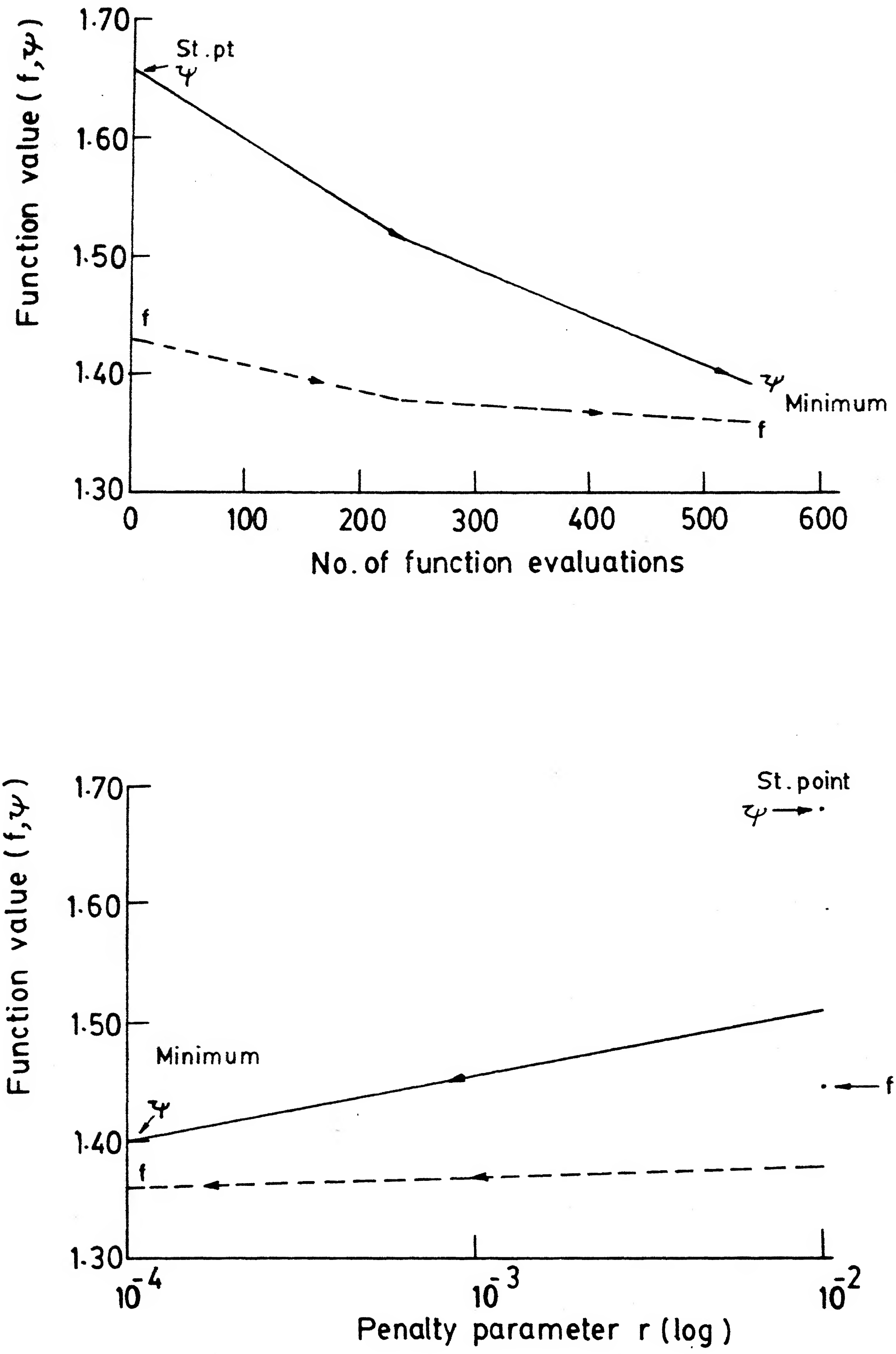


FIG. 4.3 PROGRESS OF MINIMIZATION (Indirect procedure)

in the minimization model, the Davidon-Fletcher-Powell method of unconstrained minimization coupled with the Extended penalty function formulation. The DFP method belongs to a class of unconstrained minimization methods which are based on gradients. However, in the present case the objective and the constraint functions evade explicit differentiation. The difficulty is inherent in the methods of analysis e.g., Janbu's method and the Spencer's method themselves. The finite difference technique, therefore, has been taken recourse to for numerical evaluation of (approximate) gradients. The central difference formula has been used to obtain the first partial derivatives of the ψ- function as :

$$\frac{\partial\psi(D,\ r_k)}{\partial d_i} = \frac{\psi(D^{(i+)}, r_k) - \psi(D^{(i-)}, r_k)}{2\Delta d_i}$$

where $D^{(i+)} = (d_1,\ d_2, \ldots\ldots\ldots, d_i+\Delta d_i, \ldots\ldots d_{n_{dv}})$

$D^{(i-)} = (d_1,\ d_2, \ldots\ldots\ldots, d_i-\Delta d_i, \ldots\ldots d_{n_{dv}})$

and, Δd_i = some small change in d_i, $i = 1, 2, \ldots\ldots, n_{dv}$.

The difficulty, again, is that it is not known beforehand what value of Δd_i should be taken to get accurate results. Fox (1971) has pointed out that "if Δd_i is taken too small, the result will be mostly round off noise; if Δd_i is too large, the truncation error will be high." It should, however, be smaller than the initial step length chosen in a particular case. Considering this, corresponding to the chosen common step length of Ø.Ø1 for the normalized design variables, a common value of Δd_i = Ø.ØØ5 has been adopted. δ_t and ε_o values used in the

earlier solution using Powell's method have been retained.

One Dimensional Search : Usually, the Cubic interpolation technique is employed for one dimensional minimization within the DFP method of multidimensional unconstrained minimization. Since the Cubic interpolation technique makes use of gradients in addition to function values, a good number of gradient evaluations are involved in finding the optimum (λ^*) along any direction. The success of the DFP method depends on the accuracy with which λ^* is found out.

In the present analysis also the Cubic interpolation technique was used initially. However, the DFP method was found not to converge. This is possibly due to the inaccurate determination of the λ^* values due to the inaccurately evaluated gradients. By adopting various values of Δd_i it was found that the calculated gradients were widely different. Repeated trials using various step lengths and Δd_i values did not help convergence.

As an alternative, therefore, the Quadratic interpolation technique, which does not require gradients, was used for the one dimensional search. Convergence has now been achieved. The final results are as follows :

F_{min} = 1.40 (as against 1.36 obtained using the Powell's method)

θ = 0.24825 rad.

The final value of the equality constraint

= 0.123×10^{-6} (in place of 0.5878×10^{-12} by Powell's method)

This is, thus, a much poorer convergence and inferior result as compared to what was obtained by using Powell's method. The obtained shear surfaces having minimum factors of safety and the calculated internal stress and forces are not presented here as such.

TABLE 4.5

Time Requirement in the Direct and Indirect Procedure

Procedure used	Number of Functions Evaluations	CPU time required (secs)
Indirect	512	480
Direct	2037	63

3. Baker (1980) using non-circular analysis and dynamic programming technique.

In the present studies, analyses have been carried out using both Janbu's and Spencer's methods coupled with the developed procedure based on SUMT. In the case of Spencer's method, the Direct procedure being more effective as well as more efficient than the Indirect procedure, as demonstrated earlier, has been used. All analyses are carried out with respect to general slip surface using discretization Model I.

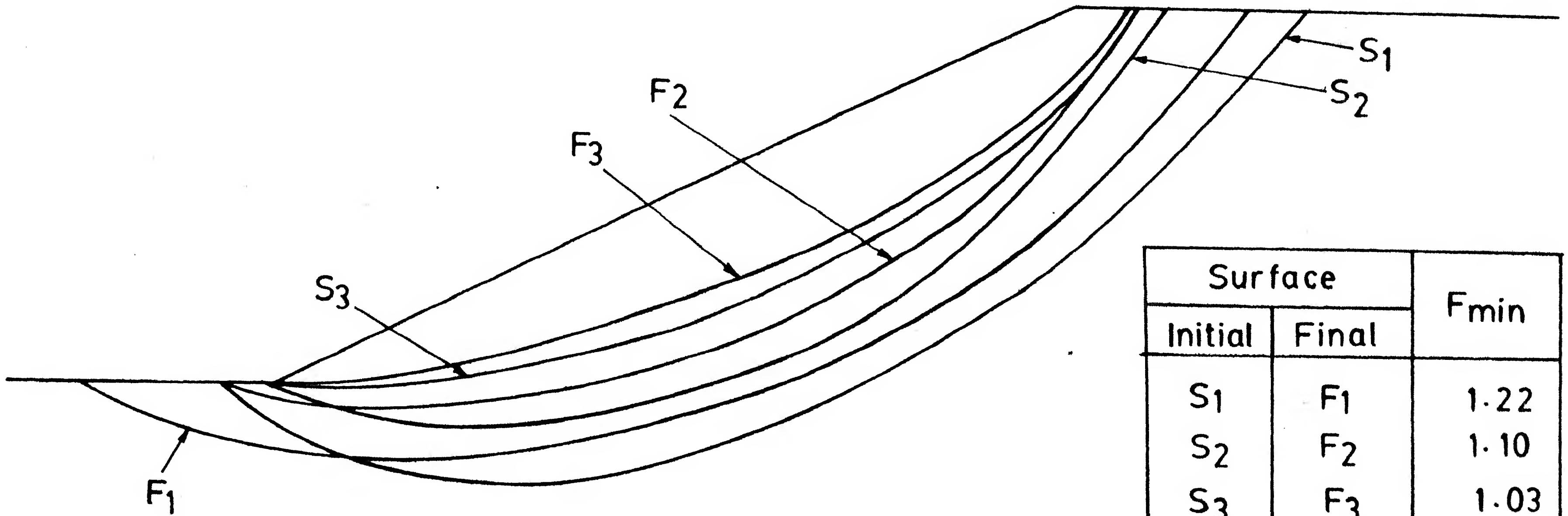
4.3.2 Solution By Spencer's Method

General

To ensure a global minimum and also to study the starting point dependence, three widely different starting points have been considered. They correspond to the slip surfaces marked S_1, S_2 and S_3 respectively in Figure 4.5. To start with, it has been assumed that k=1 throughout i.e., the interslice forces are parallel and that no tension crack is present.

Results and Discussion

The obtained final surfaces marked F_1, F_2, F_3 in Figure 4.5 correspond to the starting surfaces S_1,S_2,S_3 respectively. The corresponding minimum factors of safety are 1.28. 1.10 and 1.03 for F_1,F_2 and F_3 respectively. This clearly shows that the solution is starting point dependent as is usually the case with any nonlinear programming problem. Among the three final surfaces, F_3 yields the smallest F_{min} and hence it is most likely to be the critical surface in the present problem. On examination of the detailed output it is seen that although the



Surface		F_{min}
Initial	Final	
S_1	F_1	1·22
S_2	F_2	1·10
S_3	F_3	1·03

FIG. 4·5 STUDY OF STARTING POINT DEPENDENCE
(Example problem 4·2)

line of thrust for total stress remains entirely within the middle-third (these results, however, are not presented here), the solution is unacceptable as the effective interslice force (Z') at the last interslice boundary is found to be negative. As a corrective measure, tension crack has been introduced in the analysis with water pressure acting in it. The depth of tension crack has been treated as a design variable starting with a trial depth of 3.0 m based on Spencer's chart (Spencer, 1968). The revised solution is found to be acceptable. The value of F_{min} has reduced slightly to a value of 1.004. The corresponding θ = 0.4164 rad. The final depth of tension crack has been obtained as 4.8929 m. The critical shear surface so obtained is shown in Figure 4.6 and the associated line of thrust and other internal variables are presented in Table 4.6. It is seen from Table 4.6 that both the line of total thrust and the line of effective thrust lie within the middle-third. All the F_v values are greater than the value of F. The Table also shows that the effective normal and shear stresses at slice bases are all positive as required for an acceptable solution. Thus it is observed that the imposition of constraints on line of thrust in the form proposed in Chapter 3 together with the introduction of tension crack has resulted in an acceptable solution. The difference between the values of F_{min} in the unacceptable and the acceptable solutions in this case is about 3%.

For this last analysis leading to the acceptable solution, a total of thirteen slices have been considered; the number of design variables is 17 including the depth of tension crack. The

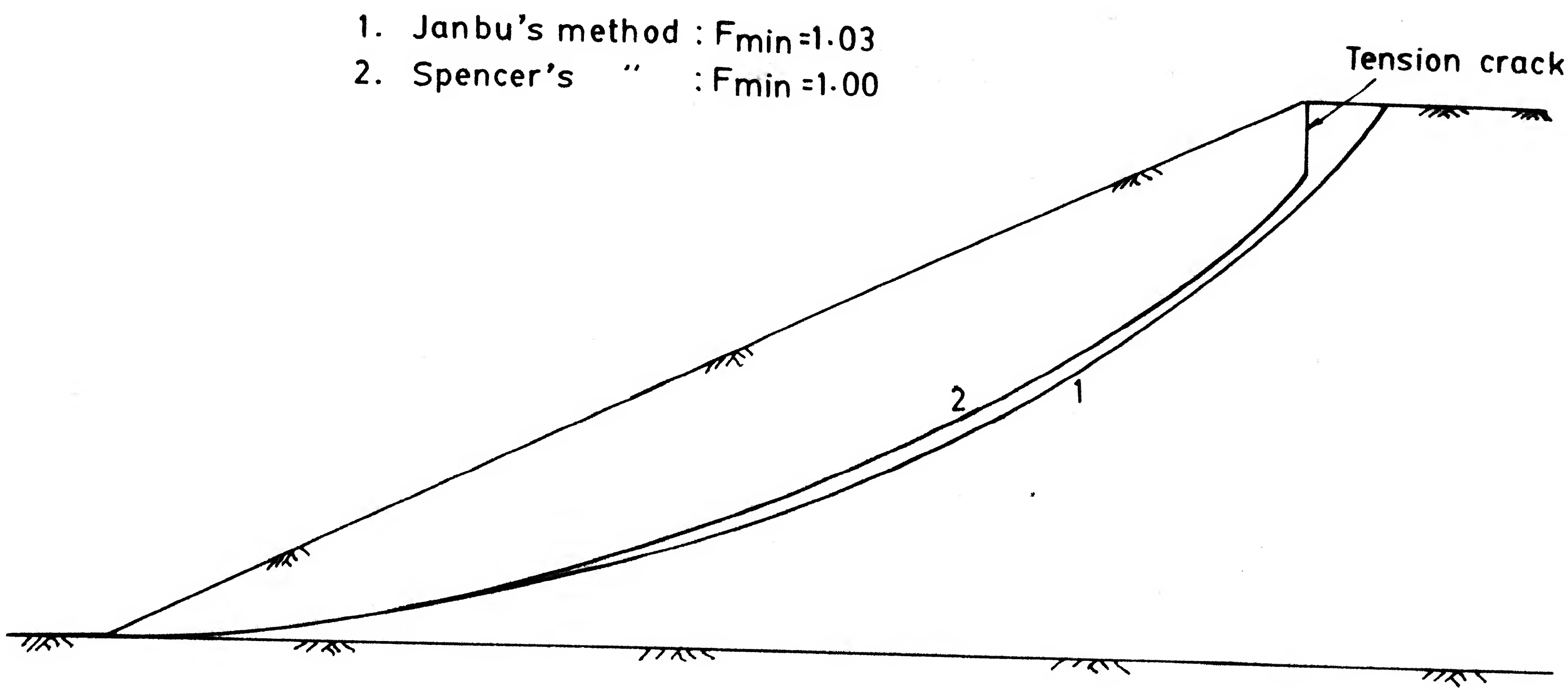


FIG. 4·6 CRITICAL SHEAR SURFACES FOR EXAMPLE PROBLEM 4·2 BY JANBU'S AND SPENCER'S METHODS OF ANALYSIS

Table 4.6

Calculated Responses Associated with the Solution to ExampleProb. 4.2 by Spencer Method

Slice No .	σ' kPa	τ kPa	L/H	L′/H	$\frac{Z'}{\gamma b H_t}$	F_v
1	21.89	30.17				
			0.57	0.61	0.05	3.43
2	44.68	49.19				
			0.42	0.45	0.09	2.96
3	55.08	58.00				
			0.39	0.41	0.13	2.71
4	59.50	62.54				
			0.37	0.39	0.16	2.56
5	65.00	64.47				
			0.36	0.37	0.18	2.46
6	63.00	64.45				
			0.35	0.36	0.18	2.40
7	60.00	62.99				
			0.35	0.36	0.17	2.38
8	58.50	60.29				
			0.34	0.35	0.15	2.39
9	56.15	55.66				
			0.34	0.35	0.13	2.45
10	44.19	48.21				
			0.34	0.35	0.09	2.61
11	41.08	45.91				
			0.35	0.36	0.06	2.83
12	29.71	35.31				
			0.37	0.48	0.03	3.45
13	14.10	21.89				

Table 4.7

Design Vector and Constraints for the Example Problem 4.2
(Solution by Spencer's Method)

Starting Point

F = 1.1000 θ = 0.4000

Design Variables (Not normalized)

-0.51	0.00	0.515	1.50	2.50	4.25	6.40	9.00	11.95
15.00	18.85	22.75	61.20	-5.00	1.10	0.40	27.50	

Constraints (inequality) :

-0.9941	-0.7775	-0.6685	-0.5857	-0.5242	-0.4698	-0.4163	-0.3618
-0.2960	-0.2342	-0.1637	-0.0953	-1.0200	-0.0050	-0.4700	-0.0150
-0.7500	-0.4000	-0.4500	-0.3500	-0.0999	-0.8000	-0.0500	-0.8500
-0.4505	-0.3854	-0.3532	-0.3424	-0.3325	-0.3316	-0.3333	-0.3408
-0.3630	-0.4186	-0.7233	-18.82	-0.5496	-0.6146	-0.6468	-0.6576
-0.6675	-0.6684	-0.6667	-0.6592	-0.6370	-0.5814	-0.2767	17.82
-0.1001	-0.3990	-0.0625					

Constraint (equality) : 0.800E-02

δ_t = 0.001 ε_o = -0.1 r_o= 0.1E-03

f = 1.1000 ψ = 1.3014 Z_n = -0.2534E+03 M_n = -0.1425E+04

Optimal Point

F = 1.0041 θ = 0.4164

Design Variables (Not normalized)

-0.2338	0.2190	1.0750	2.2580	3.7229	5.4468	7.4141	9.6194
12.1122	15.0213	17.9337	21.5400	62.1160	-0.0448	1.0041	0.4164
25.6071							

Constraints (inequality) :

-0.7357	-0.6213	-0.5381	-0.4811	-0.4286	-0.3796	-0.3329	-0.2866
-0.2389	-0.1883	-0.1322	-0.0672	-0.6866	-0.4032	-0.3270	-0.2819
-0.2590	-0.2433	-0.2381	-0.2874	-0.4164	-0.0033	-0.6939	-0.4607
-0.5682	-0.4226	-0.3864	-0.3695	-0.3592	-0.3520	-0.3467	-0.3428
-0.3404	-0.3395	-0.3451	-0.3729	-0.4318	-0.5774	-0.6136	-0.6305
-0.6408	-0.6480	-0.6533	-0.6572	-0.6596	-0.6605	-0.6549	-0.6271
-0.0141	-0.4164	-0.4611					

Constraint (equality) : 0.7723E-14

No. of r-minimization required = 7

f = 1.0041 ψ = 1.0061 Z_n = 0.3719E-04 M_n= 0.3048E-02

Note: Out of a total of 51 inequality constrants, the first 12 are bounday constraints, next 12 are curvature constraints, next 24 are constraints on the line of thrust, next 3 are side constraints

design vector and constraints at the beginning and end of minimization are given in Table 4.7. The design variables are not normalized in this final analysis as it has been previously observed that normalization does not make significant change in the final result. The table also indicates that even though the initial design vector is infeasible the proposed scheme brings out a feasible optimal solution quite efficiently.

4.3.3 Solution by Janbu's Method

General

In the analysis using Janbu's method the same number of slices(13) as in analysis using Spencer's method has been adopted. The assumed line of thrust is as given in Table 4.8. Such a line of thrust has been adopted based on Janbu's suggestion (Janbu,1973). No tension crack has been assumed to be present.

Another point of distinction between the behaviour of the two methods is that while in the analysis by Janbu's method the imposition of end constraint has been necessary for convergence, that by Spencer's method has converged without such constraints.This is possibly due to the fact that in Janbu's method the line of thrust is pre-assigned whereas in Spencer's method the line of thrust is allowed to vary its position.

Results and Discussion

As in case of solution by Spencer's method described in Section 4.3.2, in order to study the starting point dependence three widely apart starting surfaces have been considered. The factor of safety corresponding to the final surfaces have been

obtained as 1.59, 1.0318 and 1.1575 respectively. However, detailed results are not presented herein. Thus the starting point dependence is observed with Janbu's method also. Of the three trials, the smallest value obtained for F_{min} is 1.0318. The corresponding (critical) slip surface is shown in Figure 4.6. As will be demonstrated in the following, the solution satisfies all the constraints and the acceptability criteria.

Table 4.9 shows the F_v- values, the interslice forces E and T and the effective normal and shear stresses on the bases of the individual slices. It is seen that all F_v values are greater than F_{min} and the signs of the forces and stresses are admissible. The design variables and constraints at the starting and optimal points are presented in Table 4.10. The table shows that even though the initial design vector is infeasible the optimal design vector is a feasible one.

Comparison with Solution by Spencer's Method:

The minimum factors of safety obtained using the two methods are found to be nearly equal (1.03 in Janbu's method against 1.00 in Spencer's method). The critical surfaces also show close resemblance except at the upper end where they fall apart as shown in Figure 4.6. It is noticeable that an acceptable solution has been obtained using Janbu's method even without assuming tension crack.

4.3.4 Comparison with Other Solutions Involving Spencer's Method:

Table 4.11 shows the values of the minimum factors of safety F_{min} along with the interslice force inclinations θ, obtained by

Table 4.8

Assumed line of thrust in the solution by Janbu's GPS for the Example Problem 4.2

Inter slice boundary	1	2	3	4	5	6	7	8	9	10	11	12
h_t/z	0.34	0.35	0.36	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.43

Table 4.9

Internal Variables Associated with the Solution by Janbu's GPS for the Example Problem 4.2

Slice No.	σ' kPa	τ kPa	E kN/m	-T kN/m	F_v
1	4.08	14.94			
			125.6	31.8	1.416
2	18.72	26.85			
			380.4	153.9	1.032
3	28.35	35.16			
			657.5	293.7	1.032
4	38.97	43.32			
			911.5	434.7	1.033
5	58.23	58.98			
			1059.3	498.6	1.111
6	71.39	69.68			
			1096.7	504.2	1.203
7	76.03	72.64			
			1068.0	486.5	1.276
8	76.52	73.86			
			968.3	442.5	1.348
9	75.64	73.14			
			820.3	373.6	1.483
10	71.72	69.96			
			630.1	285.0	1.540
11	64.27	63.90			
			411.9	183.6	1.695
12	52.13	54.03			
			186.6	79.6	1.999
13	30.45	36.40			

Table 4.10

Design Vector and Constraints for the Example Problem 4.2
(Solution by Janbu's Method)

No. of slices = 13

No. of design variables = n_{dv} = 14

Starting Point

F = 1.0515

Design Vector (Not normalized) :

5.8458	10.4208	14.7417	18.3000	21.3500	23.8917	26.1792	27.9583
28.9750	29.9815	30.5508	31.0185	61.6000	-5.0000		

Constraints (All Inequality) :

-0.1896	-0.2557	-0.3133	-0.3460	-0.3620	-0.3614	-0.3524	
-0.3268	-0.2761	-0.2251	-0.1598	-0.0911	-0.3332		
-1.2719	-0.2541	-0.7625	-0.5094	-0.5093	-0.2542	-0.5084	
-0.7624	-0.0102	-0.4372	-0.1016	-0.9862			
-0.3269	-0.0582	0.0685	-0.0171	-0.0755	-0.0712	-0.1821	-0.1685
-0.2996	-0.3021	-0.3916	-0.4634	-0.0615			

Extended Penalty function has been used as some constraints are violated

ε = -0.100 δ_t = -0.001 r_o = 0.1×10^{-3} f = 1.0515 ψ = 1.0738

Optimal Point

F = 1.0317

Design Vector (Not normalized) :

6.1799	10.7469	14.7699	18.2655	21.0338	23.2638	25.1738
26.8176	28.2053	29.3363	30.1825	30.6691	61.0541	-4.3860

Constraints (All inequality) :

-0.1920	-0.2592	-0.3086	-0.3407	-0.3489	-0.3395	-0.3196
-0.2911	-0.2538	-0.2083	-0.1537	-0.0872		
-0.2395						
-1.6129	-0.5440	-0.5274	-0.7273	-0.5383	-0.3200	-0.2671
-0.2543	-0.2576	-0.2848	-0.3596	-0.6557		
-0.0756	-2.13E-04	-2.52E-04	-1.03E-03	-0.0718	-0.1423	-0.1911
-0.2348	-0.2798	-0.3299	-0.3912	-0.4839		
-0.0417						

No. of r-minimizations required = 6

f = 1.0317 ψ = 1.0318

It is noticeable that all the constraints are now satisfied.

Note: Out of a total of 38 constraints, first 13 are boundary constraints, including one end constraints at the top end , next 12 are curvature constraints, next 12 are on F_v, and the last one is side constraint on F

Table 4.11

Comparison of Various Solutions to Example Problem 4.2
(All the solution are based on Spencer's method)

Sl. No.	Methodology used: Shape of slip surface	Methodology used: Minimization technique	Minimum Factor of safety F_{min}	Corresponding Interslice force angle in degree θ	Reference
1	Circular	Grid search	1.07	22.50	Spencer (1967)
2.	General	Variation technique			Narayan et al. (1976)
		(i) Direct	1.13	23.41	
		(ii) Indirect	1.12	23.72	
3.	General	Dynamic programming	1.02	22.50	Baker (1980)
4.	General	SUMT			Present solution
		(i) acceptable	1.00	23.85	
		(ii) unacceptable	1.03	24.66	

Table 4.12

Line of Thrust for effective Stress for Spencer's Critical Circle
(Example Problem 4.2)

Interslice boundary	L/H	L′/H	$\frac{Z'}{\gamma b H_t}$
1	0.43	0.45	0.06
2	0.37	0.38	0.14
3	0.35	0.36	0.25
4	0.34	0.34	0.31
5	0.33	0.34	0.39
6	0.33	0.33	0.45
7	0.33	0.33	0.50
8	0.33	0.32	0.53
9	0.32	0.32	0.54
10	0.32	0.31	0.54
11	0.32	0.31	0.52
12	0.32	0.30	0.48
13	0.32	0.30	0.43
14	0.31	0.29	0.36
15	0.31	0.27	0.29
16	0.30	0.25	0.21
17	0.30	0.22	0.14
18	0.29	0.12	0.06
19	0.28	-1.27	0.005
20	0.36	0.28	-0.02

various investigators using different techniques.

From Table 4.11 it is seen that the minimum factor of safety obtained in the present solution is comparable to Baker's solution by dynamic programming and is lower than the other two solutions (Spencer, 1967; Narayan et al., 1976). However, the critical surface, as shown in Figure 4.7 is markedly different from those obtained by other methods. It seems that the introduction of tension crack has made all the difference. Now, so far as an acceptable line of thrust is concerned, except in the case of solution by variational technique by Narayan et al., only the line of thrust for total stress were reported and shown to be satisfactory. This, however, does not necessarily imply that the line of thrust for effective stress will also be satisfactory. This has been verified by reanalysing the critical slip circle reported by Spencer (Narayan et al., 1976; Spencer, 1967). Its factor of safety has been obtained as 1.07 which is exactly the same as the reported value. 21 slices have been considered. The obtained lines of thrust have been presented in Table 4.12. As seen from this table, although the line of thrust for total stress (L/H) is more or less within the middle-third, as reported, the line of thrust for effective stress indicated by L′/H values and the effective interslice forces indicated by Z′ are far from satisfactory. As regards the other acceptability criteria that the F_v values cannot be less than F for the critical surface, while the present solution and the solution by variational technique satisfy this criteria, there is no mention about it in the other two. Possibly due to these reasons it was

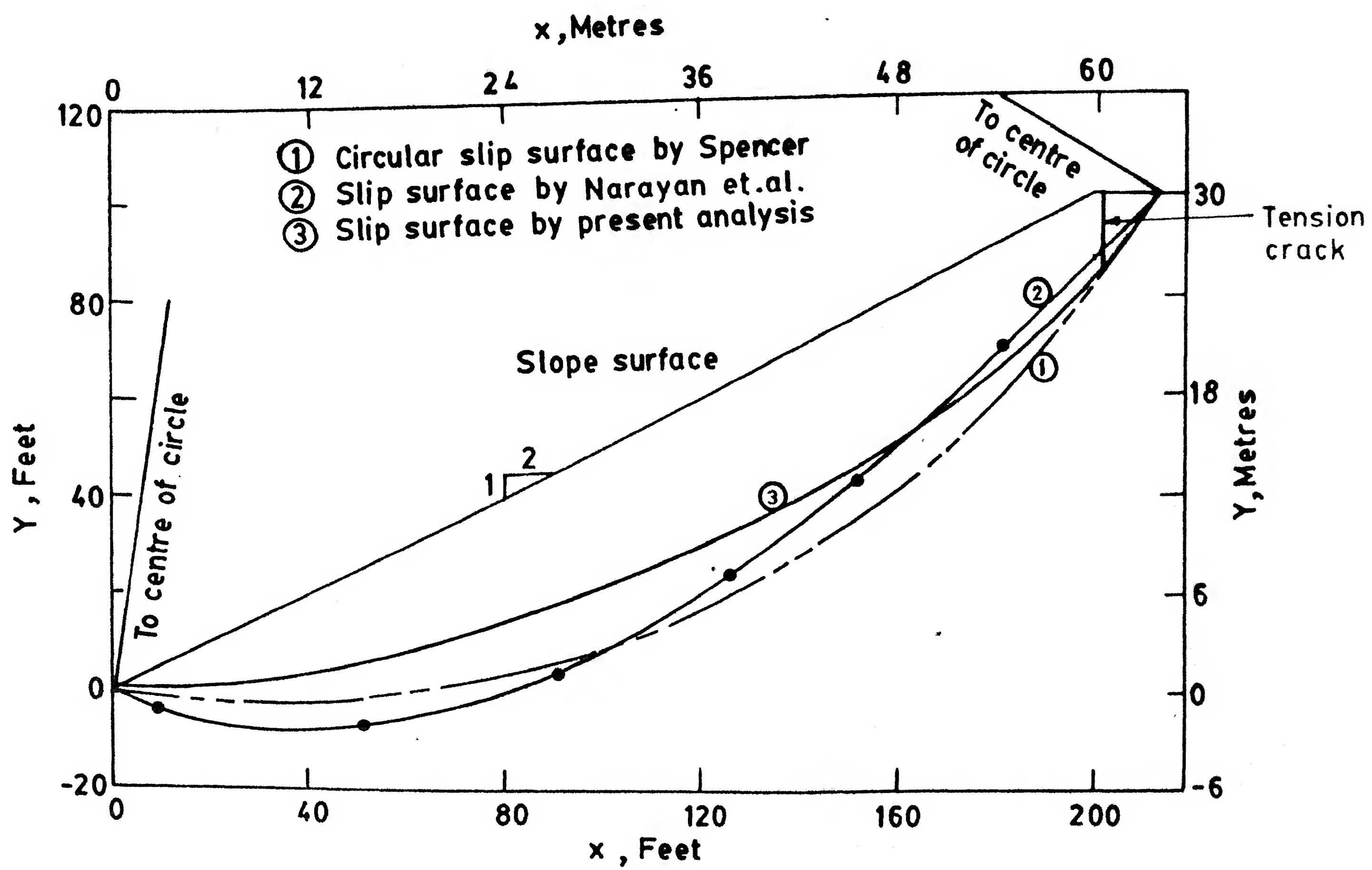


FIG. 4.7 COMPARISON OF CRITICAL SLIP SURFACES OBTAINED BY VARIOUS TECHNIQUES

not needed to assume the presence of a tension crack. In the present investigation also, a critical surface has been obtained, as mentioned earlier, without tension crack, for which the total stress line of thrust is a satisfactory one whereas the line of effective thrust is unsatisfactory.

That an acceptable solution was obtained using variational technique even without the assumption of presence of tension crack may be due to the difference in the interslice force function considered in the Narayan et al.'s solution and the present analysis. In the former the interslice force function, f(x), was based on effective stress whereas in the present case this function (k-distribution) has been based on the total stress.

4.4 EXAMPLE PROBLEM 4.3

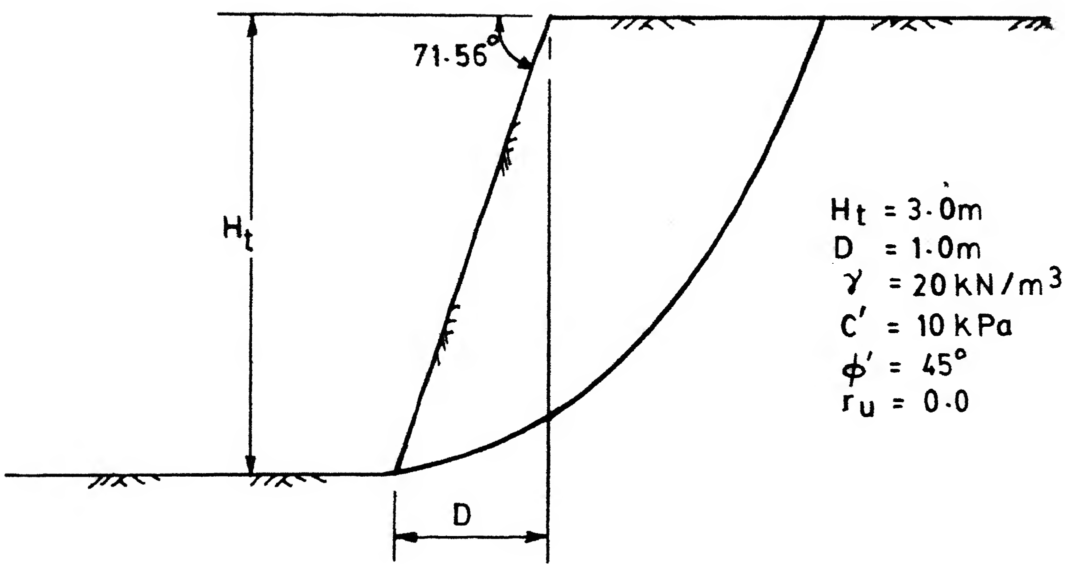


Figure 4.8 : SLOPE SECTION IN EXAMPLE PROBLEM 4.3

(Castillo and Luceno, 1982)

4.4.1 General

In the third example in this chapter a steep slope (β = 71.56^{o}) has been considered. A section of the slope along with the soil and pore pressure data has been shown in Figure 4.8. The same slope was earlier analysed by Baker and Garber and by Castillo and Revilla - as reported in the paper by Castillo and Luceno (1982) from which this example problem has been taken up. In this paper, it has been mentioned that both these analyses were carried out using variational techniques - no more details are available.

In the present studies, the developed procedure based on SUMT has been used in conjunction with both Janbu's and Spencer's method. As in Example Prob.4.1, Direct procedure for Spencer's method and Model I for discretization have been adopted.

4.4.2 Solution By Spencer's Method

To start with, the following have been adopted.

The total number of slices, n = 13

The total number of design variables n_{dv} = n+3 = 16

Interslice forces are parallel i.e., k=1, throughout

No tension crack is present

Results and Discussion:

As a first trial, the surface, ABC,passing through the toe (Figure 4.9) has been considered as the starting surface. However, even after quite a few r-minimizations, the solution did not converge. It was observed that the constraints on the line of thrust remained violated even after a number of cycles. This is not a usual occurrence. Because, as stated earlier, one of the most distinguishing features of the Extended penalty function technique is that at every stage it tends to satisfy the violated constraints, if any. Only when the problem is ill-conditioned in the sense that with the assumptions used in the analysis a feasible solution for the problem cannot be found, some of the constraints remain violated even after several cycles.

As already reported, the introduction of tension crack improves the line of thrust considerably. Considering this, tension crack of depth z_t = 1.08 m (Spencer's chart(1968) gives a

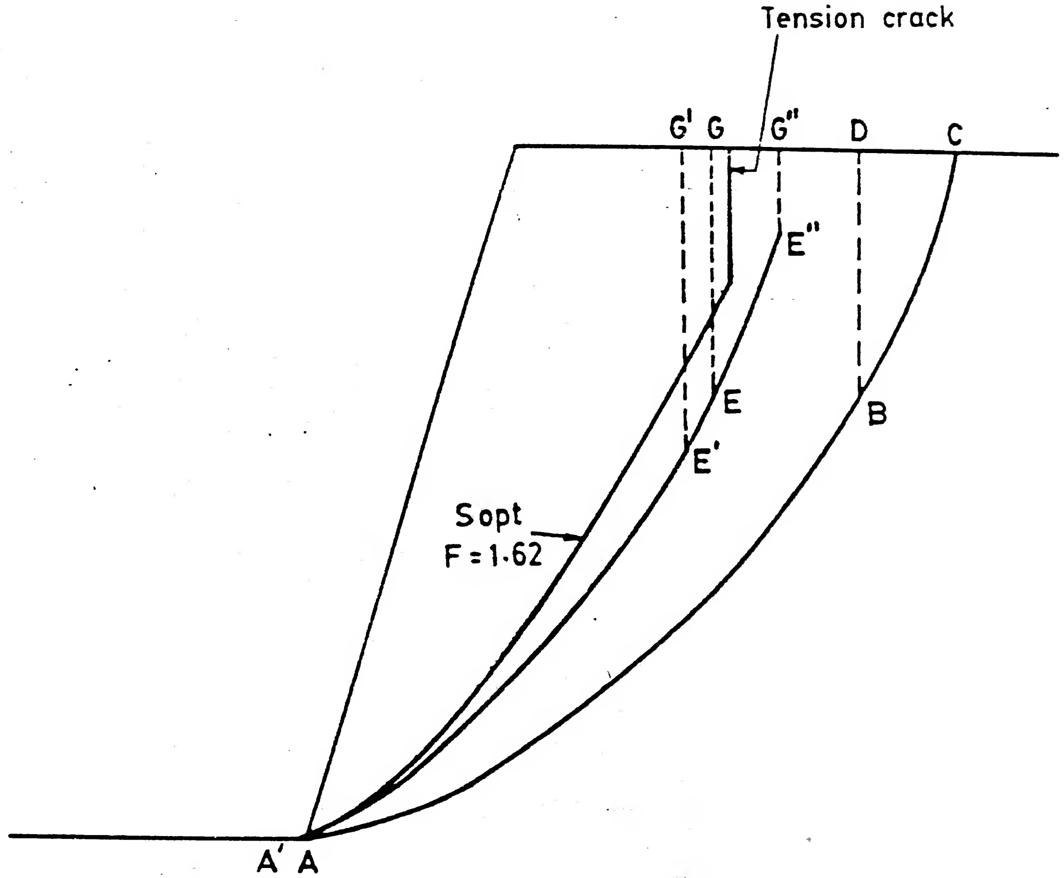


FIG. 4·9 CRITICAL SHEAR SURFACE ALONGWITH SOME TRIAL SHEAR SUREACES FOR THE EXAMPLE PROBLEM 4·3

value of about 1.2) has been assumed. The revised starting surface is marked ABD in Figure 4.9. In order to provide more flexibility z_t has been included in the design vector thus raising the number of design variables to 17. With the introduction of tension crack, the minimization progresses and converges to the final surface marked A′EG having a F_{min} = 1.54. The design vector and constraints at the starting and optimal points corresponding to this stage of analysis are presented in Table 4.13. Like the previous problems the table demonstrates the efficiency of the Extended penalty function technique in tackling infeasible design points. The depth of the tension crack increased slightly to 1.10m. Line of thrust constraints are satisfied although all L/H values are not within the middle-third. However, this surface has been found to be unacceptable as the normal stress below the base of the 13th slice at the upper end of the slip surface has a value of -11.11 kPa. This shows that when the line of thrust is not entirely within the middle-third, one needs to check the base normal stresses also.

Now, in order to satisfy the acceptability criteria, a number of trials have been performed, as explained below:

(i) Focussing the attention on the surface A′EG itself, in order to investigate what happens when the depth of the tension crack is increased still further, the surface A′E′G′ with E′G′= z_t =1.35 m was analysed to find its factor of safety and the stresses etc. as part of the solution. This result was worse than the earlier one because the value of the concerned normal

Table 4.13
Design Vector and constraints for the Example Problem 4.3
(Solution by Spencer's Method)

No. of slices = 13
No. of design variables = 17

Starting Point

F = 1.5000 θ = 0.3500

Design Variables (Not normalized)

0.0433	0.0867	0.1845	0.2833	0.4133	0.5435	0.6739	0.8333	1.0000
1.2000	1.4332	1.6667	1.0000	-1.5667	1.5000	0.3500	1.0800	

Constraints (inequality) :

-5.0000	-0.9774	-0.9548	-0.9039	-0.8524	-0.7847	-0.7169	-0.6490
-0.5660	-0.4792	-0.3750	-0.2535	-0.10E-03	-0.18E-01	-0.11E-02	
-0.31E-01	-0.2E-03	-0.2E-03	-0.28E-01	-0.72E-02	-0.33E-01	-0.34E-01	
-0.19E-01	-0.19E-01	-0.18E-01	0.15E-01	0.29E-01	0.53E-01	0.69E-01	
0.96E-01	0.1491	0.2157	0.3292	0.3105	0.7403	-0.9808	
-0.9825	-1.0154	-1.0289	-1.0529	-1.0685	-1.0962	-1.1491	
-1.2157	-1.3292	-1.5105	-1.7403	-0.5295	-0.5100	-0.3500	
-0.8990							

Constraint (equality) : 0.3093

δ_t = 0.001 ε_o = -0.10 $r_o = 0.1 \times 10^{-3}$
f = 1.5000 ψ = 1.6079 Z_n = 0.2534E+1 M_n = 0.1057E+02

Optimal Point

F = 1.5383 θ = 0.9471

Design Variables (Not normalized)

0.0447	0.1227	0.2183	0.3291	0.4536	0.5917	0.7455	0.9150
1.0972	1.2911	1.4952	1.6993	1.0698	-0.3926	1.5383	0.9471
1.0970							

Constraints (inequality) :

-5.0000	-0.9755	-0.9356	-0.8853	-0.8271	-0.7617	-0.6891	-0.6083
-0.5193	-0.4235	-0.3217	-0.2144	-0.33E-01	-0.18E-01	-0.15E-01	-0.14E-01
-0.14E-01	-0.15E-01	-0.15E-01	-0.12E-01	-0.12E-01	-0.11E-01	-0.19E-04	-0.48E-05
-0.4137	-0.2646	-0.2646	-0.2273	-0.2102	-0.2017	-0.1989	-0.2022
-0.2506	-0.3120	-0.3832	-0.4632	-0.5497	-0.5863	-0.7355	-0.7727
-0.7898	-0.7983	-0.8010	-0.7978	-0.7494	-0.5880	-0.6168	-0.5368
-0.4502	-0.4727	-0.5483	-0.9471	-0.3019	-0.65E-01		

Constraint (equality) : 0.4595E-04
No. of r-minimization required = 5

f = 1.5383 ψ = 1.5871 Z_n = 0.6100E-01 M_n= 0.1300E-03

Note: Out of a total of 54 constraints, the first 12 are bounday constraints, next 12 are curvature constraints, next 24 are constraints on the line of thrust, last 6 are side constraints on F, θ and Z_t.

stress now became -20.2 kPa. In addition to that, some of the F_v values became negative.

(ii) In the next trial, a reduced value of z_t = 0.4 m was considered and the surface A′E″G″ with E″G″ = z_t = 0.4 m was analysed. This produced a better result in the sense that the corresponding normal stress value increased (less negative, to be more precise).

(iii) In view of (ii) above, a fresh attempt was made to determine the critical surface, starting from the surface A′E″G″ and considering z_t as a design variable. The critical surface now obtained is marked S_{opt} in Figure 4.9. This has a F_{min} = 1.609. However, the concerned normal stress is still negative, σ_n = -1.64 kPa.

(iv) It is known that proper choice of k-distribution is very vital for achieving good results. Here it is relevant to note the observation made by Soriano (1976) that for steep and highly cohesive slopes the assumption of parallel side forces is not compatible with equilibrium conditions. Keeping the above in view a different k-distribution shown as 'input' in Figure 4.10 has been tried. Even with this choice the value for the concerned normal effective stress remained negative. To systematically find a suitable k-distribution, k-values have been incorporated in the design vector and the distribution as mentioned has been adopted as an initial guess. The analysis has been carried out for the surface S_{opt} as shown in the Figure 4.9.

The function evaluation scheme based on SUMT as discussed in Chapter 3 has been modified to include k-values in the design

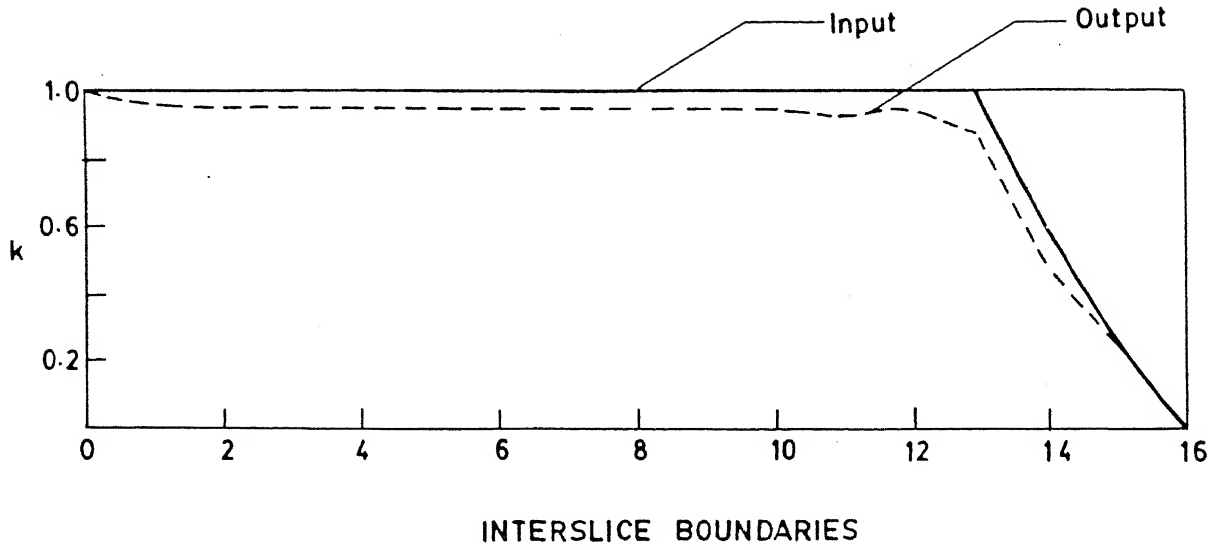


FIG. 4·10 OPTIMAL INTERSLICE FORCE FUNCTION FOR ACCEPTABLE SOLUTION (Example problem 4·3)

vector in addition to F and θ subject to the non-negativity of all the effective normal stresses at the slice bases. This has resulted in acceptable values for the stresses and other responses, shown in Table 4.14. The factor of safety has been obtained as 1.6161 and the interslice force angle θ as 0.98896. The corresponding Z_n = -0.9931E-02, M_n= 0.2931E-04. The input and output k-values have been presented in Figure 4.10. 15 slices have been considered in this analysis.

From Figure 4.10 it is seen that the obtained k-distribution is irregular. However, when the interslice force function is found by iterative manner, there are evidences of such irregular profiles (Whitman and Bailey, 1967).

The obtained values of factor of safety (1.6161) corresponding to the acceptable solution is insignificantly different from the value (1.609) corresponding to the unacceptable solution.

From Table 4.14 it is seen that although all the Z' and σ' values are positive, the line of thrust does not lie within the middle-third of the heights of most of the interslice boundaries. It may be possible to obtain better results if additional constraints on the line of thrust are incorporated in the formulation as discussed. The best results may probably be obtained if the critical surface is determined after incorporating the k-values in the design vector. However, it should be noted that the number of design variables for such an analysis will be nearly doubled and it may lead to computational difficulties.

Table 4.14

Calculated Responses Associated with the Solution to Example Prob. 4.3 by Spencer Method

Slice No.	σ' kPa	τ kPa	L/H	L′/H	$\frac{Z'}{\gamma b H_t}$	F_v
1	3.12	8.12				
			0.16	0.16	0.12	3.06
2	10.24	12.52				
			0.15	0.15	0.22	3.22
3	11.75	13.46				
			0.15	0.15	0.30	3.47
4	13.44	14.50				
			0.15	0.15	0.35	3.74
5	14.14	14.94				
			0.14	0.14	0.38	4.19
6	14.62	15.23				
			0.14	0.14	0.37	4.88
7	14.64	15.25				
			0.14	0.14	0.33	6.09
8	14.20	14.97				
			0.17	0.17	0.27	7.95
9	13.16	14.33				
			0.24	0.24	0.20	9.65
10	10.93	12.95				
			0.36	0.36	0.14	12.93
11	8.14	11.22				
			0.56	0.56	0.10	25.36
12	7.23	10.66				
			0.84	0.84	0.07	48.74
13	6.66	10.31				
			0.80	0.80	0.08	22.12
14	3.04	8.07				
			0.48	0.48	0.13	26.44
15	0.17	6.29				

Comparison with other solutions

1. Variational Techniques:

Castillo and Luceno (1982) reported that significantly different results were obtained for the present problem when different variational formulations due to i) Baker and Garber and ii) Castillo and Revilla were used — as given in Table 4.15. The wide discrepancy in the factor of safety values may be due to the manner by which the acceptability criteria have been taken care of.

2. Dynamic Programming Technique:

The program SSOPT based on dynamic programming technique developed by Baker (1979) has been used to analyse this slope. It has given a factor of safety of 1.75. However, it is observed from the partial output data presented in Table 4.16 that,

i) except for the 5 interslice boundaries in the middle portion of the slip surface, the line of thrust has coincided with the slip surface,

ii) the value of the resultant interslice force is negative (-0.27) kN at the 9^{th} interslice boundary and

iii) the value of the normal force at the base of a slice (slice No.8) is negative (-0.11) kN.

The above observations obviously indicate that the solution is unacceptable. In the course of the study of this problem it has been observed that choice of three of the user supplied parameters have marked influence on the working of the program SSOPT. These are as follows :

FSMN- the factor of safety associated with the face of the slope.

Table 4.15

Comparison of various Solutions for the Example Problem 4.3

Sl.No.	Method	F_{min}	θ	Remarks
1.	Variational formulation due to Baker and Garber	1.80	-	Not known whether acceptability criteria have been checked.
2.	Variational formulation due to Castillo and Revilla	1.60	-	The above have not been checked (Castillo, 1989).
3.	Dynamic programming (Present study using the program SSOPT)	1.75	1.52°	Acceptability criteria not satisfied.
4.	Present solution based on S.U.M.T.	1.62	55.66°	Acceptability criteria are satisfied
5.	Taylor's charts extrapolation	1.65	-	———

TABLE 4.16

A Part of the SSOPT Output for the Example Problem 4.3

THE CRITICAL PATH IS

J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
YC	0	0	0	0	0	0	0	0	8	16	18	19	20	20	20	20	20	20	20	20

SLICES DATA

J	CN	L	WT	CL	U	UTX	UTY	UM	HT	N	PR	SK	Q	Z
1	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	1	0.00	0.00
2	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	1	0.00	0.00
3	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.36	1	0.00	0.00
4	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	1	0.00	0.00
5	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	1	0.00	0.00
6	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	1	0.00	0.00
7	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	1	0.00	0.00
8	1.0293	1.89	0.20	0.71	0.00	0.00	0.00	0.00	0.30	-0.11	1.52	1	-0.27	1.30
9	1.0293	1.89	2.80	1.89	0.00	0.00	0.00	0.00	1.40	1.85	2.60	1	0.16	4.28
10	1.1193	0.57	1.44	0.57	0.00	0.00	0.00	0.00	1.80	1.09	3.58	1	0.26	3.95
11	1.1193	0.28	0.48	0.20	0.00	0.00	0.00	0.00	[illegible].20	0.33	3.87	1	0.24	3.97
12	1.1193	0.28	0.32	0.28	0.00	0.00	0.00	0.00	0.80	0.19	4.07	1	0.19	4.00
13	0.9845	0.20	0.12	0.20	0.00	0.00	0.00	0.00	0.30	0.12	4.16	1	0.00	0.00
14	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.14	1	0.00	0.00
15	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.12	1	0.00	0.00
16	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.09	1	0.00	0.00
17	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.06	1	0.00	0.00
18	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.04	1	0.00	0.00
19	0.9996	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.01	1	0.00	0.00

TET = 0.151677E+01   F = 0.174777E+01

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

END OF RUN

FO - first estimate of the minimum factor of safety of the slope.

TETO- first estimate of the inclination of the interslice forces in degrees.

The relevant study is presented in Table 4.17 which shows that the program SSOPT based on dynamic programming is starting point dependent.

3. Taylor's Chart:

As it is, Taylor's chart does not cover the case of this problem. On extrapolation, a value of approximately 1.65 is obtained which is in close agreement with the value obtained in the present analysis. However, it is interesting to note that in the typical slip surface shown for steep slopes alongside the charts, the upper portion is marked as dashed.

4.4.3 Solution by Janbu's Method

The same procedure as described in Section 4.3.3 has been followed. However, even on introduction of tension crack as design variable, no acceptable solution could be obtained although a reasonable value of the minimum factor of safety F_{min} = 1.6452 was obtained. The solution is unacceptable as some of the normal stresses are found to be negative. ii) Some of the F_v values have been found to be negative iii) Some of the vertical shear forces (T) are found to be positive.

The reason for not obtaining an acceptable solution is possibly inherent in the Janbu's GPS procedure itself. In this method the line of thrust is assumed and not determined as in Spencer's method. This may have affected the flexibility in the operation of the numerical scheme. The assumption of other line of thrust positions may yield better result. However, such a study has not been undertaken here.

Table 4.17

Starting Point Dependance of the Program SSOPT

Trial	FSMN	FO	TETO	Message
1	1.Ø	1.Ø	1.Ø	CONSTRAINTS EXCLUDE SOLUTION
2	2.Ø	1.5Ø	1Ø.Ø	NO SOLUTION FOR TET IN THE RANGE Ø < TET < PAY/2
3	2.Ø	1.5Ø	1.Ø	------do------
4	4.Ø	5.Ø	3Ø.Ø	Convergence

4.5 EXAMPLE PROBLEM 4.4

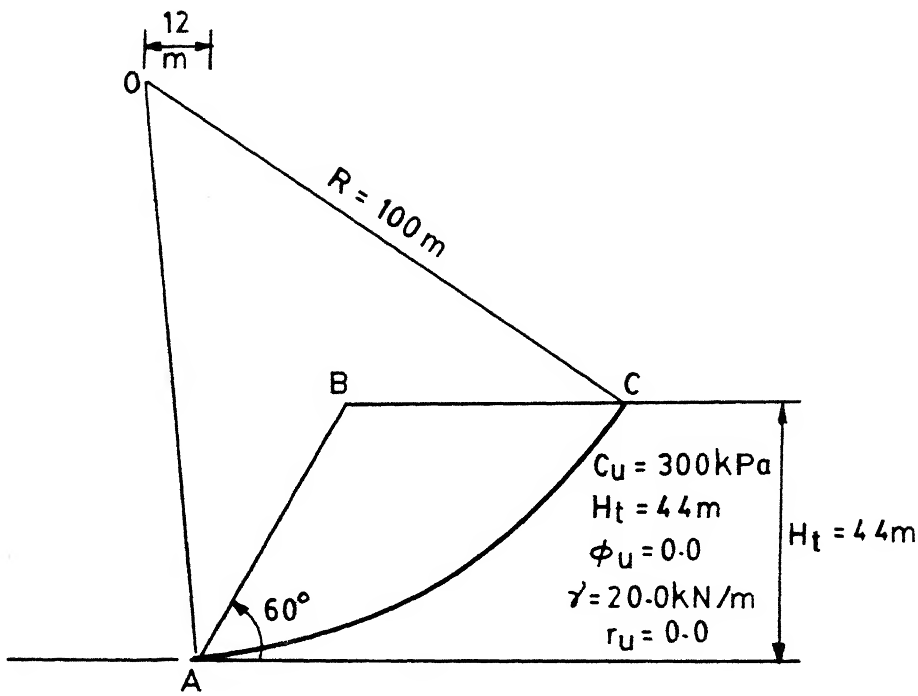


Figure 4.11 : SLOPE SECTION IN EXAMPLE PROBLEM 4.4

(Soriano, 1976)

4.5.1 General:

This example deals with the evaluation of Spencer's factor of safety (Spencer, 1973) associated with a given slip surface, for a simple slope section shown in Figure 4.11. The purpose of this example is two-fold as discussed below.

Soriano (1976) has cited the above example problem as one simple case which is not solvable by the iterative schemes suggested by Spencer (1973), Morgenstern and Price (1967) etc., when $\phi_u = 0$. He has further reported that if $c_u = 300$ kPa, the factor of safety by the force equilibrium method would be larger than F=1.89 for any value of θ, whereas by the moment equilibrium method the factor of safety is independent of θ and has the value of F = 1.86. He also observed, "Even though the factor of safety is quite well defined for this slope any scheme trying to satisfy moment and force equilibrium will fail to converge. This would very often be the case for steep and highly cohesive slopes for which the assumption of parallel side force is not compatible with equilibrium conditions."

The above observation regarding the inadequacy of the iterative scheme for a complete equilibrium method such as the Spencer method as well as the incompatibility of the assumption of parallel side forces led to the present study consisting of the following:

i) to try to solve the problem using the iterative scheme based on SUMT, as proposed in this thesis for solving Spencer's equations, assuming parallel side forces.

ii) should the above trial fails, to try with the developed scheme but using a general empirical interslice force function reported by Fredlund (1984). For ready reference the details of this function is presented in the following section.

4.5.2 General Empirical Interslice Force Function:

Fredlund has reported about a detailed study by Wilson and Fredlund (1983) and Fan (1983) on interslice force functions computed from finite element analysis. Based on a large number of analyses, a general empirical interslice force function was proposed:

$$f(x) = K \exp\left(\frac{-C^{N}\omega^{N}}{2}\right)$$

where,

K = magnitude of the interslice force function at mid slope (i.e. maximum value).

C = variable to define the inflection points.

N = variable to specify the flatness or sharpness of curvature.

ω = dimensionless x-position relative to the midpoint of the slope.

The variable K is a function of the slope inclination and the depth factor. The constants C and N are related to slope inclination. These parameters were computed for a circular slip surface. Analyses for composite or non-circular slip surfaces showed variations in magnitude for some of the constants in the above expression but the general shape of the function remained the same. The constants K, C and N may be obtained from the charts (Fredlund, 1984) available for them.

4.5.3 Results and Discussion :

(a) Solution using the developed scheme and the assumption of parallel side forces :

An attempt has been made to find the Spencer's factor of safety of the given slip circle using the proposed equation-solver. The side forces are assumed to be parallel. In the first trial run, no tension crack has been introduced. 20 slices have been adopted. In tune with Soriano's observations, the results did not converge.

In the second trial, in addition to the above, tension crack has been introduced. A tentative depth of 0.25 $H_t \approx$ 10.0 m for the water-filled tension crack has been taken in the analysis. The result now converges. The initial and the final results are as follows:

Variable/Function	Initial Value	Final Value
F	2.0	1.61
θ	0.1	0.3820
Z_n	-0.3539E+03	0.8836E-03
M_n	-0.3115E+04	0.2087E-02

However, the associated line of thrust and the resultant interslice forces have been found to be unsatisfactory and hence it is not an acceptable solution. An acceptable solution may be possible to obtain by treating k-values as design variables as in the Example problem 4.3, together with imposition of constraints on the positions of the line of thrust. However, such studies have not been undertaken here.

(b) Solution using the developed scheme and the general empirical interslice force function (Fredlund, 1984):

For evaluating the interslice force function values, the constants K,C and N have been obtained from the charts supplied as follows:

$$K = 5.0, \quad C = 3.0, \quad N = 2$$

Having obtained these values, f(x) values have been computed corresponding to different ω-values as given in Figure 4.12.

In the first trial when no tension crack was assumed, there was no convergence.

In the second trial, assuming in the above, water-filled tension crack of depth 10.0 m, convergence could be achieved.

The final results are as follows:

$$F = 1.61, \quad \theta = 0.319 \text{ rad}$$

$$Z_n = -0.6103E-04, \quad M_n = -0.7400E-03$$

The θ value here is only slightly different from that in case (a).

However, as in case (a) above, the line of thrust and the resultant interslice forces are not satisfactory and hence this also is not an acceptable solution.

Two observations can be made from the above :

1) By assuming tension crack it is possible to find a solution which satisfies both force and moment equilibrium.

2) The suggested general empirical interslice force function by Wilson et al. (1983) does not work in case of this problem.

It is expected that the special formulation incorporating k-values in the design vector as suggested in Section 4.4.2 would give better results in such cases.

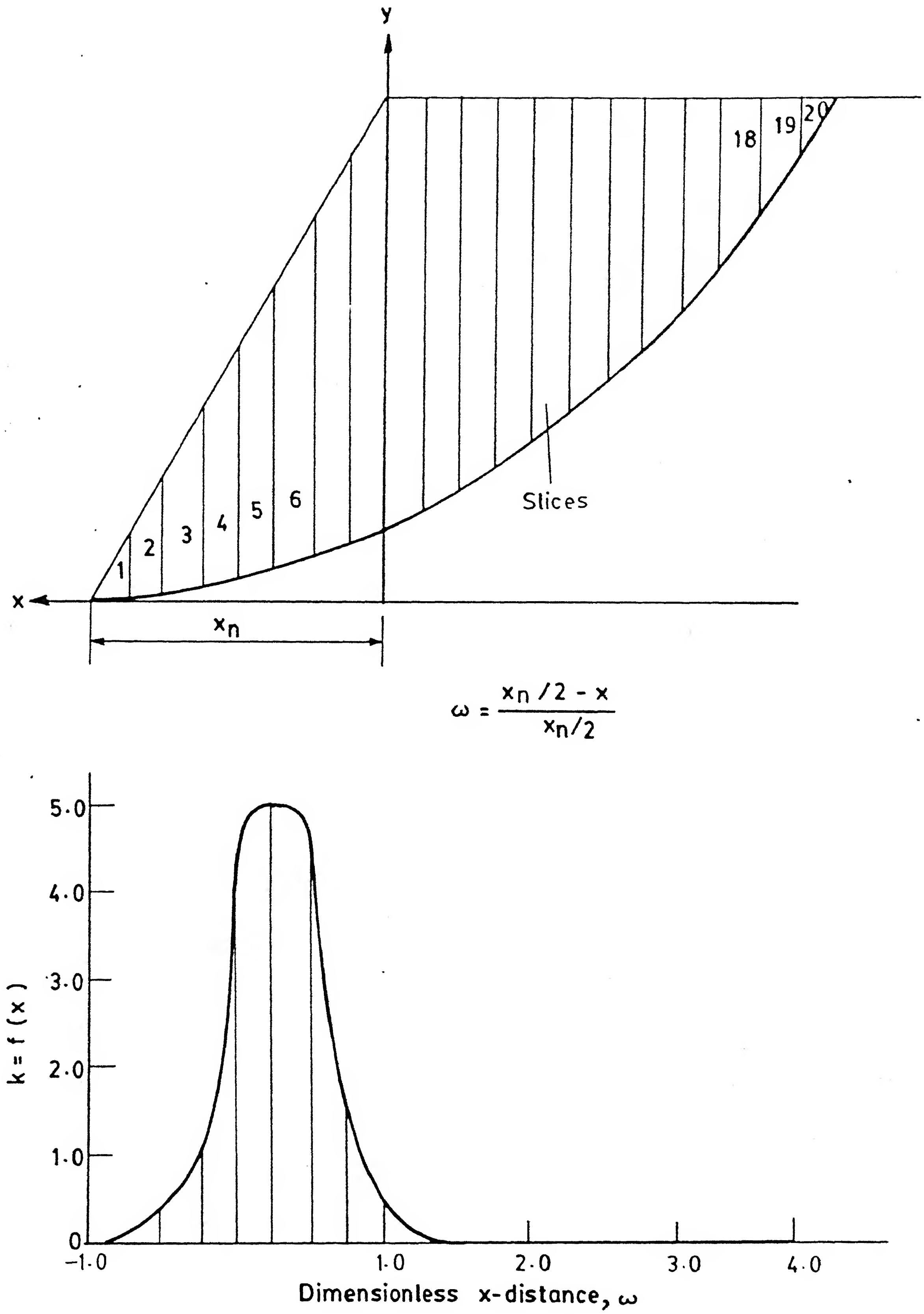


FIG. 4.12 INTERSLICE FORCE FUNCTION ASSUMED FOR EX. PROBLEM 4.4

4.6 EXAMPLE PROBLEM 4.5

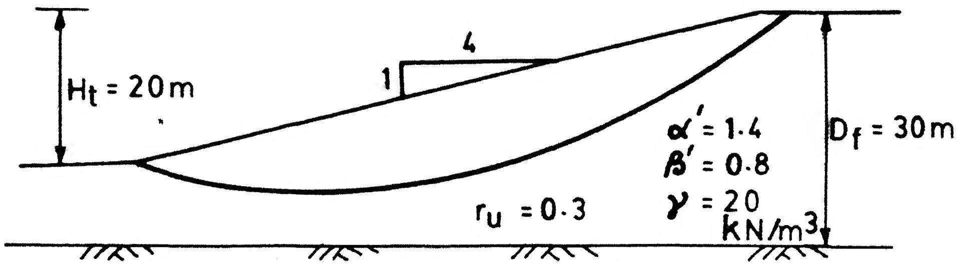


Figure 4.13 : SLOPE SECTION IN EXAMPLE PROBLEM 4.5
(Charles and Soares, 1984)

4.6.1 General:

In all the previous example problems, linear Mohr-Coulomb envelope was assumed. The principle of the stability analysis in soils with nonlinear strength envelope using the developed technique has been presented in Chapters 2 and 3. To demonstrate the application of the developed technique to such situations, the simple example shown in Figure 4.13 has been chosen. It is a slope in cutting excavated at 1 in 4 in London clay with De Mello's (1977) nonlinear shear strength parameters $\alpha' = 1.4$ and $\beta' = 0.8$ and bulk unit weight of 20 kN/m^3. The clay layer is 30 m deep and overlies bedrock. The average pore pressure coefficient r_u within the slope is 0.3. This problem was studied earlier by the following investigators :

1. Charles and Soares (1984) using the simplified Bishop method for circular analysis.
2. Yudhbir et. al. (1987) using the Janbu's GPS procedure for non-circular analysis.

In the present studies, analysis by Spencer method for a general non-circular slip surface has been carried out using the Direct procedure to determine the critical surface and the corresponding minimum factor of safety. Discretization Model I has been used. Results have been obtained for both nonlinear strength envelope and the idealised linear strength envelope and compared. Comparison has also been drawn with the results reported earlier.

4.6.2 Results and Discussion

Case I : Nonlinear Strength Envelope

At the beginning, a surface in the vicinity of the critical slip circle reported by Charles and Soares (1984) was chosen as the starting slip surface. This is shown as the surface oabc in Figure 4.14(a). The initial values for c′ and ϕ' were chosen arbitrarily as 60 kN/m^2 and 17° respectively for all of the 17 slices into which the sliding mass was divided. The obtained final surface has a F_{min} = 1.74. However, detailed output showed that the horizontal effective interslice force (Z′) at the last interslice boundary was negative and hence the solution was unacceptable. Therefore, the problem was solved again after introducing water-filled tension crack. The depth of tension crack was treated as a design variable with an initial depth of

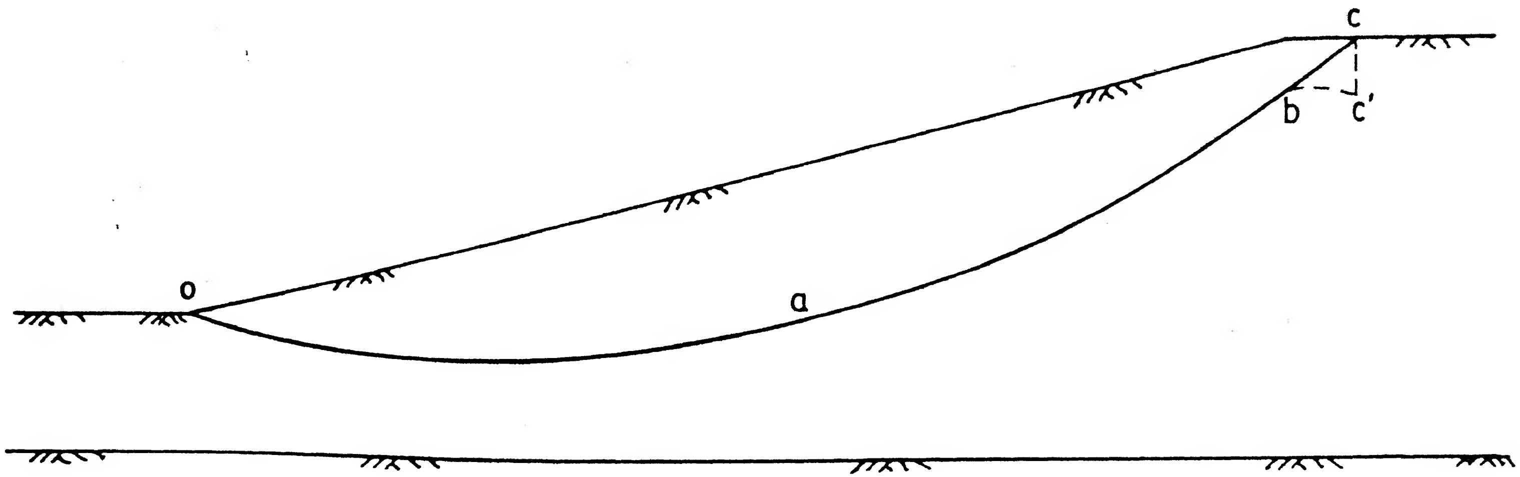


FIG. 4.14 (a) STARTING SLIP SURFACES FOR EX. PROB 4.5

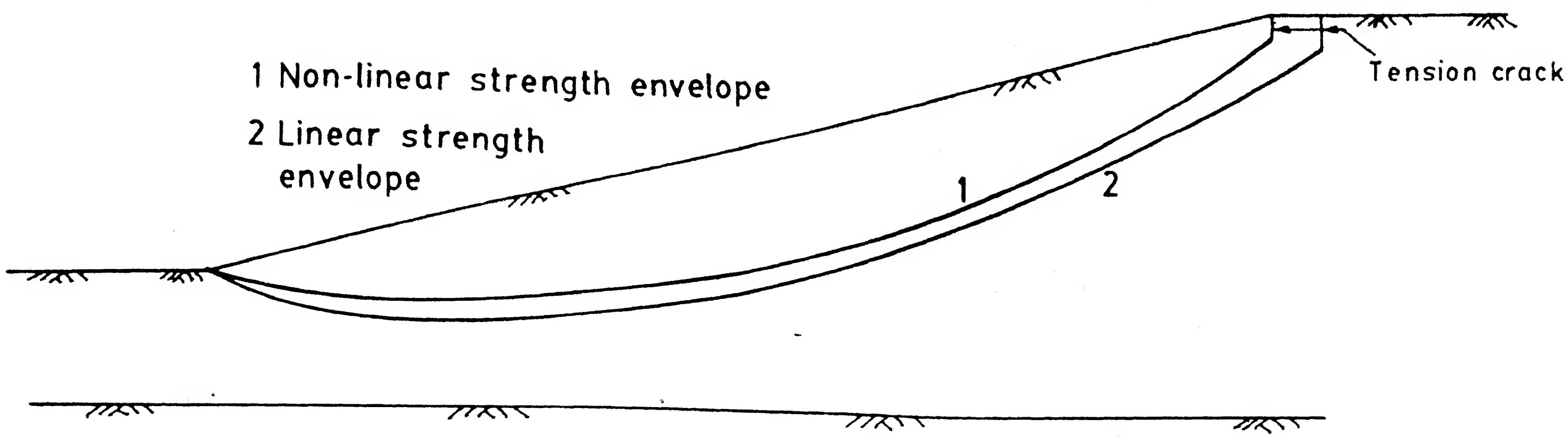


FIG. 4.14 (b) CRITICAL SLIP SURFACES FOR EX. PROB 4.5

Table 4.18
Calculated Responses Associated with the solution to Example Problem 4.5 by Spencer Method

Slice No.	σ' kPa	τ kPa	L/H	L′/H	$\frac{Z'}{\gamma b H_t}$
1	24.32	10.77			
			0.47	0.51	0.04
2	56.44	21.12			
			0.38	0.39	0.10
3	78.59	27.52			
			0.36	0.36	0.16
4	94.97	32.02			
			0.35	0.35	0.21
5	107.0	37.46			
			0.34	0.35	0.26
6	115.53	38.90			
			0.34	0.34	0.30
7	121.11	39.57			
			0.34	0.34	0.32
8	123.73	39.54			
			0.34	0.34	0.33
9	123.62	38.93			
			0.34	0.34	0.32
10	121.22	37.64			
			0.34	0.34	0.30
11	116.21	35.72			
			0.34	0.34	0.28
12	108.85	33.18			
			0.34	0.34	0.24
13	99.29	30.08			
			0.34	0.34	0.19
14	87.82	25.98			
			0.35	0.34	0.14
15	73.13	20.42			
			0.35	0.36	0.09
16	54.11	11.69			
			0.38	0.41	0.04
17	26.96				

Table 4.19

Design Vector and Constraints for the Nonlinear case for the Example Prob. 4.5

No. of slices = 17

No. of design variables = 21

Starting Point

F = 1.2500 θ = 0.1500

Design Variables (Not normalized)

-1.00	-2.00	-2.80	-3.00	-3.00	-2.20	-1.70	-1.00	0.00	1.50	3.00
5.00	7.00	9.60	13.00	16.50	80.00	-5.00	2.25	0.15	4.00	

Constraints (Inequality)

-0.9000	-0.8000	-0.7200	-0.7000	-0.7000	-0.7800	-0.8300	-0.9000
-1.0000	-0.9063	-0.8125	-0.6875	-0.5625	-0.4000	-0.1875	
-0.0000	-0.2000	-0.6000	-0.2000	-0.8000	-0.3000	-0.2000	-0.3000
-0.5000	-0.0000	-0.5000	-0.0000	-0.6000	-0.8000	-0.0999	-4.0000
				-0.3901	-0.2926	-0.2806	-0.3093
-0.3492	-1.0857	1.8939	0.3397	0.1675	0.1476	0.1640	0.2350
-0.3232	0.5187	1.0209	2.2532	-0.6099	-0.7074	-0.7194	-0.6907
-0.6508	0.0857	-2.8940	-1.3397	-1.1676	-1.1476	-1.1640	-1.2350
-1.3232	1.5187	-2.0210	-3.2532	-0.2600	-0.1500	-0.0950	

Constraint (Equality) : 0.1495E+02

δ_t = 0.0001 ε = -0.100 $r_o = 0.1 \times 10^{-4}$

f = 2.2500 ψ = 2.2809 Z_n = -0.2909E+04 M_n = -0.2104E+05

Optimal Point

F = 1.6741 θ = 0.2235

Design Variables (Not normalized)

-1.2461	-1.8510	-2.1252	-2.1266	-1.8911	-1.4429	-0.8011	0.0380
1.0744	2.2971	3.7223	5.3551	7.1979	9.2451	11.5922	14.4081
79.9003	-0.22E-04	1.6741	0.2235	1.7365			

Constraints (Inequality)

-1.0000	-0.8754	-0.8149	-0.7875	-0.7873	-0.8109	-0.8557	-0.9199
-0.9979	-0.9412	-0.8742	-0.7962	-0.7068	-0.6059	-0.4938	-0.3653
-0.6412	-0.3308	-0.2727	-0.2369	-0.2128	-0.1935	-0.1974	-0.1972
-0.1863	-0.2025	-0.2075	-0.2100	-0.2044	-0.2999	-0.4688	-1.0395
-0.4731	-0.3784	-0.3572	-0.3487	-0.3443	-0.3416	-0.3397	-0.3384
-0.3374	-0.3367	-0.3363	-0.3363	-0.3369	-0.3394	-0.3471	-0.3787
-0.5269	-0.6216	-0.6428	-0.6513	-0.6557	-0.6585	-0.6603	-0.6616
-0.6626	-0.6633	-0.6637	-0.6637	-0.6631	-0.6606	-0.6529	-0.6213
-0.6841	-0.2235	-0.0214					

Constraint (Equality): 0.7513E-13

No. of r-minimizations = 9

f = 1.6741 ψ = 1.6743 Z_n = 0.4694E-03 M_n = 0.8965E-03

Note: Out of a total of 67 constraints, first 16 are upper boundary, constraints, next 8 are lower boundary constraints, next 16 are curvature constraints, next 32 are constraints on line of thrust and last three are side constraints 1 cm F and two on θ respectively.

Case II : Idealised Linear Strength Envelope :

In order to investigate the effect of nonlinearity, analysis has also been done using the strength parameters obtained from a linear idealisation of the curved strength envelope. Yudhbir et. al. (1987) considered an idealisation which gives $c' = 33.0$ kN/m^2 and $\phi' = 19^\circ$.

The minimum factor of safety in this case has been obtained as 2.03. As in the case of nonlinear strength envelope, it was required to introduce tension crack with water pressure in it as otherwise there was no convergence. The same initial surface as described earlier with initial $z_t = 4.0$m was used. The optimal depth of tension crack has been found to be 2.81m against 1.74m obtained earlier. The line of thrust and other internal variables show that the solution is an acceptable one; however, these results are not presented here. The critical surface obtained in this case is also shown in Figure 4.14(b) along with that obtained in the nonlinear case. It is noticeable that compared to the nonlinear case, a deeper critical surface has been obtained using linear approximation. Similar observation was reported by Yudhbir et al. (1987) with regard to their analysis using Janbu's GPS; for the same linear idealisation, they reported a F_{min} of 1.96.

Overestimation in factor of safety:

The result of a conventional stability analysis of the same slope carried out using a linear approximation of the curved failure envelope would depend on the particular linear approximation adopted in the analysis. As far as the

approximation used by Yudhbir et al. (1987) and in the present studies is concerned, the linear approximation results in an overestimation in the factor of safety by about 22 percent. Figure 4.15 shows that for the nonlinear case the paths followed by the function values f and ψ from the starting point to the respective minima in terms of variations with the number of function evaluations as well as the penalty parameter r. It is observed that the trend of convergence is the same as demonstrated earlier for the Direct procedure with linear strength envelope in the case of Example problem 4.1.

4.6.3 Comparison with Earlier Solutions:

Figure 4.16 shows the critical shear surfaces considering nonlinear strength envelope obtained in the present solution as well as those reported by Charles and Soares (1984) and Yudhbir et al. (1987). The surfaces are seen to be quite different from one another and the critical surface obtained in the present analysis is observed to be much shallower compared to the other two. The associated minimum factors of safety are as given in Table 4.20. It is seen that although the surfaces are quite different, the factor of safety values are quite close.

4.7 CONCLUSIONS

From the studies contained in this chapter the following conclusions are drawn :

1. The proposed technique based on SUMT is very effective in analysing both flat and steep slopes in homogeneous soils. However, as is usual with any nonlinear programming problem,

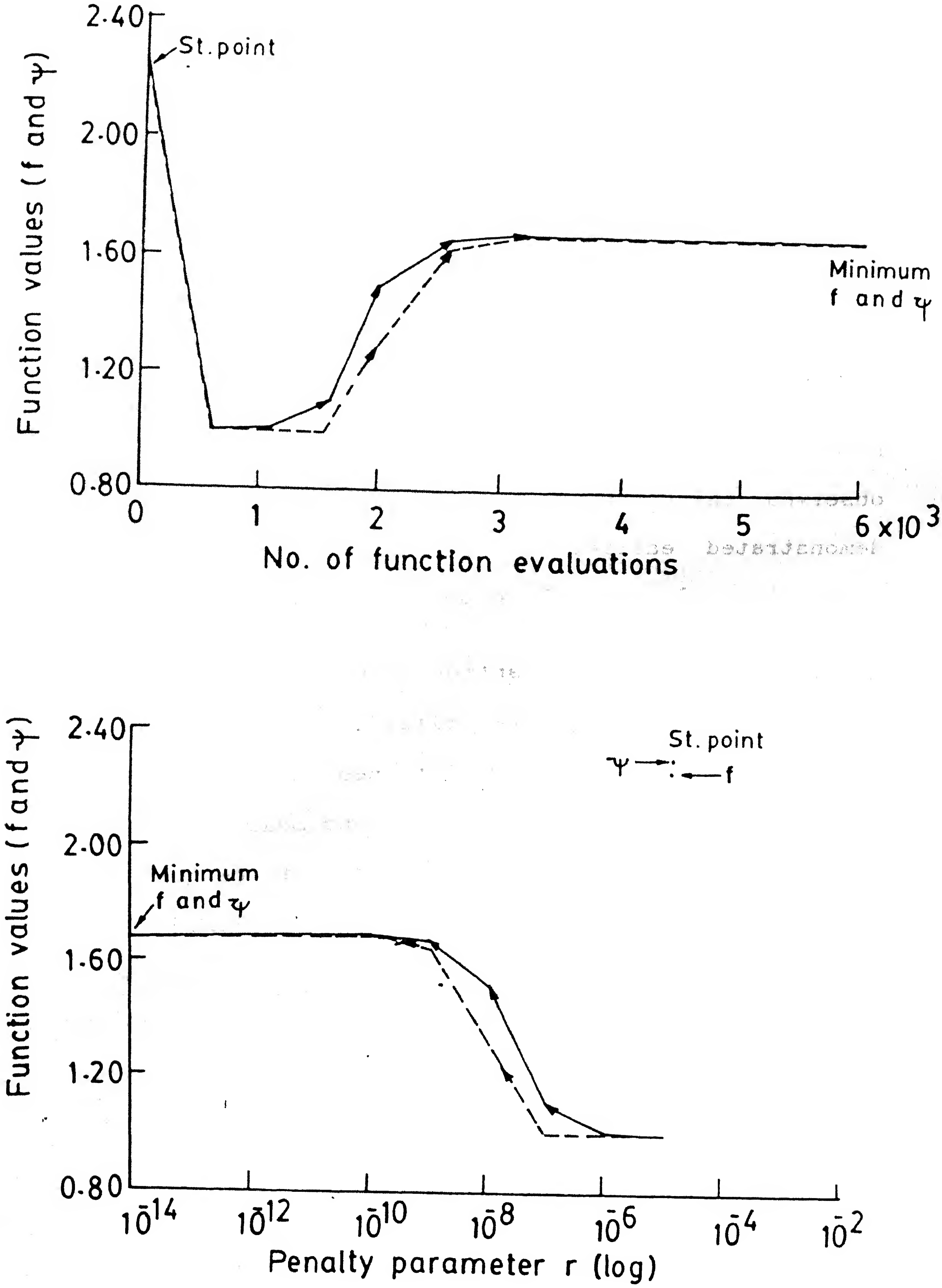


FIG. 4·15 PROGRESS OF MINIMIZATION IN THE EXAMPLE PROBLEM 4·5 WITH NONLINEAR STRENGTH ENVELOPE

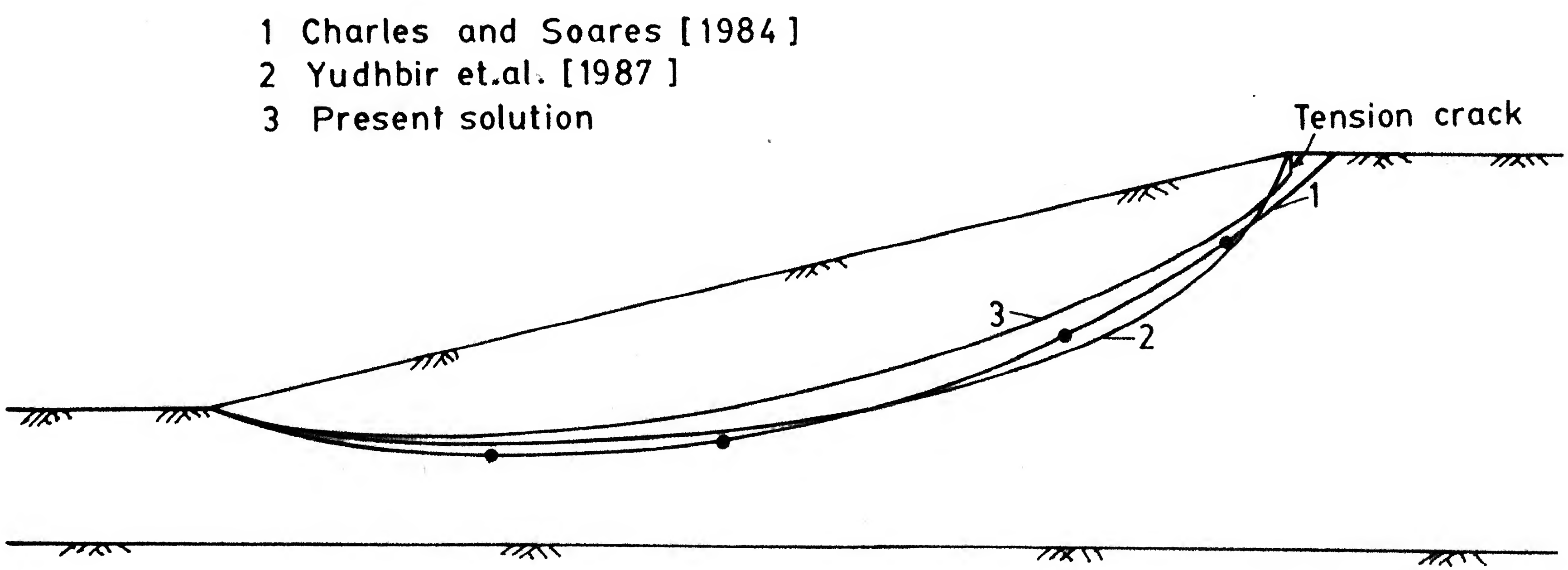


FIG. 4·16 COMPARISONS OF THE RESULTS OBTAINED USING NONLINEAR STRENGTH ENVELOPE FOR THE EX. PROBLEM 4·5

Table 4.20

Comparison of the Minimum factor of safety

(Example Problem 4.5)

Sl. No.	Method of analysis	F_{min}	Remarks
1.	Simplified Bishop - Charles and Soares (1984) nonlinear strength envelope	1.66	Circular slip surface No tension crack assumed
2.	Janbu's GPS -Yudhbir et al. (1987) (i) nonlinear strength envelope (ii) linear strength envelope	 1.70 1.96	General Slip surface No tension crack assumed Overestimation in linear case = 15 %
3.	Spencer (1973) method - Present solution (i) nonlinear strength envelope (ii) linear strength envelope	 1.67 2.03	General slip surface. Water-filled tension crack assumed Overestimation in linear case = 22 %

the scheme is starting point dependent. As such, to ensure global minimum a number of initial design vectors should be tried.

2. The Direct procedure is more efficient than the Indirect procedure.
3. The Extended interior penalty function method is very efficient in handling infeasible design points for this class of problems.
4. The results obtained by using the proposed procedure compares very well with those obtained by using other techniques e.g., calculus of variation and dynamic programming. However, the present method is more flexible in taking care of the acceptability criteria in the analysis.
5. In most of the analysis using Spencer's method, introduction of tension crack is necessary to get an acceptable critical shear surface. In some cases even for getting convergent solution of Spencer's stability equations for a given shear surface, introduction of such tension cracks become necessary.
6. The proposed Direct procedure successfully adopts the nonlinear strength envelope of soils in the standard slope stability computational methods.
7. Contrary to general belief even simple slopes in homogeneous soils may pose considerable difficulty in finding acceptable solution. The present approach appears to be the most promising in tackling such problems.

CHAPTER 5

ANALYSIS OF SLOPES IN SOILS HAVING NONHOMOGENEOUS AND ANISOTROPIC STRENGTH

5.1 INTRODUCTION

In nature most soils exhibit anisotropy because of the mode of deposition, the stress metamorphosis after deposition, or both. Consequently, the undrained shear strength of some soils vary with the direction of the slip plane with respect to a reference direction. Since the slip surfaces are generally curved, it becomes imperative to take the strength anisotropy into account in the calculation of factors of safety of slopes in embankments and cuttings. The reason is explained as follows :

In the conventional methods of stability analysis, the effect of the rotation of principal stresses along the failure arc is not considered. However, near the top of the slope, the direction of the principal stress, σ_1 is nearly vertical, while near the toe, σ_1, acts in an almost horizontal direction — the direction of rotation being clockwise, as shown in Fig. 5.1. When the soil is isotropic it does not matter because strength will be the same whatever the principal stress direction may be. But when the soil is anisotropic, the strength c_{i_1} at the location where the major principal stress σ_1 makes an angle i_1 with the vertical is different from c_{i_2} at the location where the corresponding angle is i_2.

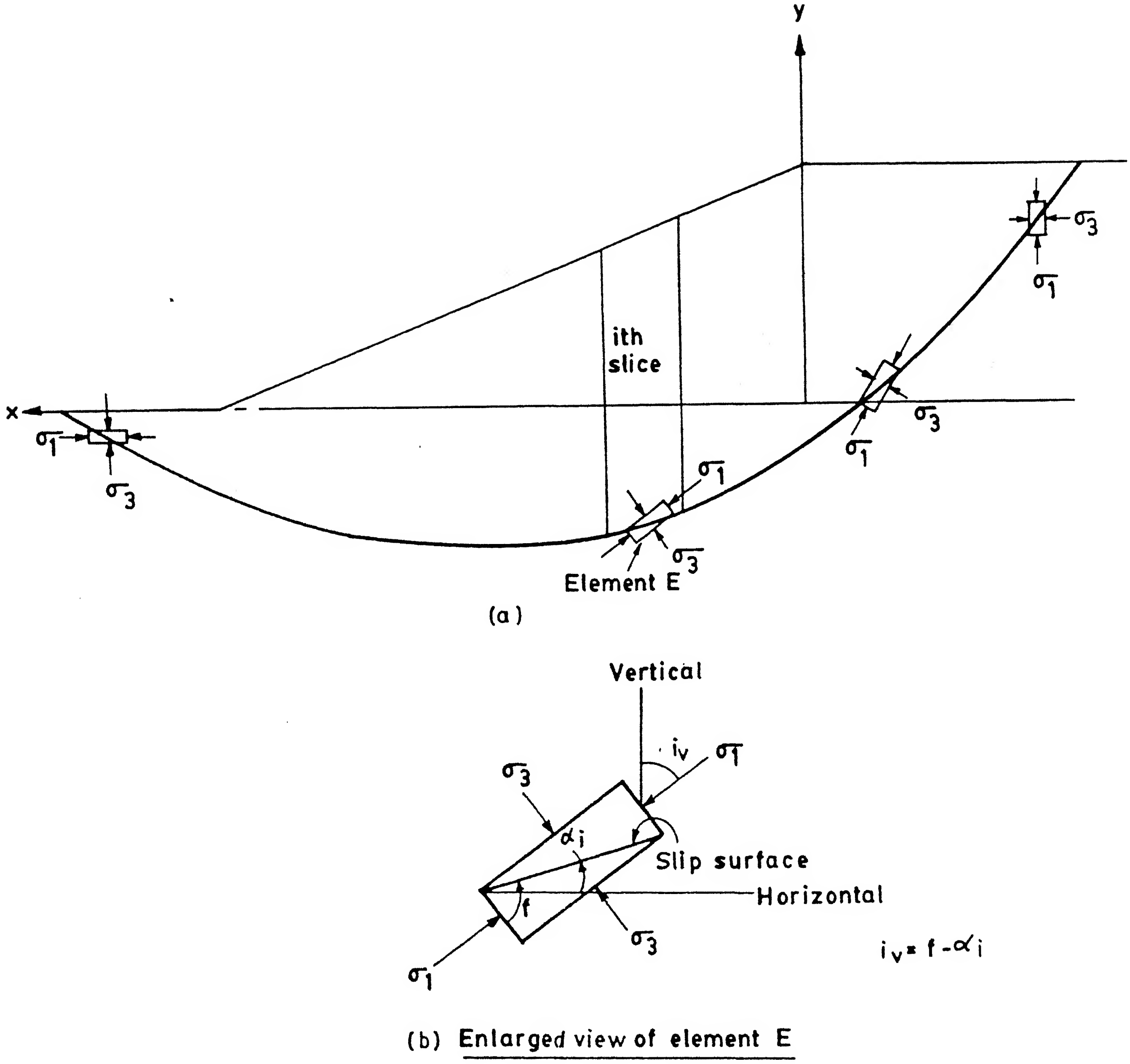


FIG. 5.1 ROTATION OF PRINCIPAL STRESSES ALONG THE SLIP SURFACE

The term 'nonhomogeneous soil' herein means only that the undrained shear strength of the soil varies with depth. It has been found from field investigations that, in general, the in situ undrained shear strength of normally consolidated clay increases linearly with depth or with vertical effective stress, p, due to overburden; and specifically the shear strength ratio c_u/p is found to be a constant. c_u/p is chiefly dependent on the geological history of the clay. Skempton (1948a, 1948b) correlated c_u/p with plasticity index for normally consolidated marine clays. Hansen (1952) introduced the following relationship for the in situ shear strength of a clay mass :

$$c_u = c_o + \lambda_o \gamma z \tag{5.1}$$

where c_o and λ_o are soil parameters determined empirically. λ_o is a function of the coefficient of earth pressure at rest, a 'cohesion factor', the effective angle of internal friction and a factor λ introduced by Skempton (1948b). Given the value of λ_o, c_o is chosen in such a way that c_u gives the approximate vane shear strength at different depths.

It has also been observed that for a limited depth below the ground surface the in situ undrained shear strength is usually higher than the strength obtained by assuming a linear distribution of strength with depth. This is due to desiccation of that zone caused by fluctuation of water table. James et al. (1969), based on published evidence, has postulated an empirical relationship to account for increased undrained shear strength in the desiccated zone. This relationship is expressed as follows :

$$c_t(z) = c_o + \psi \gamma z + F_o c_o \exp\left[-\left\{z/\beta_a b\right\}^n\right] \tag{5.2}$$

where,

$c_t(z)$ = the undrained shear strength at a point located at a distance z below the ground surface

c_o = the zero intercept on the strength axis of the linear portion of the composite strength curve

γ = the unit weight of soil which is equal to γ_{sub} for submerged condition

z = depth below ground surface

b = a parameter having the dimension of length.

ψ, F_o, β_a and n are dimensionless empirical coefficients.

The general strength distribution expressed by Equation (5.2) covers the cases where the undrained shear strength is constant, increases or decreases with depth. If ψ equals zero, the analysis will be similar to the conventional one with full consideration to any desiccation in the upper zone. If ψ and F_o both vanish, the medium is homogeneous with constant value of undrained shear strength parameter c_o.

Taylor (1937) used an average constant shear strength for the soil assuming the validity of $\phi_u = 0$ analysis. This method leads to inaccurate results especially for flat slopes. Gibson and Morgenstern (1962) established a relationship between safety factor and slope inclination for a cutting in a soil whose strength is zero at the ground surface and increases linearly with depth. Kenney (1963) also presented a solution using the above assumptions. Koppula (1984b) extended the theoretical investigation of Gibson and Morgenstern (1962) to consider the

general case of a normally consolidated clay whose undrained shear strength is finite at the top and varies linearly with depth. Majumdar (1971), Prater (1979) and Koppula (1984a) have included the effect of earthquakes in the above mentioned kind of analysis.

None of the above mentioned studies considered anisotropy in the undrained shear strength. Lo (1965), Chen et al. (1975), Ranganathan and Srinivasulu (1979) have presented very useful contribution in the direction of studying the effect of strength anisotropy on the conventional factor of safety in the design of earth slopes and cuts.

Basudhar (1976) and Basudhar et al.,(1986) have considered both anisotropy and nonhomogeneity in their analysis. The authors have used Casagrande and Carrillo's model (Casagrande and Carrilo , 1944) for strength anisotropy and James et al.'s model for nonhomogeneous strength variation. Unlike the earlier studies in which the trivial and time-consuming grid search technique was used for finding the minimum factor of safety and the corresponding critical slip circle, Basudhar (1976) is among the earliest researchers to use efficient optimization algorithms for the purpose. Specifically, the author formulated the problem as one of nonlinear programming and made use of the powerful unconstrained search algorithms such as the Powell's, DFP and Fletcher-Reeves methods. However, his study was based on the assumption of circular slip surface as in most of the earlier studies.

A few attempts have been made to study the influence of strength anisotropy on the search for critical non-circular slip

surface (Arai and Nakagawa, 1986; Greco, 1987) based on the nonlinear programming problem formulation. These studies, however, have not considered nonhomogeneous strength variation. Further, they are based on the simplified Janbu's method which involves approximations.

The present study essentially pertains to the application of penalty function method in locating the critical slip surface for slopes in cutting in nonhomogeneous and anisotropic soils in conjunction with Spencer's method (Spencer, 1973) for factor of safety computations. The formulation of the problem as one of nonlinear programming has been discussed in Chapter 3. With a view to check the criticality of the circular slip surface assumed a priori, the results of the present analysis have been compared with those reported by Lo (1965).

5.2 ANALYSIS

5.2.1 Assumptions

1. A $\phi_u = 0$ analysis is valid
2. The degree of anisotropy is a function of depth.

5.2.2 Model For Nonhomogeneity

The empirical rule suggested by James et al. as given by Equation (5.2) is considered in the analysis. According to this rule, the undrained shear strength at any depth z below the ground surface in horizontal and vertical directions, c_v and c_h respectively, are given by :

$$c_v = c_o + \psi_1 \gamma\ z + F_1 c_o \exp\left\{ - \left(\frac{z}{\beta_1 H_t} \right)^{n_1} \right\} \qquad (5.3)$$

$$c_h = p_o + \psi_2 \gamma\ z + F_2 c_o \exp\left\{ - \left(\frac{z}{\beta_2 H_t} \right)^{n_2} \right\} \qquad (5.4)$$

where,

c_o and p_o are the zero depth intercepts on the strength axis of the linear portion of the composite strength curve. ψ_1 and ψ_2 denote the slopes of the linear portion of the composite strength curve.

F_1, F_2, β_1, β_2, n_1 and n_2 are empirical coefficients.

The distributions given by Equations (5.3) and (5.4) are shown in Figures 5.2(a) and 5.2(b) respectively. These distribution rules are generally enough to include cases where undrained shear strength is constant, increasing or decreasing with depth. Based on the findings of many investigators over the years, James et al. (1969) have suggested the most probable range of values for various parameters for normally consolidated and slightly overconsolidated soils. The degree of anisotropy as a function of depth is defined as :

$$K_z = \frac{c_v}{c_h} \qquad (5.5)$$

Basudhar (1976) and Basudhar et al.,(1986) have presented a detailed study on the effects of variation of the above parameters on the factor of safety using the Ordinary Method of Slices.

In the present thesis, the developed computer program has the provision for using the generalized distribution rules given by Equations (5.3) and (5.4). However, for the purpose of analysis carried out in this study, the simple case of a linear variation with zero strength at the surface has been selected. This has

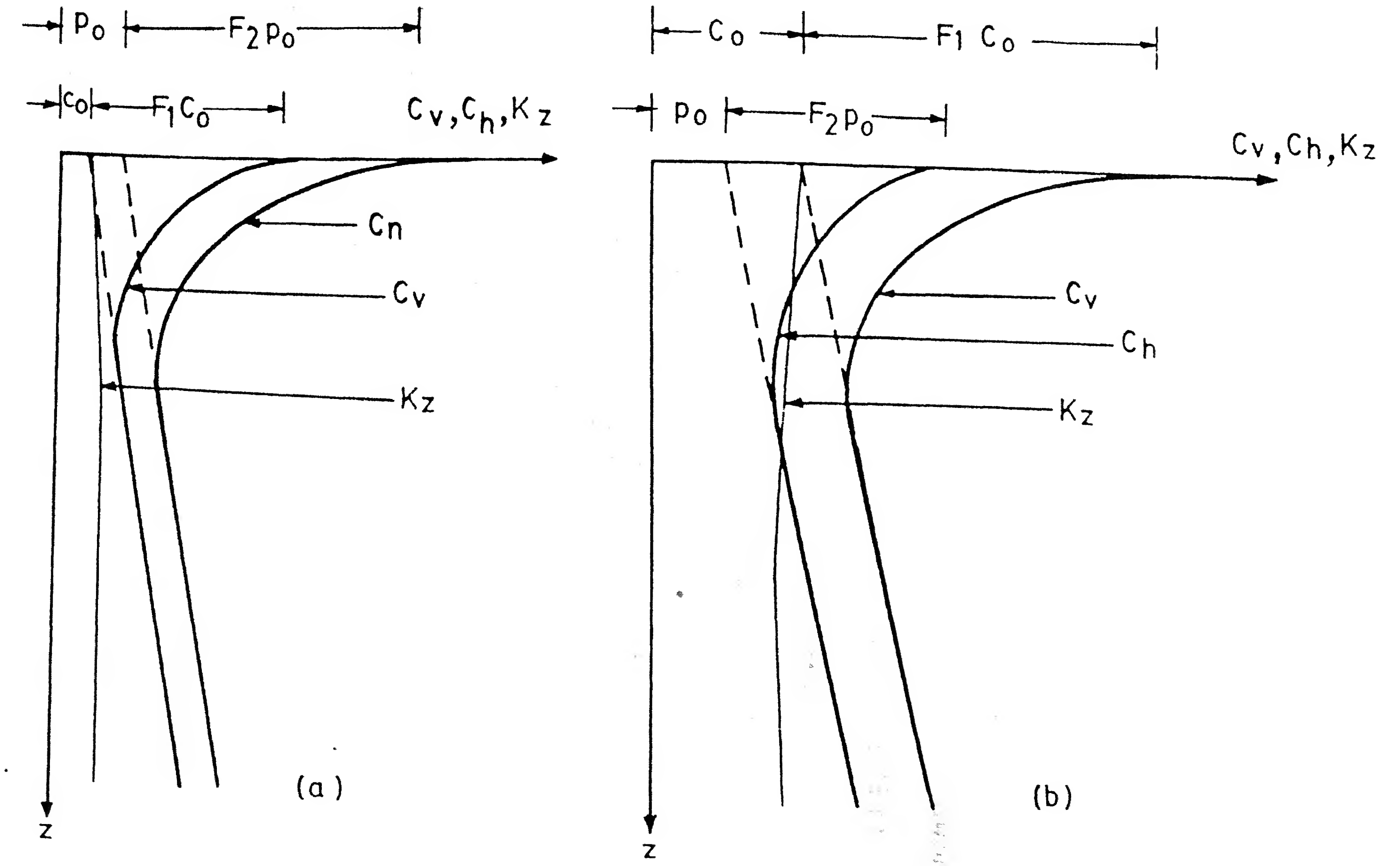


FIG. 5.2 VARIATION OF C_v, C_h AND K_z WITH DEPTH (James et al.)

been done to compare the obtained results with those of Lo (1965). The linear variation mentioned above is obtained as a sub-case of the generalized James et al.'s model as follows :

Substituting $c_o = \emptyset = p_o$

and $F_1 = \emptyset = F_2$

Equations (5.3) through (5.5) become

$$c_v = \psi_1 \gamma z \tag{5.6}$$

$$c_h = \psi_2 \gamma z \tag{5.7}$$

$$K_z = \frac{\psi_2}{\psi_1} = \text{constant} = K \text{ (say)} \tag{5.8}$$

5.2.3 Model For Anisotropy

Casagrande and Carrilo (1944) proposed a mathematical relationship between the directional strength, c_i and the angle of deviation i_v, of the major principal stress at failure from the vertical as follows :

$$c_i = c_h + (c_v - c_h) \cos^2 i_v \tag{5.9}$$

where,

c_v, c_h = the undrained shear strength in the vertical and the horizontal directions i.e., the strength determined when a sample is tested with the direction of the major principal stress coinciding with the vertical and the horizontal directions also known as "principal directions", respectively.

The above variation is also shown in Figure 5.3. Although the above relationship was proposed by Casagrande and Carrilo on an intuitive basis, Lo (1965) has reported that there are experimental evidence to suggest that Equation (5.9) may be applied to the commonly encountered soils. The model has since

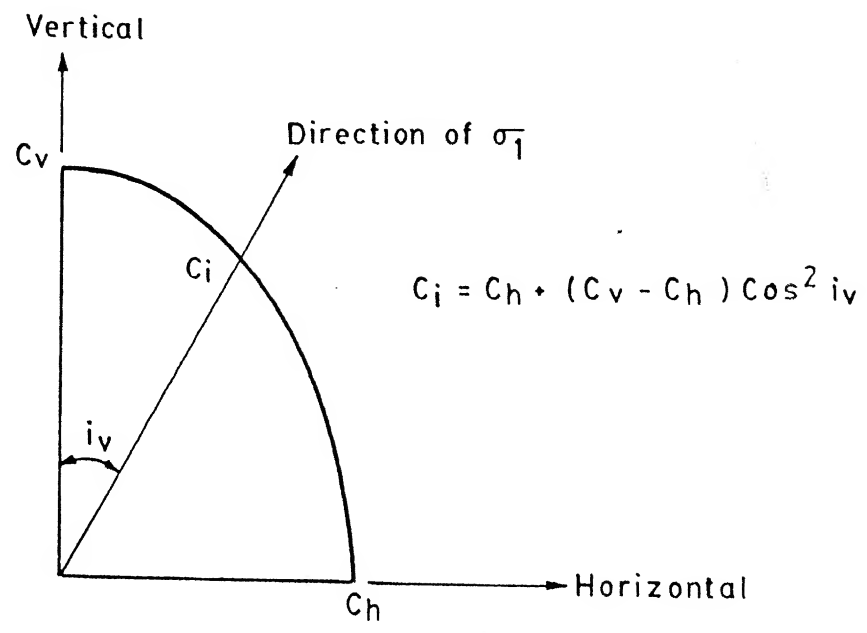


FIG. 5.3 ASSUMED ANISOTROPIC STRENGTH VARIATION (Casagrande and Carrillo, 1944)

been used by many investigators (Lo, 1965; Basudhar, 1976; Law, 1978; Greco, 1987).

In the present investigation, to take into account the point to point variation of the undrained shear strength along a shear surface, use is made of the Equation (5.9) to choose the appropriate shear strength applicable to each slice. This requires values of c_v, c_h and i_v to be known. c_v and c_h are obtained from Equations (5.3) and (5.4) and i_v is obtained as follows :

The angle i_v :

Referring to Figure 5.1(b), the angle i_v which the major principal stress direction makes with the vertical is obtained slicewise as,

$$i_v = f - \alpha_i \tag{5.10}$$

where, α_i = the angle between the slip surface (i.e., the base of the slice) and the horizontal direction for the i^{th} slice.

and, f = the angle between the slip surface (i.e., the base of the slice) and the direction of the minor principal stress.

In deriving the relationship (5.10). It has been assumed here that the width of a slice is small and hence the variation of i_v between the two ends of its base is negligible.

In case of London clay, experiments performed by Lo (1965) showed that the angle of inclination of the failure plane is independent of the orientation of the major principal stress at failure and has a mean value of 56°. Based on this observation,

it is assumed in the present analysis that the angle f is constant for all the slices. This enables one to obtain values of i_v for different slices using the Equation (5.10).

5.2.4 The Objective Function, Design Vector and Constraints

The objective function, the design vector and the constraints in this analysis remain essentially the same as those in the general formulation presented in Chapter 3 with reference to homogeneous slopes. Specifically, the Direct procedure of critical slip surface determination using Spencer's method has been used for the current analysis.

5.2.5 Lo's Stability Number

For the case of anisotropic and linearly increasing undrained strength, Lo (1965) has derived the following expression for the factor of safety

$$F = \frac{c_v}{\gamma z} N_s \tag{5..11}$$

where, c_v is the undrained shear strength in the vertical direction at depth z and N_s is the stability number given by :

$$N_s = \frac{3[\frac{1+K}{2} \cot\lambda + \frac{\alpha(1+K)}{2} (1-\cot\lambda \cot\alpha) + \frac{1 - K}{4 \sin\alpha \sin\lambda} X]}{\sin^2\alpha \sin^2\lambda \ (Y-Z)} \qquad ...(5.12)$$

where,

$$X = \sin\alpha \cos(2f-\lambda)-\sin2\alpha \cos(\alpha+\lambda) \cos(2f-2\lambda)+\frac{1}{3} \sin3\alpha \cos(2f-3\lambda) \qquad ...(5.13)$$

$$Y = 1 - 2\cot^2\beta + 3\cot\lambda \cot\beta + 3\cot\alpha \cot\lambda - 3\cot\alpha \cot\beta \qquad (5.14)$$

$$\text{and } Z = 6n(n + \cot\beta - \cot\lambda + \cot\alpha) \qquad (5.15)$$

where, K = the degree of anisotropy, c_h/c_v

β = the slope angle

f = the angle between the failure plane and the plane normal to the direction of the major principal stress.

α, λ and n = the characteristic parameters for a circular slip surface -- these three parameters completely define its location and are demonstrated in Figure 5.4

Basis of Comparison of Present solution with Lo's :

As per notations used in the present analysis,

$$\frac{c_v}{\gamma z} = \psi_1 = \text{the rate of increase of strength with depth.}$$

Equation (5.11), therefore, reduces to :

$$F = \psi_1 \; N_s \tag{5.16}$$

or

$$N_s = \frac{F}{\psi_1} \tag{5.17}$$

In the present analysis, the (minimum) factor of safety is obtained through the minimization of the objective function. The equivalent stability number can then be obtained from Equation (5.17) and this value may be compared with the stability numbers reported by Lo. Alternatively, the factor of safety corresponding to Lo's N_s may be obtained from Equation (5.16) and this value can be compared with the minimum factor of safety obtained in the present solution.

5.3 RESULTS AND DISCUSSION

Solutions have been obtained for slopes with inclinations (β) of 15^o, 30^o, 45^o, 60^o and 90^o and degrees of anisotropy (K) equal to 0.5 and 1.0 (isotropic case). A value of 0.2 has been taken for ψ_1. The results are presented in nondimensional form.

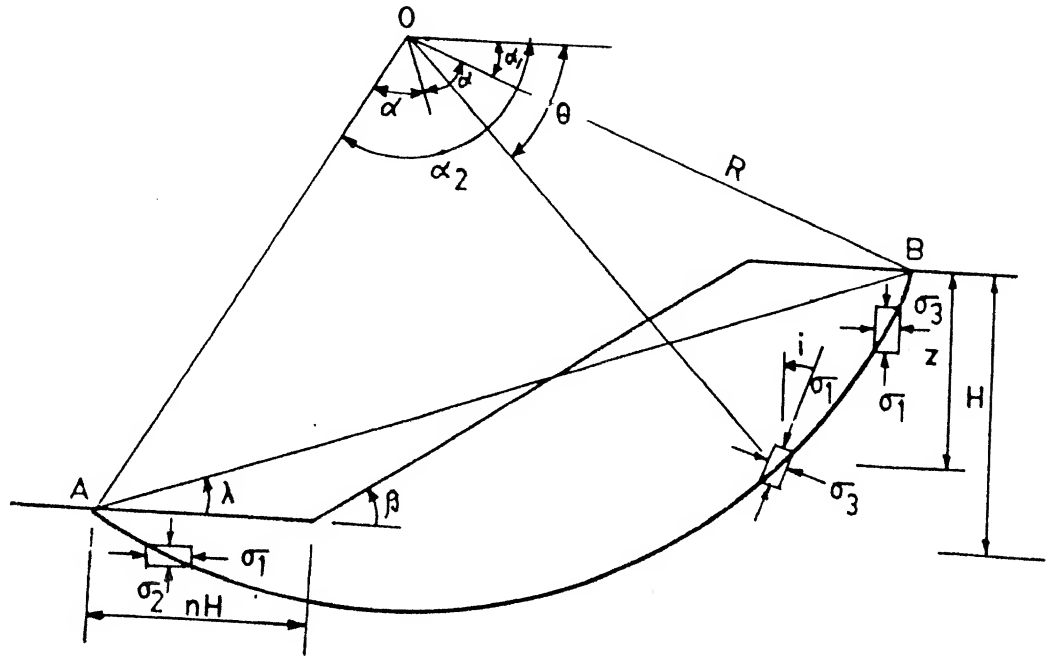


FIG. 5.4 DEFINITION OF GEOMETRIC PARAMETERS OF A TYPICAL SLIP CIRCLE [LO,1965]

The Initial Slip Surface

For any iterative numerical scheme it is essential to have a good initial approximation of the solution vector to arrive at the final results most efficiently. Since Lo's critical circle is expected to serve as a good starting slip surface for the determination of the critical non-circular slip surface, the same has been adopted. Although the critical slip circle which corresponds to Lo's stability number has not been explicitly reported in the paper, the same can be found out in an indirect manner as explained in the following.

It can be seen from Equation (5.12) that the Lo's stability number N_s is a function of K, f, β, α, λ and n out of which f has been assumed by Lo as 55°. Thus, for particular values of β and K, N_s reduces to a function of α, λ and n. Lo also observed that for given values of K and β, minimum value of N_s is obtained for the case n = Ø. As reported by Lo, the slip surfaces are, therefore, an infinite number of circles passing through or above the toe of the slope, defined by a set of values of λ and α. The particular values λ_c, α_c of λ, α which define the critical circle can be found out using an unconstrained search method such as Powell's method after formulating it as an optimization problem stated as follows :

Find λ and α which minimizes N_s given by Equation (5.12) (for given values of β, K, f and n) subject to the nonnegativity of the values of λ and α.

Results obtained using Powell's method has been presented in Table 5.1. After obtaining the optimal solutions using Powell's method the same have been checked for attainment of global minima

TABLE 5.1

α and λ Values For Lo's Critical Circle

Sl. No	β degree	K	α radian	λ radian	N_s
1	15	1.0	0.2882	0.1767	9.7485
			0.6478	0.2105	8.5105
		0.5	0.6510	0.2064	6.1094
2	30	1.0	0.6074	0.3761	5.4484
		0.5	0.6454	0.3629	4.1997
3	45	1.0	0.5006	0.5118	4.1064
		0.5	0.5632	0.4860	3.4025
4	60	1.0	0.3570	0.6227	3.2341
		0.5	0.4245	0.5825	2.8704
5	90	1.0	0.5516	0.7862	1.9964
		0.5	0.7682	0.7333	1.9601

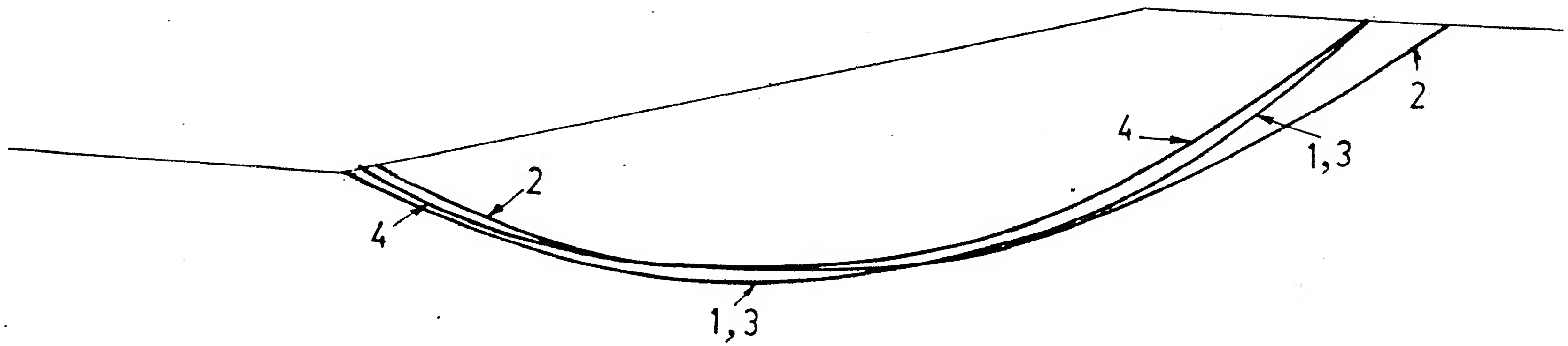
by comparing the obtained minimum function values with the Lo' stability numbers read from the chart. It has been observed tha in all cases the obtained minima have tallied with Lo's N values. This shows that global minima have been reached and th obtained (λ, α) values give Lo's critical circle.

It is worthwhile to note that to solve a problem of the same nature, namely, to find α and λ which gives minimum factor of safety, Koppula (1984a) has reported that the Grid-search technique is preferable to other sophisticated optimization techniques. He argues that only Grid-search technique can ensure a global minimum. However, it has been demonstrated here that starting from arbitrary initial guess values, global minima can be obtained by using Powell's method of unconstrained minimization, even in the case of such complicated trigonometric functions as the Lo's N_s.

Critical Non-circular Surfaces

Figures 5.5 through 5.8 show the critical non-circular surfaces obtained by the present analysis for $\beta = 15^{\circ}$, 30°, 45° and 60° respectively. In the same Figures, critical slip circles corresponding to Lo's solution as first obtained in the present analysis and later used as starting surfaces are also shown. In each figure, two sets of starting and critical surfaces are shown, one set for K = 1 and the other for K = 0.5. 15 slices have been generally used for the analysis by the Direct procedure.

For $\beta = 15^{\circ}$, 30°, 45° and 60°, Lo's critical circles are observed to pass through the toe. Although no restriction has been imposed, the corresponding critical non-circular surfaces



1 : LO'S SLIP CIRCLE FOR K = 1
3 : LO'S " " " K = 0.5
2 : GENERAL SLIP SURFACE FOR K = 1
4 : GENERAL " " " K = 0.5

FIG. 5.5 CRITICAL SLIP SURFACES FOR $\beta = 15°$

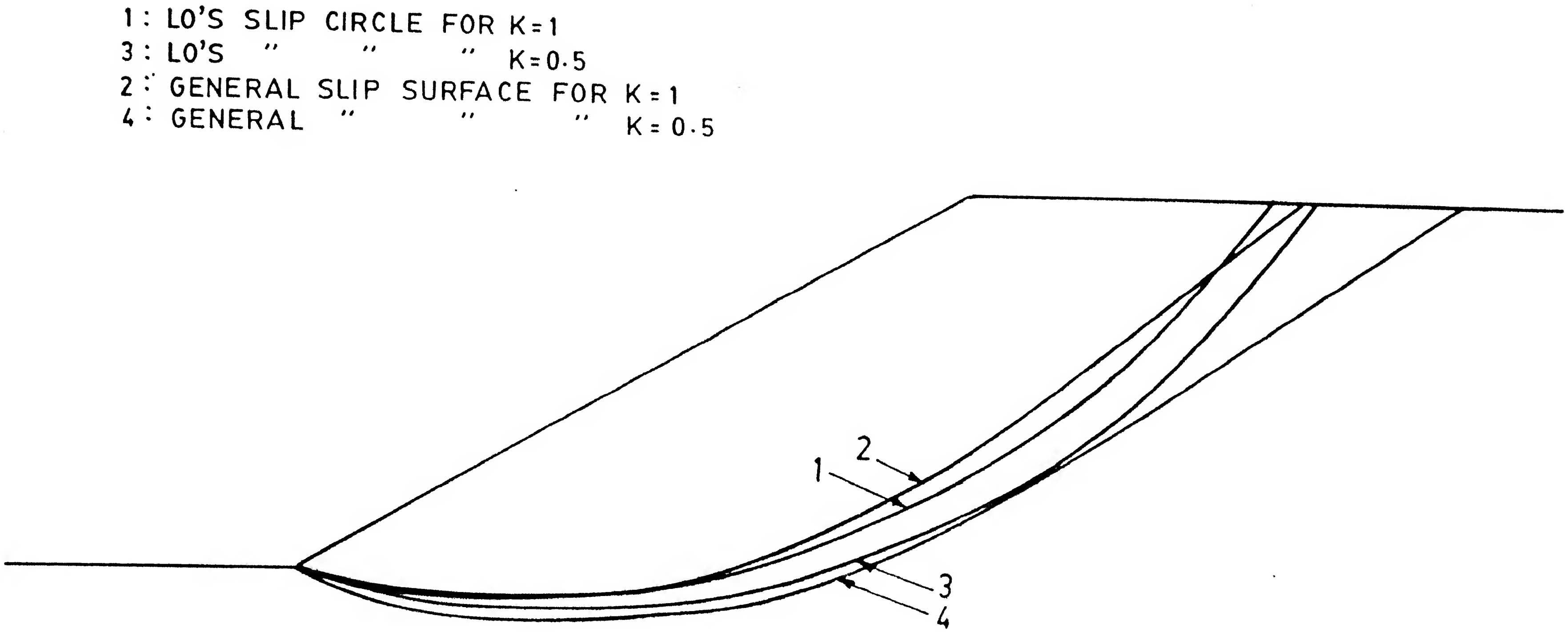


FIG. 5.6 CRITICAL SLIP SURFACE FOR $\beta = 30^{\circ}$

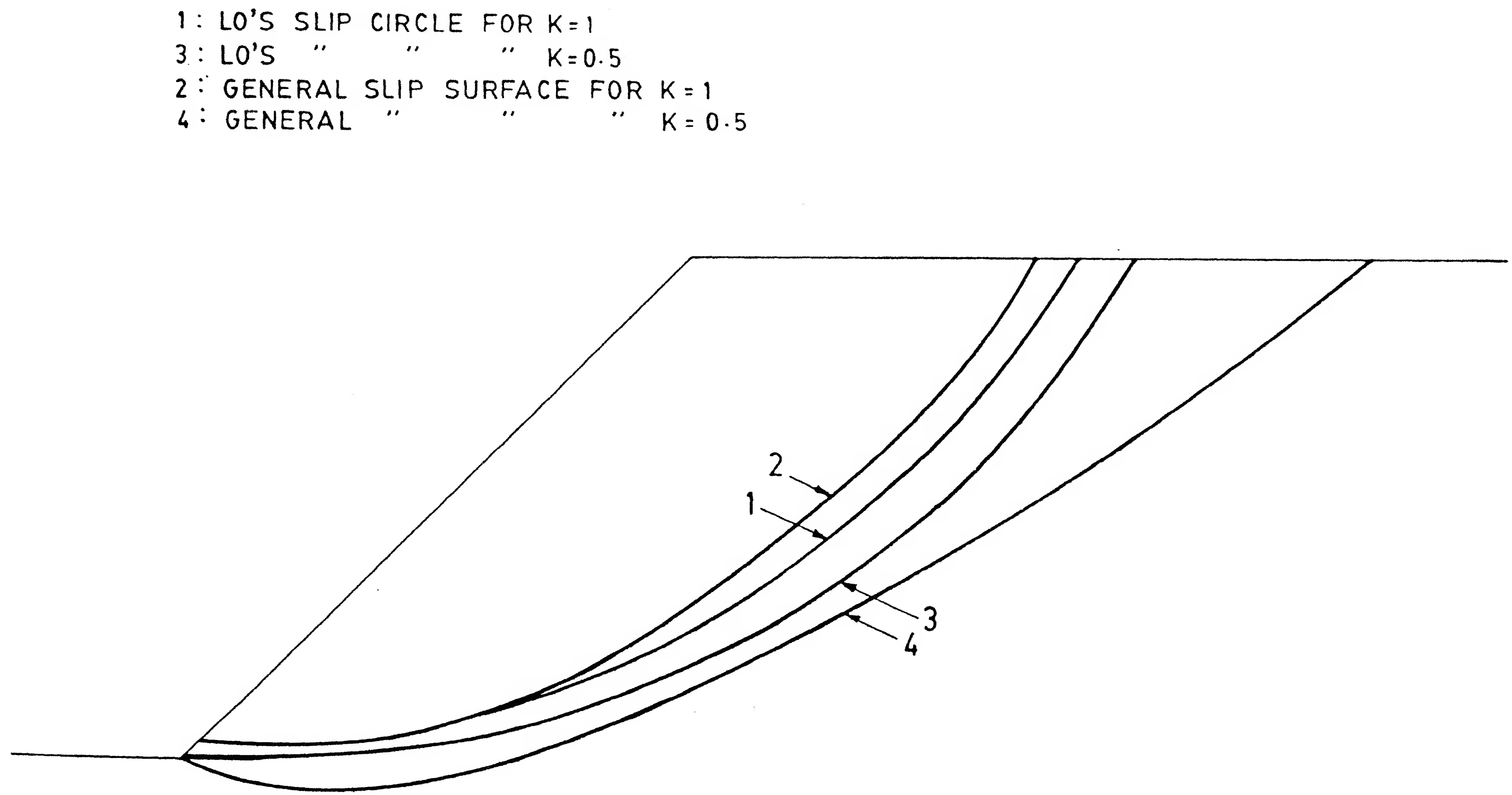


FIG. 5·7 CRITICAL SLIP SURFACES FOR $\beta = 45^{\circ}$

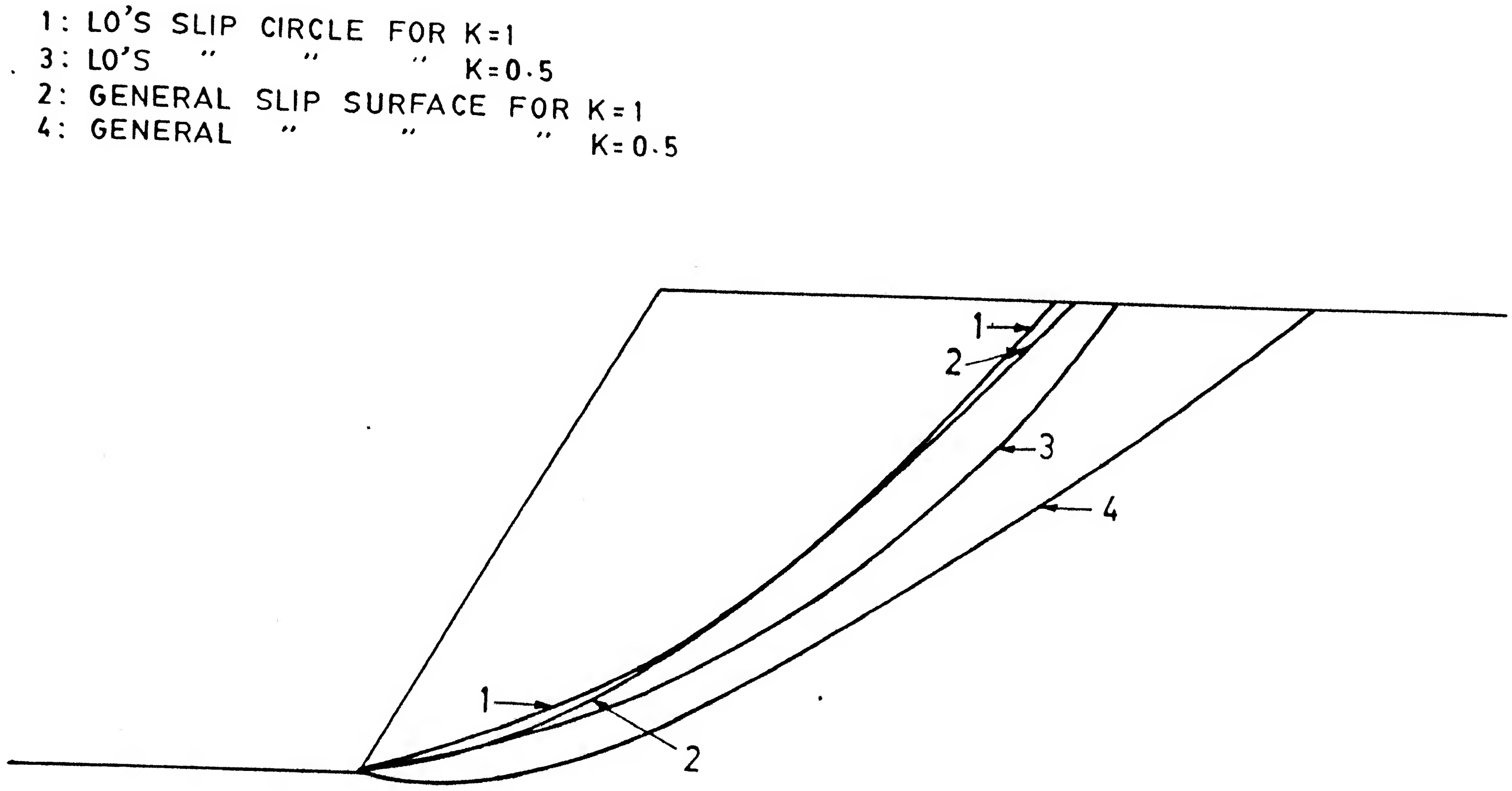


FIG. 5.8 CRITICAL SLIP SURFACES FOR β = 60°

are also observed, from Figures 5.5 through 5.8, to pass through the toe. In the case of vertical cut i.e., $\beta = 90^{\circ}$ (Figure 5.9), however, the shear surface has been forced through the toe in view of the following :

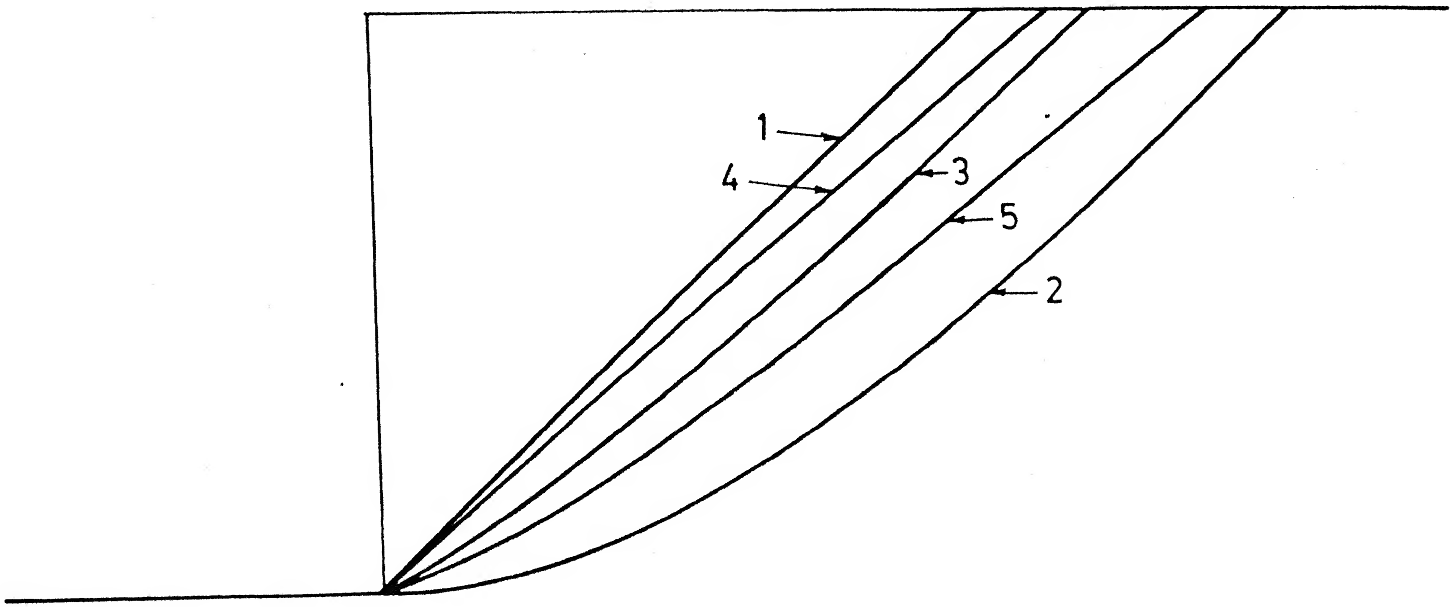
(a) Lo (1965) made the observation that the critical slip circle passes through the toe.

(b) Basudhar's (1976) observation are also along the same lines.

(c) While there is hardly any chance of a base failure considering the fact that there is higher and higher strength below, the investigation of the possibility of a slope failure has revealed that the potential shear surface shows a tendency to approach the slope surface as the optimization progresses until, finally, it ends up in passing through the top corner of the slope. This is meaningless and only suggests that a slope failure is perhaps not a possibility here and a toe failure is the one most likely to occur. The above observations have been made by Basudhar (1976) as well as in the present analysis.

Unlike other slope angles, the non-circular analysis for $\beta=90^{\circ}$ could not be successfully carried out when Lo's critical surface was used as the initial slip surface due to the following:

Lo's critical slip circle for $\beta = 90^{\circ}$ is essentially a straight line through the toe and inclined at 45° with the horizontal. Now, in the present formulations there are curvature constraints imposed on the shape of the general slip surface. It is noticeable that such constraints (g_j) are only marginally satisfied (i.e., they become active constraints) such that a



1 : LO'S SLIP CIRCLE FOR K = 1
4 : LO'S " " " K = 0.5
3 : GENERAL SLIP SURFACE FOR K = 1
5 : GENERAL " " " K = 0.5
2 : INPUT FOR GENERAL SLIP SURFACE

FIG. 5.9 CRITICAL SLIP SURFACES FOR $\beta = 90^\circ$

small shift in the co-ordinates of any point on the slip surface, representing a design variable, causes violation of the curvature constraint around the said point and thus incurs heavy penalty. As the Extended Penalty function method tends to bring the solution back to the feasible region (to maintain the concavity of the shear surface), it has been observed that a few points towards the upper end of the slip surface cross the slope boundary, which means these points violate the boundary constraints. The process of bringing the solution back to the feasible regions again causes violation of a few curvature constraints. As this process goes on repeating, the minimization scheme cannot converge to any solution even after many r - minimization cycles. However, these results are not presented here.

By analysing the Lo's critical surface by Spencer's method using the proposed equation-solver, it has also been observed that while the obtained factor of safety matches with Lo's value, the line of thrust is highly unsatisfactory (Table 5.2). This indicates that the said surface is not acceptable.

Eventually, a different starting surface with any reasonable concavity (Figure 5.9) has been tried. With this changed initial design vector a convergent solution has been achieved. considering the relatively short stretch of the slip surface in the case of $\beta = 90^{\circ}$, a total of 10 slices have been considered. The obtained critical slip surface is also shown in Figure 5.9.

The lines of thrust associated with the critical surfaces obtained in the present analysis remain within the sliding mass

TABLE 5.2

Calculated Responses Corresponding to the Critical Shear Surfaces for the Cases : $\beta = 30^{\circ}$, K = 1 AND $\beta = 30^{\circ}$, K = 0.5

Slice No.	$\beta = 30^{\circ}$, K = 1				$\beta = 30^{\circ}$, K = 0.5				$\beta=90^{\circ}$, K=1
	σ	τ	L/H	$Z/\gamma bH_t$	σ	τ	L/H	$Z/\gamma bH_t$	L/H*
1	1.90	1.80			2.02	1.33			
			0.35	0.23			0.35	0.21	-0.02
2	3.30	1.90			3.64	1.51			
			0.33	0.49			0.30	0.44	-0.05
3	3.90	2.00			4.85	1.69			
			0.30	0.69			0.28	0.64	-0.03
4	4.80	2.00			5.79	1.82			
			0.29	0.88			0.26	0.82	-0.09
5	5.00	1.90			6.49	1.92			
			0.28	0.98			0.25	0.94	-0.16
6	5.70	1.90			6.94	1.99			
			0.27	1.07			0.23	1.00	-0.25
7	5.80	1.80			7.13	2.02			
			0.25	1.08			0.22	0.98	-0.35
8	5.70	1.70			7.01	2.00			
			0.24	1.00			0.20	0.87	-0.33
9	5.50	1.50			6.57	1.87			
			0.22	0.86			0.19	0.68	-0.39
10	5.10	1.30			5.75	1.64			
			0.19	0.63			0.19	0.49	-0.59
11	4.40	1.10			4.71	1.36			
			0.20	0.41			0.19	0.32	-0.90
12	3.40	0.90			3.67	1.08			
			0.20	0.23			0.20	0.18	-1.40
13	2.40	0.60			2.63	0.78			
			0.21	0.10			0.22	0.08	-2.25
14	1.50	0.40			1.58	0.47			
			0.29	0.03			0.28	0.02	-5.12
15	0.50	0.10			0.53	0.16			

* These data are obtained through analysis of Lo's critical slip surface by Spencer method.

though not strictly within the middle-third of the interslice heights. However, it has been generally observed that as the slope angle β increases, there is a tendency for the line of thrust to increasingly fall below the lower-third positions. Introduction of tension crack and/or assumptions of different k-distribution may improve the line of thrust; however, such trials have not been done here. The interslice forces as well as the normal and shear stresses at the slice bases are, however, positive in all cases. Typical values of these responses have been presented in Table 5.2 for the case of $\beta = 30^{\circ}$ for both K=1 and K = 0.5.

Typical design vectors and constraints before and after minimization for the case of $\beta=30^{\circ}$, K=1 are presented in Table 5.3. It is seen that the initially violated constraints are finally satisfied.

Figure. 5.10 and Figure 5.11 show the path followed by the f and ψ functions with the progress of minimization for the cases $\beta=30^{\circ}$, K = 1 and $\beta=30^{\circ}$, K = 0.5 respectively. It is seen that they follow the same trend as has been observed in the case of homogeneous slopes.

A summary of the results obtained (F_{min} and N_s) in the present analysis is presented in Table 5.4. Values obtained from Lo's chart are also presented side by side for the sake of comparison. The Table shows that as far as the magnitude of the factor of safety is concerned, there is practically no difference between the non-circular analysis in the present study and the circular analysis by Lo.

TABLE 5.3

Design Vector and Constraints ($\beta = 30^0$, $K = 1$)

NUMBER OF SLICES = 15
NUMBER OF DESIGN VARIABLES = 18

STARTING POINT

F = 1.2000 θ = 0.4000

DESIGN VARIABLES (not normalized)

-0.0333	-0.05430	-0.0631	-0.0600	-0.0449	-0.0175	0.0226	0.0761
0.1439	0.2274	0.3286	0.4505	0.5973	0.7761	1.7320	-0.7999
1.2000	0.4000						

Constraints (Inequality)

-1.3424	-1.2786	-1.2159	-1.1539	-1.0921	-1.0299	-0.9668	-0.9024
-0.8360	-0.7668	-0.6714	-0.5495	-0.4027	-0.2239	-0.0124	-0.0121
-0.0120	-0.0120	-0.0122	-0.0127	-0.0134	-0.0143	-0.0157	-0.0177
-0.0206	-0.0249	-0.0320	-0.0450	-0.4003	-0.3804	-0.3706	-0.3654
-0.3637	-0.3659	-0.3732	-0.3889	-0.4206	-0.4890	-0.7155	-1.4164
-6.0791	13.7528	-0.5997	-0.6196	-0.6294	-0.6346	-0.6363	-0.6341
-0.6268	-0.6111	-0.5794	-0.5110	-0.2845	0.4164	5.0791	-14.7528
-0.4500	-0.4000	-0.1236					

Constraint (Equality) 0.2220E+00

ϵ_o = -0.10, δ_t = 0.0001 f = 1.200 = 1.2236
Z_n = -0.3414E-01 M_n = -0.3593E+00

OPTIMAL POINT

F = 1.0843 θ = 0.3208

DESIGN VARIABLES (not normalized)

-0.0344	-0.0670	-0.0668	-0.0659	-0.0345	-0.00305	0.0468	0.1203
0.2141	0.3336	0.4661	0.5995	0.7330	0.8665	1.7204	-0.8610
1.0843	0.3208						

Constraints (Inequality)

-1.3241	-1.3261	-1.2193	-1.1630	-1.0685	-1.0050	-0.9334	-0.8499
-0.7623	-0.6664	-0.5339	-0.4005	-0.2670	-0.1335	-0.0085	-0.0327
-0.84E-03	-0.0304	-0.91E-04	-0.0184	-0.0237	-0.0203	-0.0256	-0.0130
-0.95E-03	-0.51E-04	-0.15E-05	-0.23E-05	-0.3498	-0.3292	-0.3036	-0.2935
-0.2752	-0.2658	-0.2542	-0.2379	-0.2186	-0.1938	-0.1965	-0.2011
-0.2144	-0.2859	-0.6502	-0.6708	-0.6964	-0.7065	-0.7248	-0.7342
-0.7458	-0.7621	-0.7814	-0.8062	-0.8035	-0.7989	-0.7856	-0.7140
-0.3343	-0.3208	-0.2028					

Constraint (Equality) 0.2570E-11

No. of r-minmizations required = 6
f = 1.0843 = 1.0845 Z_n = -0.2517E-06 M_n = -0.6520E-07

Note : Out of the 59 inequality constraints, first 14 are boundary constraints, the next 14 are curvature constraints, the next 28 are the constraints on the line of thrust and the last 3 are the side constraints on F and θ respectively. Lower limit on F is 0.75 here.

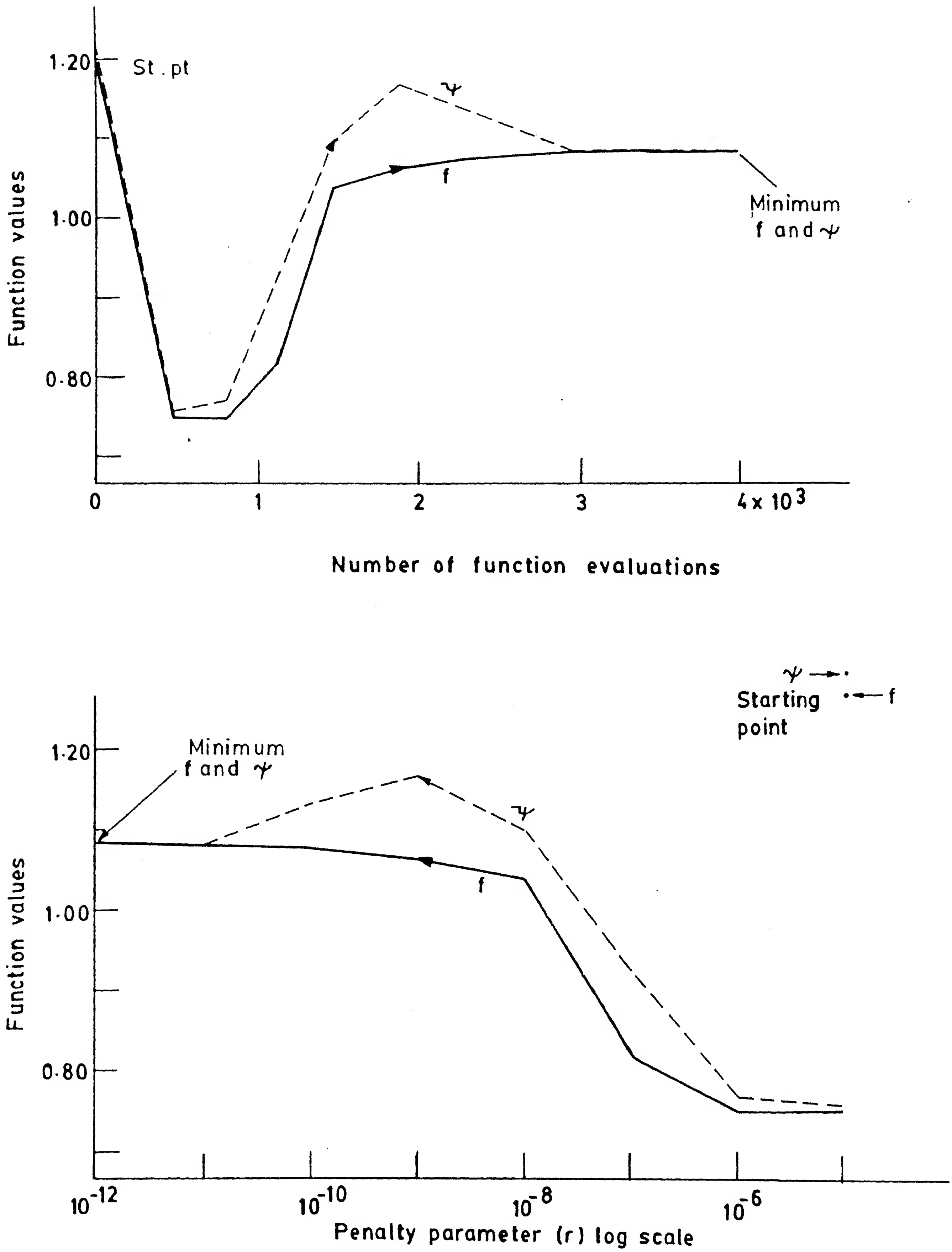


FIG. 5·10 PROGRESS OF MINIMIZATION IN THE CASE $\beta = 30°$, K = 1

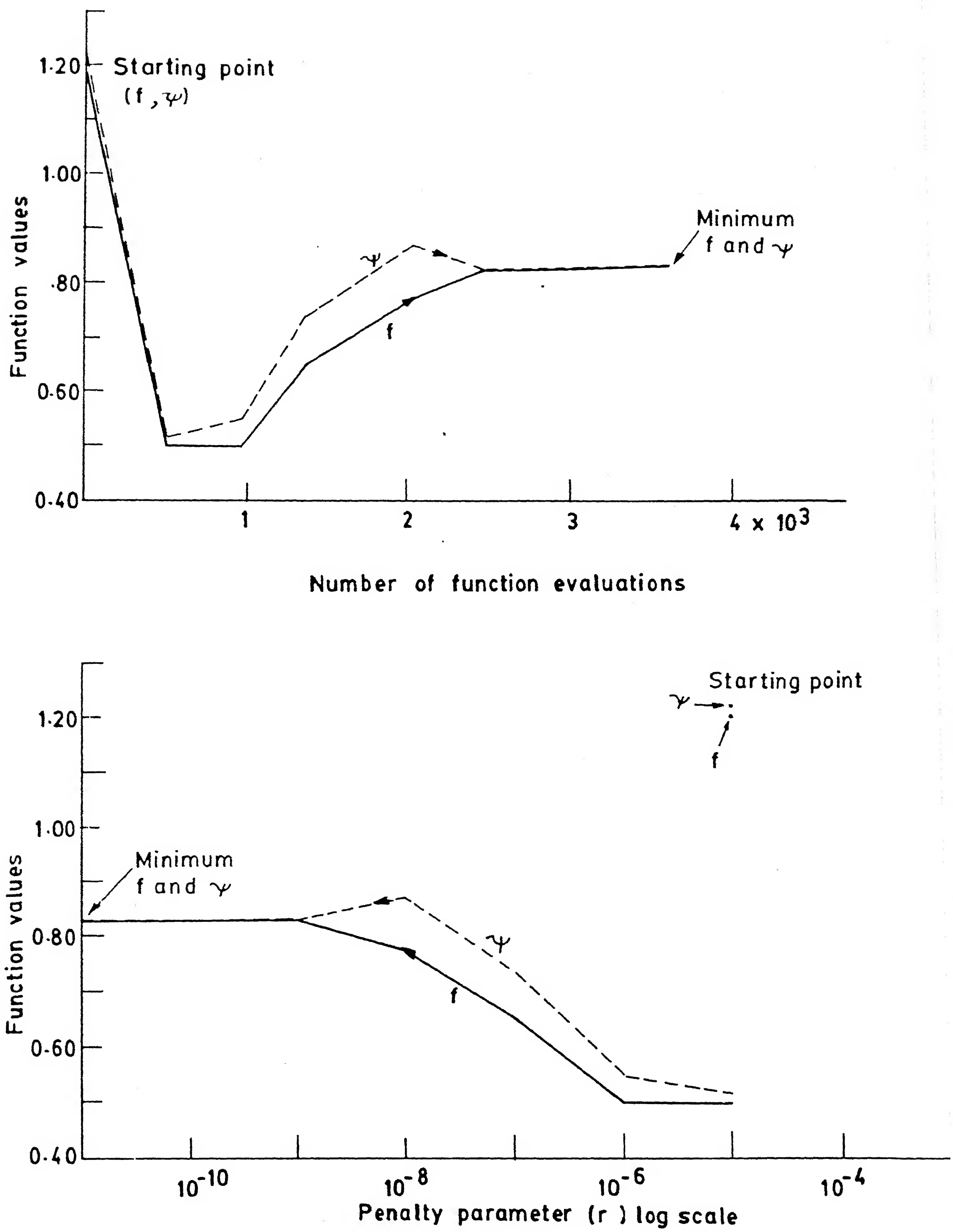


FIG. 5.11 PROGRESS OF MINIMIZATION IN THE CASE $\beta = 30^\circ, K = 0$

TABLE 5.4

Comparison of Present Solution with LO'S Solution

Sl. No.	Slope angle	Degree of anisotropy	Min. Factor of Safety: Present soln.	Min. Factor of Safety: Lo's soln.	Stability Number: Present soln.	Stability Number: Lo's soln.
1	15°	1.0	1.7138	1.7021	8.5692	8.5105
		0.5	1.2220	1.2220	6.1100	6.1094
2	30°	1.0	1.0843	1.0897	5.4215	5.4484
		0.5	0.8280	0.8399	4.1398	4.1997
3	45°	1.0	0.8105	0.8213	4.0527	4.1064
		0.5	0.6677	0.6805	3.3386	3.4025
4	60°	1.0	0.6338	0.6468	3.1669	3.2341
		0.5	0.5614	0.5741	2.8071	2.8704
5	90°	1.0	0.4188	0.4000	2.0942	1.9965
		0.5	0.4136	0.3920	2.0680	1.9601

To study the effect of anisotropy on the shape and location of the critical slip surface and the corresponding minimum factor of safety for various slope angles, Table 5.5 present a form of quantitative comparison of the critical non-circular slip surfaces obtained for the cases K=1 and K=0.5. The Table is formed as explained below:

A number of vertical sections at equal intervals (which are positioned very close to the slice boundaries used in the analysis) have been considered and these are numbered serially starting from the section passing through the toe towards the upper intersection point of the slip surface with the ground surface (from left to right as per Figures 5.5 through 5.9). The depths of the slip surfaces (from the horizontal crest level), at their intersections with these discrete vertical sections, are measured from Figures 5.5 through 5.9. The ratios of such depths give an idea of the relative shallowness or deepness or proximity of these surfaces. Similarly, the ratio of the x co-ordinates of the corresponding ends of the two critical slip surfaces give an idea about the relative lateral deviation. Alongside this, the F_{min} values are also compared.

To study the effect of the a priori assumption regarding the shape of the slip surface, a similar kind of comparison is presented in Table 5.6 between the critical non-circular slip surface and the Lo's critical slip circle.

The notations used in Tables 5.5 and 5.6 have the following meanings (Figures 5.4 through 5.8)

x_I^B, x_I^T = the x-coordinates of the slip surface at its bottom (toe) and top intersection points with the ground surface respectively, in the isotropic case i.e., K = 1. these are measured from left to right

x_A^B, x_A^T = the above corresponding to K = 0.5.

d_I, d_A = the depths from the original ground surface, of the points on the slip surface at its intersection with the discrete vertical sections in the case of K = 1 (isotropic)and K = 0.5 (anisotropic) respectively.

F_I, F_A = the values of the minimum factor of safety for K = 1 and K = 0.5 respectively

x_P^B, x_P^T = the x co-ordinates of the critical slip surface obtained in the present solution at its bottom and top intersection points.

x_L^B, x_L^T = the above corresponding to Lo's solution.

d_P, d_L = the depths from the original ground surface, of the points on the critical slip surface at its intersection with the discrete vertical sections in the present solution and Lo's solution respectively.

F_P, F_L = the values of the minimum factors of safety in the present and Lo's solutions respectively.

Table 5.5 shows that the ratio $\frac{d_A}{d_I}$ assumes a value as high as 5-10 (corresponding to the 13th vertical section for the case of β=45^o) which indicates that the critical non-circular surfaces corresponding to the isotropic and anisotropic cases are quite apart. Detailed observation shows the following:

At the section 1, $\frac{d_A}{d_I}$ and x_A^B/x_I^B are very close to 1.0 for all the β-values, indicating that all the surfaces pass through the

toe. As one goes from left to right, the ratio $\frac{d_A}{d_I}$ increases (or remains constant) which means that the critical surfaces for the anisotropic cases are much deeper. As β increases, $\frac{d_A}{d_I}$ increases at a faster rate. The ratio x_A^T/x_I^T is generally greater than 1.0 which means the critical surface for the anisotropic cases deviates outwardly from the corresponding isotropic surface.

An exception to this trend is the case with the flat slope of $\beta=15^o$. It is seen that contrary to the above the ratio d_A/d_I starts decreasing from the middle towards the upper intersection point, which means that in that part the surface corresponding to anisotropic case moves upward and inward relative to that corresponding to the isotropic case.

The ratio F_A/F_I is the least (0.71) for $\beta=15^o$ and it increases as β increases and assumes a value of 0.99 at $\beta=90^o$. This shows that as far as the magnitude of the factor of safety is concerned, it is the flatter slopes which are most affected due to anisotropy. For $\beta=90^o$, the ratio is nearly 1 which signifies that in vertical cuts anisotropy has no significant influence.

In Table 5.6 a detailed comparison between the solutions obtained by Lo (1965) and the present solution in respect of the critical slip surface and the corresponding factor of safety has been presented. From the ratio (x_P^B/x_L^B), (x_P^T/x_L^T) and (dP/dL) designating the ratios of the starting and end x-coordinates and the intermediate depths with respect to the corresponding values of Lo's critical surface, it can be observed that the obtained slip surfaces are significantly different from Lo's critical circular surfaces especially towards the right intersection

TABLE 5.5

Detailed Comparison Between Solution Obtained for K = 1 and K = 0.5

β deg.	x_A^B / x_I^B	x_A^T / x_I^T		Discrete Sections Considered Along x-axis (L to R) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	F_A / F_I
15	1.02	0.74	d_A / d_I	1.04	1.03	1.01	1.00	1.00	1.00	1.00	0.95	0.94	0.90	0.85	0.79	0.62	0.19	0.71
30	1.00	1.46	d_A / d_I	1.01	1.04	1.06	1.06	1.06	1.10	1.15	1.17	1.25	1.36	1.52	1.88	2.50	-	0.76
45	1.04	1.95	d_A / d_I	1.06	1.09	1.10	1.11	1.13	1.14	1.18	1.24	1.32	1.51	1.75	2.31	5.10	-	0.82
60	1.00	1.59	d_A / d_I	1.06	1.09	1.13	1.17	1.25	1.38	1.58	1.93	3.05	-	-	-	-	-	0.89
90	1.00	1.14	d_A / d_I	1.02	1.04	1.06	1.14	1.16	1.27	1.44	1.72	-	-	-	-	-	-	0.99

TABLE 5.6

Detailed Comparison Between Present and Lo's Solutions

β deg.	K	$\frac{x_P^B}{x_L^B}$	$\frac{x_P^T}{x_L^T}$		Discrete Sections Considered Along x-axis (L to R) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\frac{F_P}{F_L}$
15	1.0	0.95	1.42	$\frac{d_P}{d_L}$	0.93	0.94	0.95	0.95	0.95	0.95	0.97	1.02	1.01	1.02	1.08	1.15	1.43	9.00	-	1.01
15	0.5	0.97	1.05	$\frac{d_P}{d_L}$	0.97	0.97	0.97	0.96	0.95	0.95	0.96	0.97	0.95	0.91	0.92	0.91	0.88	1.70	-	1.00
30	1.0	1.0	1.10	$\frac{d_P}{d_L}$	1.00	1.02	1.01	1.01	1.02	1.00	0.97	0.95	0.93	0.87	0.83	0.81	1.00	-	-	1.00
30	0.5	1.0	1.40	$\frac{d_P}{d_L}$	1.01	1.04	1.03	1.02	1.03	1.03	1.03	1.03	1.02	1.02	1.03	1.14	1.38	3.60	-	0.99
45	1.0	0.96	0.89	$\frac{d_P}{d_L}$	0.98	1.00	1.00	1.00	1.00	1.00	0.97	0.94	0.92	0.86	0.80	0.74	0.49	-	-	0.99
45	0.5	1.0	1.51	$\frac{d_P}{d_L}$	1.04	1.06	1.07	1.07	1.07	1.06	1.06	1.05	1.07	1.11	1.13	1.26	1.47	2.15	19.0	0.98
60	1.0	1.0	1.05	$\frac{d_P}{d_L}$	1.01	1.03	1.03	1.02	1.0	1.0	1.0	1.0	1.17	-	-	-	-	-	-	0.98
60	0.5	1.0	1.46	$\frac{d_P}{d_L}$	1.05	1.08	1.10	1.10	1.10	1.14	1.19	1.30	1.56	3.13	-	-	-	-	-	0.98
90	1.0	1.0	1.19	$\frac{d_P}{d_L}$	1.06	1.12	1.21	1.29	1.45	1.68	2.4	2.5	-	-	-	-	-	-	-	1.05
90	0.5	1.0	1.24	$\frac{d_P}{d_L}$	1.08	1.14	1.21	1.34	1.46	1.67	2.09	3.33	-	-	-	-	-	-	-	1.06

point. The difference increases as the slope angle and the degree of anisotropy increase. However, there is no appreciable difference between the corresponding factor of safety values as is evident from the ratios (F_P/F_L).

5.4 CONCLUSIONS

Based on the studies conducted in this chapter the following conclusions are drawn :

1. The generalised search procedure based on the Sequential Unconstrained Minimization Technique (SUMT) can be successfully combined with models of anisotropy and nonhomogeneity to study the influence of undrained shear strength variation with depth and direction on the stability of slopes.
2. Powell's method can be successfully used to minimize such complicated trigonometric functions as the expression for Lo's Stability Number.
3. The critical shear surface and the corresponding factor of safety for anisotropic soil significantly differ from those for the isotropic soil; the effect of anisotropy on the factor of safety is more pronounced in flat slopes. This finding agrees with Lo's observations.
4. The obtained non-circular critical surfaces are significantly different from Lo's critical circular slip surfaces especially towards the upper intersection point. The difference increases as the slope angle as well as the degree of anisotropy increases. However there is no appreciable difference between the corresponding factor of safety values.

CHAPTER 6

ANALYSIS OF ZONED EMBANKMENTS AND DAMS

6.1 INTRODUCTION

The proposed method based on the Sequential unconstrained minimization technique has been successfully applied to slopes in homogeneous soils as demonstrated in Chapter 4. In this chapter the methodology has been extended to the analysis of zoned embankments and dams founded on layered deposits. The review of the available literature pertaining to the analysis of zoned dams has already been presented in Chapter 1.

In extending the basic methodology as discussed in Chapter 3 to heterogeneous slope sections, proper care needs to be taken in the calculation of the weight, the mobilized shearing stress and pore water pressure for each slice. Once this is done, the same procedure as formulated for homogeneous slopes and presented in Chapter 3 can also be applied to the non-homogeneous slopes. For dams founded on thin shear zone a special formulation has also been presented in this chapter.

Several example problems have been chosen to study the effectiveness of the method in dealing with heterogeneous sections. Example problem 6.1 has been included with a view to compare the solutions obtained by using the developed program with those given by other currently available programs as reported in the literature. The purpose of the Example problem 6.2 is to study the multiplicity of solutions obtained by using a

complete equilibrium method, in particular, the Spencer's method. Example problems 6.3, 6.4 and 6.5 deal with re-analysis of some well known dams by using the developed technique and comparison of the same with other reported solutions.

6.2 ANALYSIS

6.2.1 General

In the extension of the proposed technique of stability analysis to the analysis of zoned embankments and dams, the basic principles of the general formulations presented in Chapter 3 with reference to homogeneous slopes remain essentially the same. The Direct and the Indirect procedures as well as the two discretization models remain essentially unchanged. However, some modifications are required in the formulation of the objective function, design variables and design constraints. These are discussed in the following sections.

6.2.2 The objective Function, the Design Variables and the Constraints:

6.2.2.1 General

As before, the factor of safety of the slope is the objective function and the slip surface co-ordinates form the design vector and, in the Direct formulation, the design vector consists of the factor of safety and the interslice force inclination in addition to the slip surface co-ordinates. The

design variables are subjected to the same constraints as defined and discussed earlier namely, the boundary, curvature, acceptability and side constraints. Only some of the boundary constraints need modifications in their mathematical forms to take care of a more complex slope geometry of a heterogeneous embankment or dam section.

6.2.2.2 Modified Boundary Constraints:

The modified versions of the constraint equations have been derived for both circular and non-circular shear surfaces as follows:

A. Circular Shear Surface:

Figure 6.1 shows a typical section of a zoned dam along with a trial circular shear surface. Three subsoil layers have been shown in the figure. There can be, however, any number of layers with any arbitrary alignment.

As mentioned in Chapter 3, the upper boundary constraint requires that at least a portion of the slip circle lies within the slope to be analysed. For the slope surface ABCD (Figure 6.1) which consists of more than one segments, the above requirement will be satisfied if the following conditions are met :

(i) The radius, R, of the slip circle is larger than the perpendicular distance of the sloping surface from the center of the circle.

Since the sloping surface ABCD has three segments AB, BC and CD, the said perpendicular distance is actually the maximum of

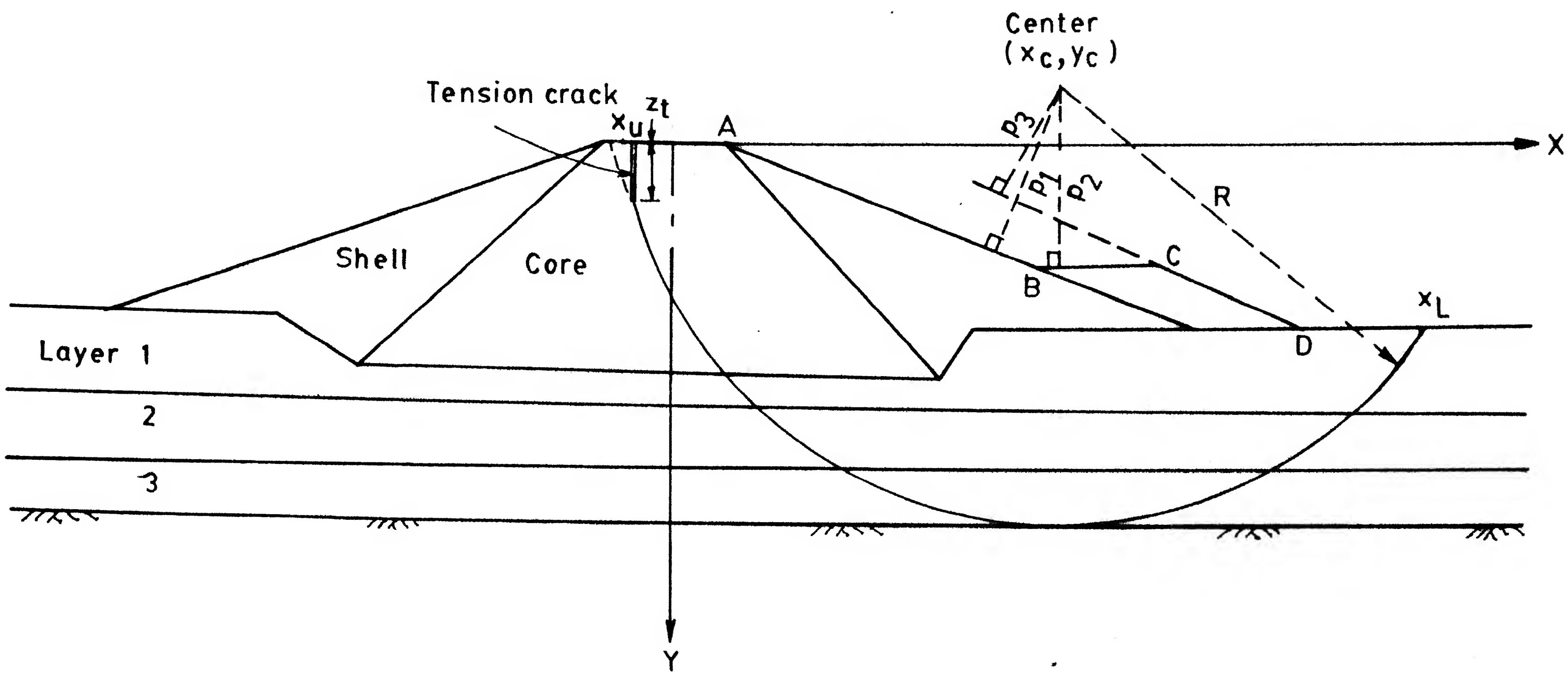


FIG. 6.1 TYPICAL SECTION OF A ZONED DAM ALONG WITH A TRIAL CIRCULAR SHEAR SURFACE

the three perpendicular distances p_1, p_2, p_3. Calling this as p_{max}, the above condition may be mathematically expressed as,

$$p_{max} - R \leq 0$$

i.e., $$g_j(D) = p_{max} - R$$

or, normalizing, $$g_j(D) = \frac{p_{max}}{R} - 1 \tag{6.1}$$

(ii) Referring again to Figure 6.1

$$x_L \geq 0$$

i.e., $$g_j(D) = - x_L$$

or, normalizing, $$g_j(D) = - \frac{x_L}{R} \tag{6.2}$$

(i) $$x_C \geq 0$$

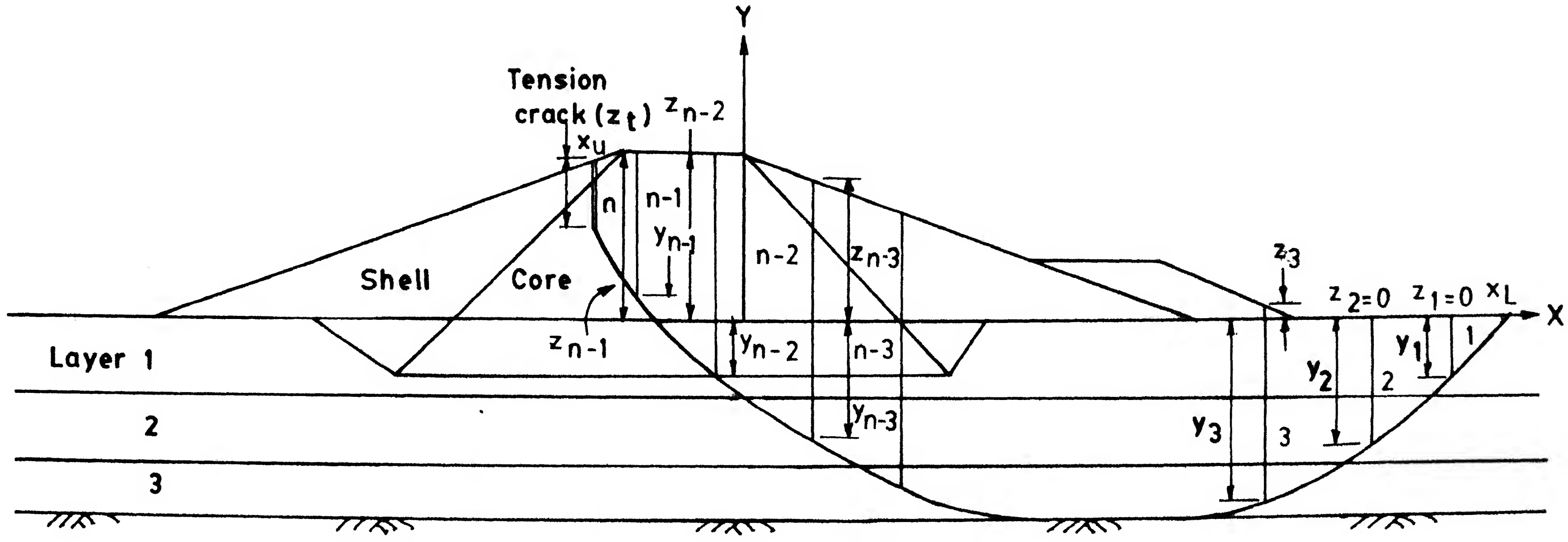
i.e., $$g_j(D) = -x_C$$

or, normalizing, $$g_j(D) = - \frac{x_C}{R} \tag{6.3}$$

where, x_C and x_L are the x-coordinates of the center and the lower intersection point of the slip circle respectively. The perpendicular distances p_1, p_2, p_3 are calculated using the known equations to the straight line segments AB, BC, CD etc. and the known co-ordinates of the center.

B. Non-circular Shear Surface:

Figure 6.2 shows the same dam section with a non-circular slip surface. The axis system and the numbering of slices shown in the figure are as generally used in the analysis by Spencer method while those followed in the analysis by Janbu's method are as shown in Figure 6.3. Referring to these figures, y and z represent the ordinates of the points on the shear surface and on



Axis system and numbering of slices shown in the figure are as generally used for analysis by Spencer method

FIG. 6.2 TYPICAL SECTION OF A ZONED DAM ALONG WITH A TRIAL NON-CIRCULAR SHEAR SURFACE

the slope boundary respectively corresponding to a particular x and the upper boundary constraints can be expressed as:

$y_i \leq z_i$ for analysis by Spencer method (Figure 6.2)

$y_i \geq z_i$ for analysis by Janbu's method (Figure 6.3)

where $i = 1,2,\ldots\ldots,n-1$

and n = the total number of slices

Thus, for analysis using Spencer's method,

$$g_j(D) = y_i - z_i$$

or, normalizing,
$$g_j(D) = \frac{y_i}{z_i} - 1 \tag{6.4a}$$

And, for analysis using Janbu's method,

$$g_j(D) = z_i - y_i$$

or, normalizing,
$$g_j(D) = \frac{z_i}{y_i} - 1 \tag{6.4b}$$

The forms of lower boundary constraints, however, remain the same as presented in Chapter 3.

6.2.2.3 Constraint on the Depth of Tension Crack (z_t)

In the general formulation presented in Chapter 3, a constraint on the depth of tension crack has been included in the analysis treating z_t as a design variable. The constraint limits the value of z_t to z_o, the depth of zero active earth pressure, as given by Equation 2.31. Such an expression, however, is not valid for the calculation of z_o in a layered soil and hence no such constraint has been considered in the relevant cases in the studies contained in this chapter. Elsewhere, tension cracks, if introduced, have been assumed to be of constant depth.

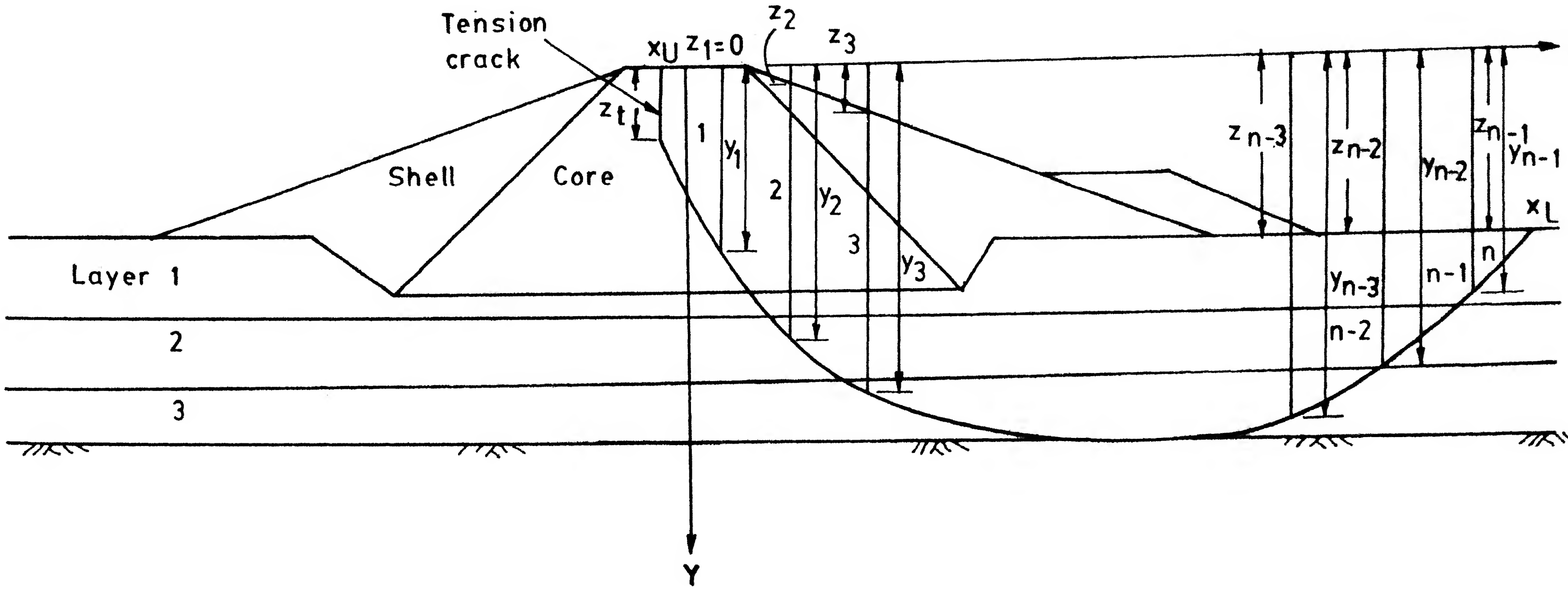


FIG. 6.3 TYPICAL SECTION OF A ZONED DAM WITH A NONCIRCULAR SLIP SURFACE
(Axis system and numbering of slices shown in the figure are as used in analysis by Janbu's method)

6.2.3 Additional Considerations Required in the Analysis of Heterogeneous Slope Sections

6.2.3.1 General

In the computation of the factor of safety of a heterogeneous dam or embankment section using the method of slices, complications arise in the calculation of weight, mobilised shear force and pore water pressure for each slice. This is because of the fact that the base of a particular slice is most likely to lie in more than one zone or region and below a number of layer or zone boundaries starting from the slope boundary. It is, therefore, required to locate the two end points of the base of a slice so that appropriate values for the soil and pore pressure properties can be used in the calculation of the above mentioned slice characteristics.

6.2.3.2 A Generalised Procedure for Computing the Slice Characteristics

A generalised procedure for the computation of the weight, the cohesive and frictional components of the shear strength and the resultant pore water pressure for each slice has been developed. It consists of the following steps:

1. To draw the slope section to a convenient scale.

2. To introduce a co-ordinate system with the following conventions:

(a) The horizontal x-coordinate increases in the direction of slide.

(b) The vertical y-coordinate increases in the direction of gravity.

(c) The origin is conveniently chosen at the mid-point of the top width of the embankment or dam section so that the y-coordinates of all points in the search region are positive.

3. The trial or given shear surface is drawn and the sliding mass is divided into the selected number of slices which need not be of uniform widths. Thus, the coordinates of the points on the shear surface at its intersections with the interslice boundaries are known. The slope of the base of the k^{th} slice can be calculated from:

$$\tan \alpha_k = \frac{y_{k+1} - y_k}{\Delta x_k} \qquad (6.5)$$

where,

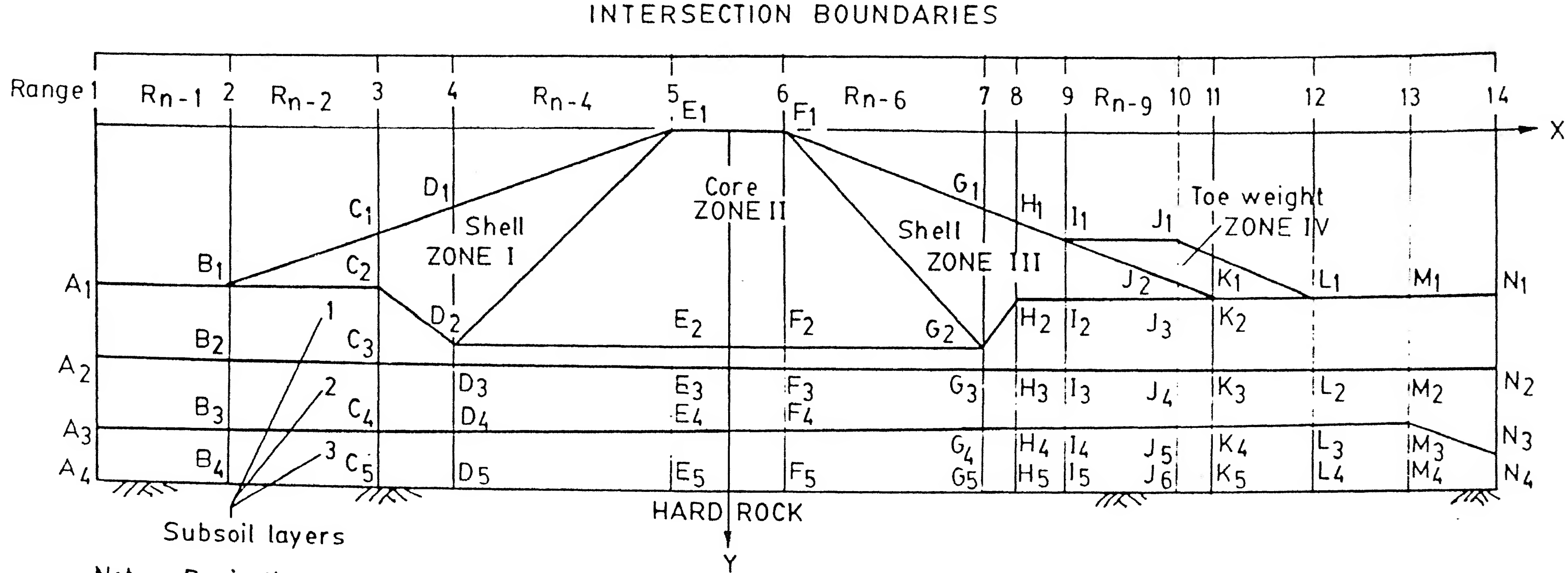
α_k = the angle made by the base of the k^{th} slice with the horizontal.

Δx_k = the width of the k^{th} slice

y_k, y_{k+1} = the ordinates of the points of intersection between the shear surface and the k^{th} and the $(k+1)^{th}$ slice boundaries respectively i.e., the ordinates of the two ends of the base of the k^{th} slice.

4. To introduce a vertical boundary at every point where the slope surface and the boundary between layers change directions. Such points are called the 'intersection points' of the slope section. When either the slope surface or any of the layer boundaries has a vertical section, two vertical boundaries are introduced at the same x i.e., a strip of zero thickness is assumed in between (calculation of slice weight etc. in such a situation has been discussed in detail in Appendix-B). Thus the entire search region is divided into a number of parts along the horizontal x-direction. Each of these parts is herein termed as a 'range' and each of the vertical boundaries mentioned above as an Intersection boundary. For any slope section the number of ranges is fixed.

5. The slope surface as well as the layer boundaries are specified by listing the y-coordinates of the points at which they are intersected by the intersection boundaries. Various zones present in a zoned dam are also treated as "layers". The geometry of any layer is specified by the co-ordinates of its lower boundary. If in a particular range (segment) a layer (zone) does not exist it has to be treated as a layer of zero thickness in that range or a number of ranges. Layers are numbered from the face of the slope downwards. To illustrate the numbering of the layers, the dam section shown in Figure 6.4 is chosen again for which the layer numbering is as given in tabular form below the dam section. For example, the layer number 1 which includes the toe weight marked Zone IV has thickness of J_1J_2 and K_1K_2 at the 10^{th} and the 11^{th} intersection boundaries



Note :- R_n is the abreviation used for range

Int. boundary → / Layer number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Slope surface	A_1	B_1	C_1	D_1	E_1	F_1	G_1	H_1	I_1	J_1	K_1	L_1	M_1	N_1
Layer 1	A_1	B_1	C_1	D_1	E_1	F_1	G_1	H_1	I_1	J_2	K_2	L_1	M_1	N_1
2	A_1	B_1	C_1	D_1	E_1	F_1	G_2	H_2	I_2	J_3	K_2	L_1	M_1	N_1
3	A_1	B_1	C_2	D_2	E_1	F_1	G_2	H_2	I_2	J_3	K_2	L_1	M_1	N_1
4	A_1	B_1	C_2	D_2	E_2	F_2	G_2	H_2	I_2	J_3	K_2	L_1	M_1	N_1
5	A_2	B_2	C_3	D_3	E_3	F_3	G_3	H_3	I_3	J_4	K_3	L_2	M_2	N_2
6	A_3	B_3	C_4	D_4	E_4	F_4	G_4	H_4	I_4	J_5	K_4	L_3	M_3	N_3
7	A_4	B_4	C_5	D_5	E_5	F_5	G_5	H_5	I_5	J_6	K_5	L_4	M_4	N_4

FIG. 6.4 SPECIFICATION OF GENERAL LAYER SYSTEM IN A TYPICAL HETEROGENEOUS SECTION

respectively and zero thickness at all other boundaries as at those boundaries it coincides with the slope surface. Similarly, the layer number 2 which includes the shell marked zone III has thickness of G_1G_2, H_1H_2, I_1I_2 and J_2J_3 at the 7^{th}, 8^{th}, 9^{th} and 10^{th} intersection boundaries respectively and zero thickness elsewhere. The idea of treating a zone of a dam as part of a layer or a stratum has been used earlier by Baker (1979).

From the known values of the co-ordinates of the intersection points of the slope geometry the values of the gradient $\bar{m}$ and the intercept $\bar{c}$ for each of the ranges are calculated using the following expressions:

$$\bar{m} = \bar{m}_{i,j} = \frac{\bar{y}_{i+1,j} - \bar{y}_{i,j}}{\bar{x}_{i+1} - \bar{x}_i} \tag{6.6}$$

and,
$$\bar{c} = \bar{c}_{i,j} = \frac{\bar{x}_{i+1}\,\bar{y}_{i,j} - \bar{x}_i\,\bar{y}_{i+1,j}}{\bar{x}_{i+1} - \bar{x}_i} \tag{6.7}$$

where the equation of the i^{th} range of the j^{th} layer is given by:

$$Y = \bar{m}_{i,j}\,X + \bar{c}_{i,j} \tag{6.8}$$

$$i = 1,\ 2,\ \ldots,\ n_R$$
$$j = 1,\ 2,\ \ldots,(n_L+1)$$

where

(X,Y)= co-ordinate of any point within the search domain.

$\bar{x}_i$ = the x-coordinate of the i^{th} intersection boundary

$\bar{y}_{i,j}$ = the y-coordinate of the i^{th} intersection point of the j^{th} layer

$\bar{m}_{i,j}$ = the gradient of the i^{th} range of the j^{th} layer

$\bar{c}_{i,j}$ = the intercept made by the i^{th} range of the j^{th} layer on the y-axis

n_R = the total number of ranges

n_L = the total number of layers

In the above, j=1 refers to the slope surface, j=2 refers to the layer number 1 and so on.

6. It is required to locate the position of the base of a slice so that appropriate soil and pore pressure properties can be ascribed to it. To do so it is required to locate the two end points of the base. The technique adopted to locate any point (x,y) on the shear surface is explained as follows:

(a) The range i within which the concerned point lies can be found such that

$$\bar{x}_i \le x \le \bar{x}_{i+1} \qquad i = 1,\ 2,\ \ldots,\ n_R$$

(b) Similarly, the layer j containing the point can be found such that,

$$y_{d_j} \le y \le y_{d_{j+1}} \qquad j = 1,\ 2,\ \ldots,\ n_L$$

where, $y_{d_{j+1}}$ is the y-coordinate of the point of intersection of the j^{th} layer boundary with an imaginary vertical line through the given point (x,y) on the shear surface and is calculated as:

$$y_{d_{j+1}} = \bar{m}_{i,j}\ x + \bar{c}_{i,j} \qquad (6.9)$$

By repeating (a) and (b) above, the following are found out for a slice:

(i) the range/ranges covering the two of its sides.

(ii) the layer/layers containing the two ends of its base.

7. The number of intersection boundaries falling within the width of a slice is noted. If this number is n_B, the entire slice is thus divided into (n_B+1) number of strips or sub-slices of varying widths. Each of these sub-slices lies entirely within one range. An example of such a slice is shown in Figure 6.5(a) in which case $n_B = 5$.

8. The y-coordinates of the two ends of the base of a sub-slice can be calculated from the already known values of (i) the y-coordinates of the ends of the base of the original slice, (ii) x-distances of the intersection boundaries and (iii) the inclination of the base of the original slice. Next, following step 6, the layers containing the two ends of the base of each of the sub slices are also found out. Next, taking a sub-slice it is to be checked whether its base has been intersected by any layer boundary. If the two bottom ends of the concerned sub slice lie in the same layer, then there is obviously no such intersection. Otherwise, there may be one or more intersections with one or more than one layer boundaries. These are found out by turns. The co-ordinates of such intersection points are calculated by using the known equations of the straight lines representing the base and the particular range of the concerned layer boundary. If the number of such base intersection is n_F,

then the number of subdivisions of this sub-slice will be (n_F+1). These are the most elementary units constituting an original slice; there cannot be any possibility of such a unit being subdivided. Any such unit lies entirely within a range and its base also lies entirely within one layer. Referring to Figure 6.5(a), it is seen that the sub-slice marked S_5 has been intersected once by a layer boundary, namely, layer 1. So for S_5, the number of elementary units is 2 as shown separately in Figure 6.5(b).

9. *Weight of a slice:*

The weight of an elementary unit described above is the sum total of the weights contributed by each of the layers it has passed through. Referring to Figures 6.5(c) and 6.5(d), the weight contributed by the j^{th} layer will be the area of the trapezium ABCD multiplied by the unit weight of that layer.

Let, b_F = the width of the unit slice.

n_D = the deepest layer or the number of layers the unit slice passes through.

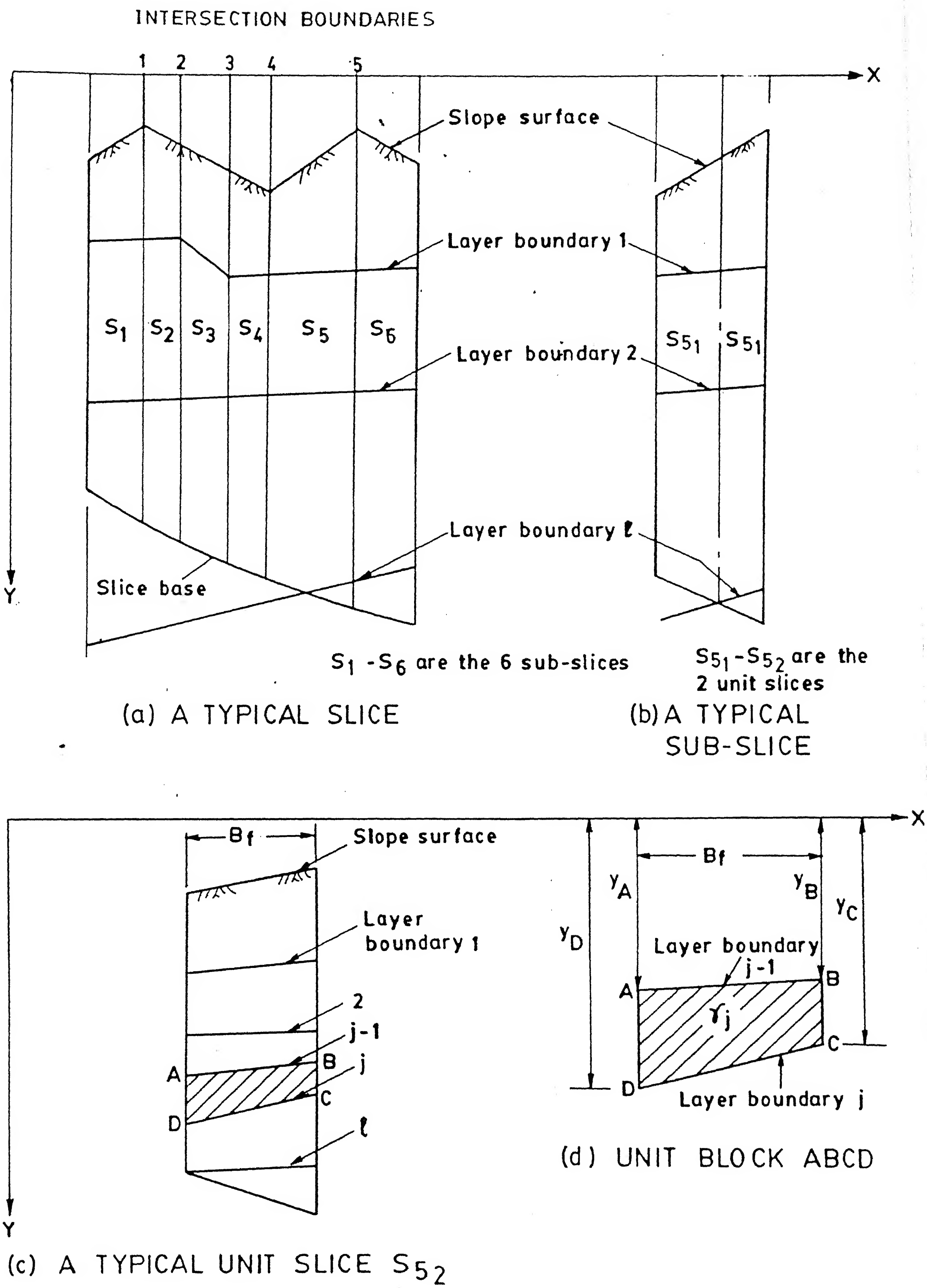
γ_j = the unit weight of the soil of the j^{th} layer

W_{ABCD} = the weight of the block ABCD

The weight of the k^{th} elementary slice is, then, given by,

$$W_{ele_k} = \sum_{j=1}^{n_D} W_{ABCD} \qquad [k = 2 \text{ for } S_{5_2} \text{ in Figure 6.5(c)}]$$

$$= \sum_{j=1}^{n_D} \frac{1}{2}\gamma_j b_F \left\{ \left[Y_D - Y_A \right] + \left[Y_C - Y_B \right] \right\} \qquad (6.10)$$



INTERSECTION BOUNDARIES
1 2 3 4 5
X
Y
Slope surface
Layer boundary 1
Layer boundary 2
Layer boundary ℓ
Slice base
S_1 S_2 S_3 S_4 S_5 S_6
S_{5_1} S_{5_1}
S_1 - S_6 are the 6 sub-slices
S_{5_1} - S_{5_2} are the 2 unit slices
(a) A TYPICAL SLICE
(b) A TYPICAL SUB-SLICE
Bf
Slope surface
Layer boundary 1
2
j-1
j
ℓ
A
B
C
D
(c) A TYPICAL UNIT SLICE S_{5_2}
y_A
y_B
y_C
y_D
Layer boundary j-1
γ_j
Layer boundary j
(d) UNIT BLOCK ABCD

FIG. 6.5

In certain cases, e.g., the bottom most block of the unit slice S_{5_2} in Figure 6.5(c) is triangular. Since a triangle is only a special case of a trapezium, the above formula (6.10) holds for all cases.

The weight of one sub-slice (say k^{th}) is obtained as:

$$W_{ss_k} = \sum_{k=1}^{n_F+1} W_{ele_k} \tag{6.11}$$

Finally, the weight of an entire slice is obtained as:

$$W = \sum_{k=1}^{n_B+1} W_{ss_k} \tag{6.12}$$

10. *Shear Strength Components:*

Considering that the two ends of a slice-base may lie in different layers, the magnitudes of cohesive and frictional components of shear strength available is likely to vary along a slice-base. However, as the base of an elementary unit described in the previous step lies entirely in a single layer, the cohesive and friction components calculated for an elementary unit may be summed up to obtain the corresponding values for the entire slice, as follows:

$$\text{Cohesive force component} = \sum_{i=1}^{n_E} c'_i \, \Delta l_i \tag{6.13}$$

$$\text{Frictional force component} = \sum_{i=1}^{n_E} \tan \phi'_i \, \Delta l_i \tag{6.14}$$

where,

c'_i, ϕ'_i = the shear strength parameters corresponding to the i^{th} elemental unit.

Δl_i = the length of the base of the i^{th} elemental unit

n_E = total number of such elemental units in the enti slice

If, for the slice concerned, the average strength parameters a: required, they can be obtained as:

$$c'_{av} = \frac{\sum_{i=1}^{n_E} c'_i \, \Delta l_i}{\sum_{i=1}^{n_E} \Delta l_i} \tag{6.15}$$

and,

$$\phi'_{av} = \tan^{-1}\left[\frac{\sum_{i=1}^{n_E} \tan\phi'_i \, \Delta l_i}{\sum_{i=1}^{n_E} \Delta l_i}\right] \tag{6.16}$$

11. *Pore Water Pressure:*

The developed computer program for the proposed procedure has the provisions for calculation of pore water pressure using two different methods:

(i) Method 1 using Bishop's pore pressure coefficient, r_u

(ii) Method 2 using given piezometric surface.

Method 1: This method can be applied to situations where values of r_u are available for all the zones of the dam or embankment section. Following the same principle as discussed in Step 10 in connection with the calculations of cohesive and frictional components of shear strength for an entire slice, the total force, U_b due to pore pressure acting on a slice base may be

obtained as follows:

$$U_b = \sum_{i=1}^{n_E} r_{u_i} \, \sigma_{b_i} \, \Delta l_i \tag{6.17}$$

where, r_{u_i} = r_u value corresponding to the i^{th} unit slice.

σ_{b_i} = average overburden pressure on the base of such a unit.

If, for the slice concerned, the average pore pressure on the base u_{av} is required, it can be obtained as:

$$u_{av} = \frac{\sum_{i=1}^{n_E} r_{u_i} \, \sigma_{b_i} \, \Delta l_i}{\sum_{i=1}^{n_E} \Delta l_i} \tag{6.18}$$

Method 2: This method is applicable where the piezometric line or surface is available. The pore water pressures are generated using the vertical distances from the piezometric surface to the shear surface.

In this method, a new set of intersection boundaries are introduced at every point where the given piezometric surface changes directions. These vertical boundaries thus form a number of vertical strips. The distance between any two consecutive boundaries is also referred to as a 'range' with reference to the pore pressure calculation. The original slices may be quite arbitrarily positioned with respect to the vertical strips.

In situations where the phreatic line or the top flow line is available in place of the piezometric surface, calculation of pore pressure can be carried out treating the phreatic surface as

the piezometric surface. This approximation, however, introdu some error. A detailed discussion on this and in general on methods of pore pressure calculations has been presented by Lai (1989) and as such they are not stated here.

When the phreatic surface is to be used, the first step using the Method 2 is to approximate the curved surface by series of straight line segments and then the new set intersection boundaries are introduced as described above. Fo example, Figure 6.6 shows a dam section with an assumed phreati line which has been approximated by 7 linear pieces namely, ab bc, cd, de, ef, fg, gh and hi. Now, regarding the positioning o the piezometric surface with respect to the bases of the variou slices, the following four cases may arise:

Case I: Within the strip, the piezometric surface lies entirely above the shear surface (Figure 6.7a). The force due to pore pressure acting at the base of a single strip or unit is given by

$$\Delta U_i = \frac{1}{2} \gamma_w (h_L + h_R) \Delta b \sec (\Delta\alpha) \qquad (6.19)$$

Case II: Within the strip the piezometric surface intersects the base of the strip with its right end going down (Figure 6.7b).

Here the pressure will be caused by only that part of the piezometric surface which is above the base and is given by

$$\Delta U_i = \frac{1}{2} \gamma_w h_L x_p \sec (\Delta\alpha) \qquad (6.20)$$

The horizontal distance, x_p, is given by

$$x_p = \frac{(c_2 - c_1)}{(m_1 - m_2)}$$

where m_1, c_1 and m_2, c_2 are the constants in the following two equations representing the base of the strip and the

PHREATIC LINE INTERSECTION BOUNDARY

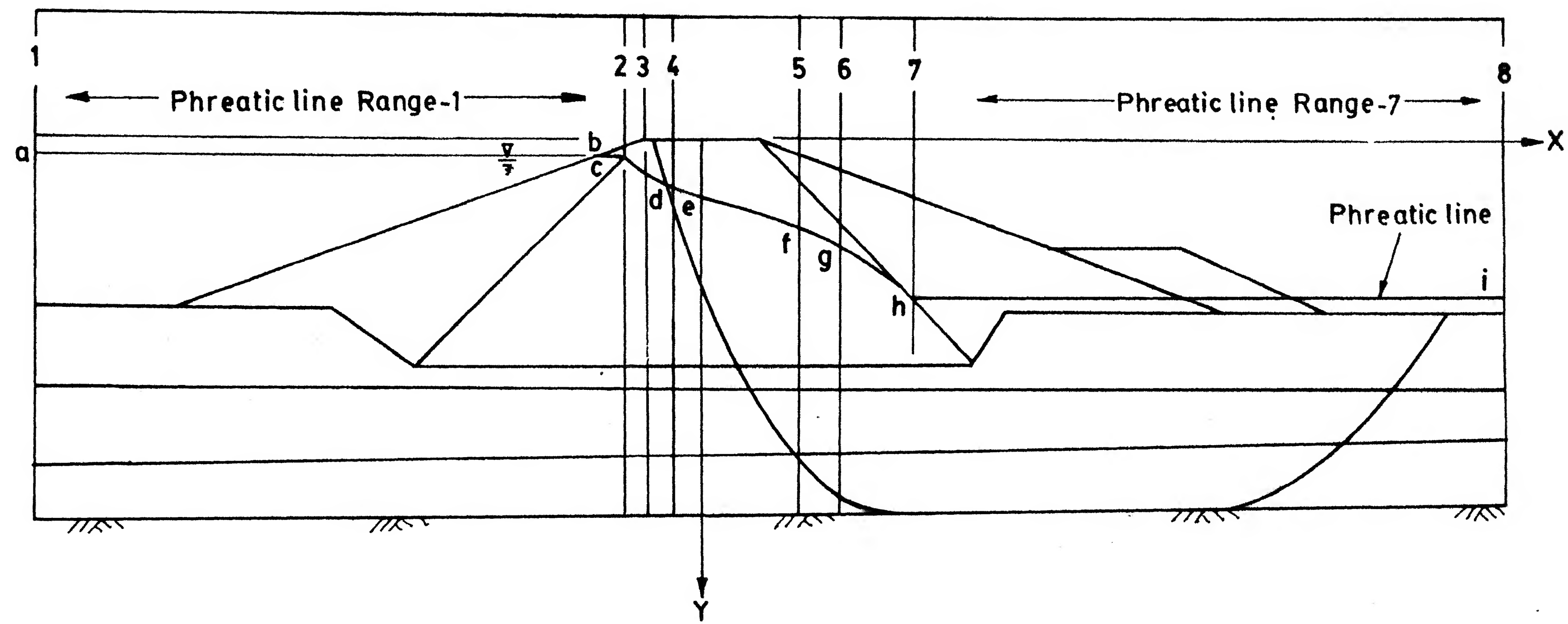


FIG. 6·6 TYPICAL SECTION OF A ZONED DAM ALONG WITH THE PHREATIC LINE

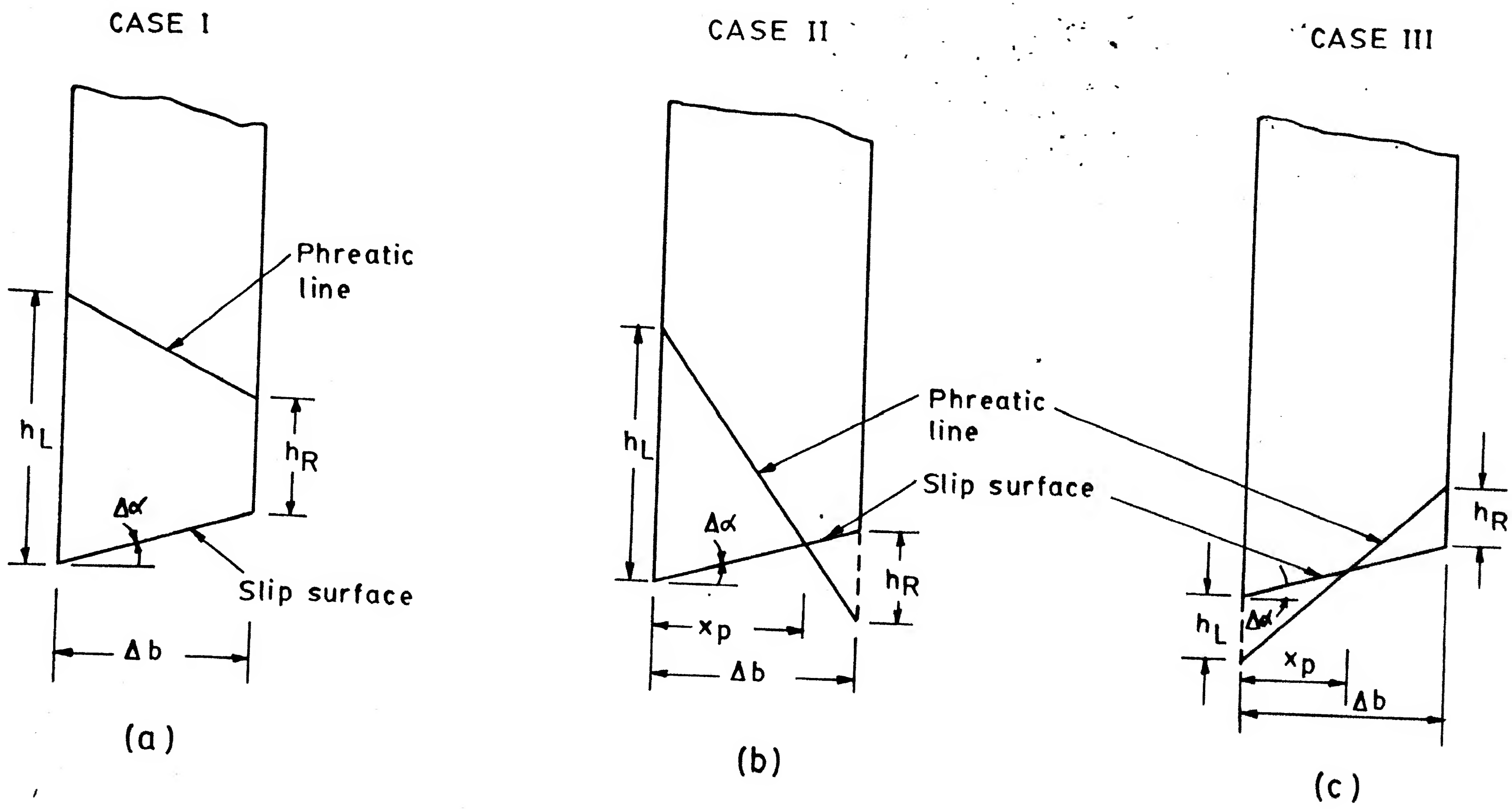


FIG. 6.7 VARIOUS POSITIONS OF THE PHREATIC LINE WITH RESPECT TO THE SLIP SURFACE

piezometric line respectively:

$$Y = m_1X + c_1$$

and

$$Y = m_2X + c_2$$

Case III: The piezometric surface intersects the base with the left end going down (Figure 6.7c). Following the same notations as used above,

$$\Delta U_i = \frac{1}{2} \gamma_w h_R (\Delta b - x_p) \sec (\Delta\alpha) \qquad (6.21)$$

Case IV: The piezometric surface lies entirely below the base of the strip:

In this case, for the strip concerned, $\Delta U_i = \emptyset$

Resultant Force Due to Pore Pressure:

The resultant force due to pore pressure acting at the base of an original slice is then obtained by adding all the components. If n_p is the number of strip or unit per slice, the resultant force is given by:

$$U_b = \sum_{j=1}^{n_p} \Delta U_i \qquad (6.22)$$

12. *Forces Due to Water Ponding on the Face of the Slope:*

The forces exerted by the water ponding on the face of the slope are computed slice-wise. Referring to Figure 6.8(a) it can be seen that two cases may arise and are discussed as follows:

Case I: This case corresponds to slices such as marked 'I' in Figure 6.8(a). This slice has been separately shown in Figure 6.8(b). Let h_1 and h_2 be the vertical distances of the two top ends of the slice from the water line measured downward. The

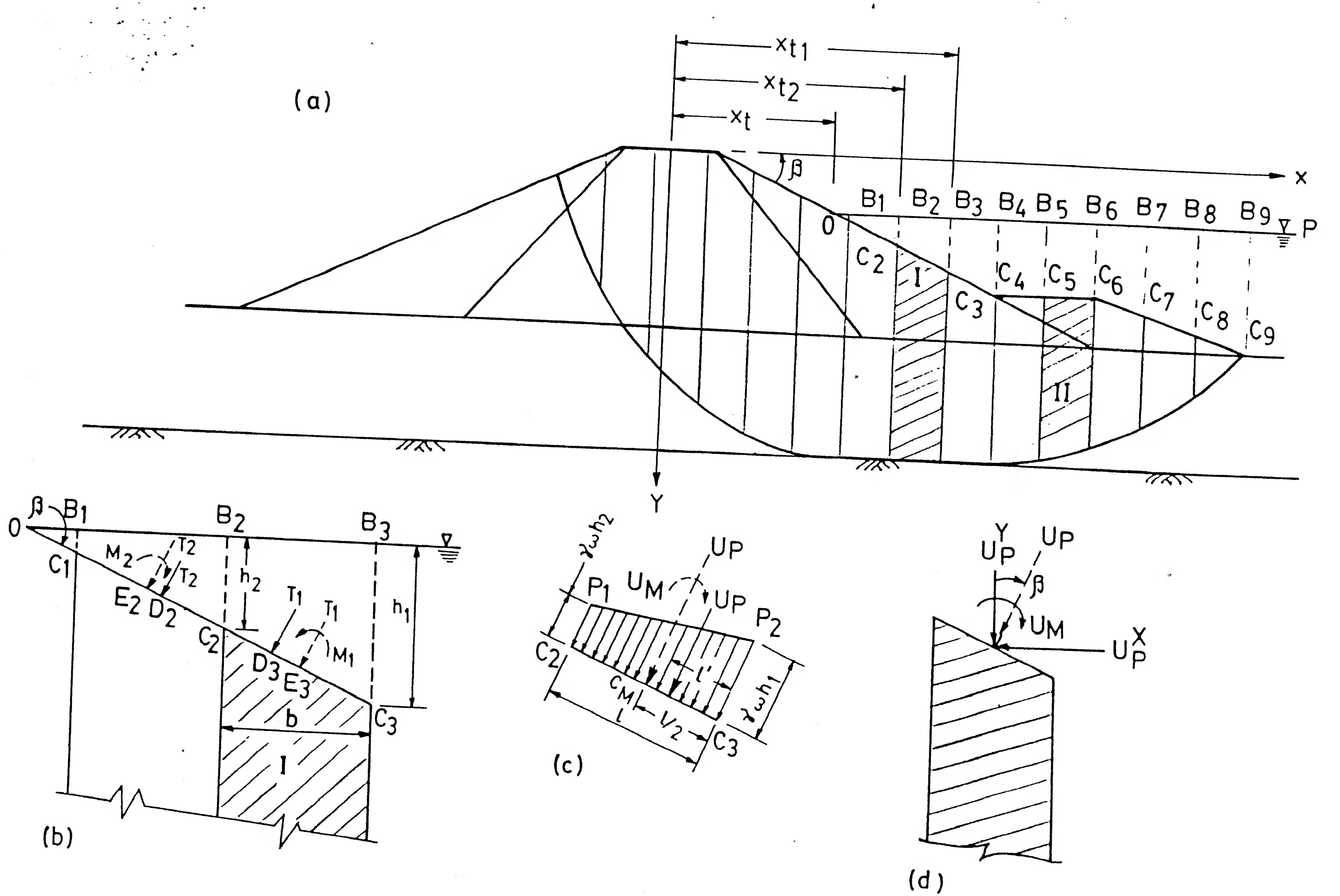


FIG. 6.8 FORCES DUE TO WATER PONDING ON THE FACE OF A SLOPE

trapezoidal pressure diagram has been shown in Figure 6.8(c). The resultant thrust on the slice is given by:

$$U_p = \text{Area of the trapezium } P_1 \, C_2 \, P_2 \, C_3$$
$$= \frac{1}{2} \gamma_w (h_1 + h_2) l \qquad (6.23)$$

where, $l = \sqrt{b^2 + (h_1 - h_2)^2}$

b = width of the slice

and it acts normally at a distance l' from C_3

where $l' = \frac{l}{3} \left[\frac{h_1 + 2h_2}{h_1 + h_2} \right]$

This is equivalent to a force U_p acting at the mid-point C_M together with a couple in the clockwise sense of moment

$$U_M = U_p \left(\frac{l}{2} - l'\right) \qquad (6.24)$$

Case II: This case corresponds to slices marked II in Figure 6.8(a) which are bounded by the horizontal ground surface. For this case,

$h_1 = h_2$ and hence $l = b$ and $l' = \frac{l}{2}$

This makes $U_M = \emptyset$

For the sake of convenience in including these forces in the formulation of Janbu's and Spencer's methods, U_P has been resolved horizontally and vertically, as shown in Figure 6.8(d), giving:

$$U_p^X = U_p \sin \beta \qquad (6.25)$$

$$U_p^Y = U_p \cos \beta \qquad (6.26)$$

13. *Earthquake Force Considerations:*

General:

For slopes subjected to earthquake, the program has the provision for a pseudo-static analysis by incorporating, slicewise, additional static forces in the horizontal and the vertical directions. The magnitudes of these forces (Figure 6.9) at the i^{th} slice are given by:

$$\Delta Q_i = \alpha_h W_i \tag{6.27}$$

$$\Delta V_i = \alpha_v W_i \tag{6.28}$$

where, W_i is the weight of the slice and α_h and α_v are the so-called seismic coefficients in the horizontal and the vertical directions respectively. The points of application are the centres of gravity of the respective slices. Although the pseudo-static analysis has several limitations (Seed, 1966), it is still the most commonly used approach because of its simplicity. The forces ΔQ_i and ΔV_i can be easily accounted for in Janbu's analysis and the generalised Spencer's analysis methods used in this thesis as both methods include in their formulations a body force each in the horizontal and the vertical direction.

From the known co-ordinates of a slip surface at its intersections with the interslice boundaries, the C.G. of a slice is calculated by the program using the following algorithms:

Calculation of C.G.'s of Slices:

As shown in Figure 6.9(a), the slices can be categorised into two types - Type I slices are those which have negative base slopes while Type II slices are those which have positive base

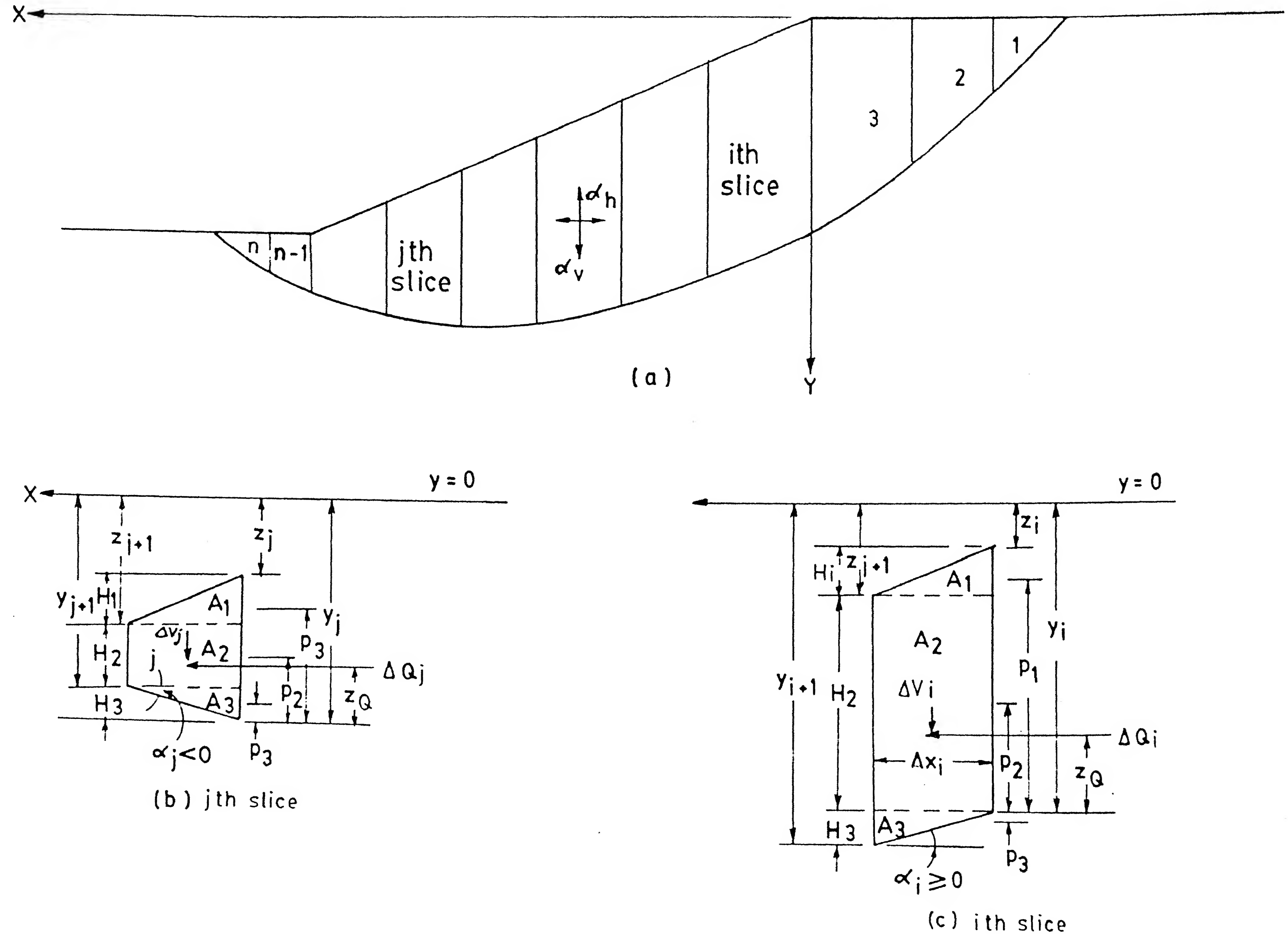


FIG. 6.9 POINT OF APPLICATION OF EARTHQUAKE FORCES

slopes. Each slice is divided, for the purpose of computation, into three areas A_1, A_2 and A_3; and p_1, p_2 and p_3 denote the vertical distances of the C.G.'s of A_1, A_2 and A_3 from the bottom most point of each slice respectively. Determination of the C.G. of each types is presented as follows:

Calculation of p_1, p_2, p_3:

Type I Slices: A typical slice of this category is the j^{th} slice in Figures 6.9(a) and (b), having $\alpha_j < 0$

Referring to Figure 6.9(b),

$$h_j = y_j - z_j \tag{6.29a}$$

$$h_{j+1} = y_{j+1} - z_{j+1} \tag{6.29b}$$

$$H_1 = z_{j+1} - z_j \tag{6.29c}$$

$$H_3 = y_j - y_{j+1} \tag{6.29d}$$

$$H_2 = h_j - (H_1 + H_3) \tag{6.29e}$$

$$p_1 = H_1/3 + H_2 + H_3 \tag{6.29f}$$

$$p_2 = H_2/2 + H_3 \tag{6.29g}$$

$$p_3 = 2H_3/3 \tag{6.29h}$$

Type II Slices: a typical slice of this category is the i^{th} slice in Figures 6.9(a) and (c), having $\alpha_i \geq 0$. Expressions (6.30a) through (6.30d) also hold for these types. Further, referring to Figure 6.9(c),

$$H_2 = h_j - H_1 \tag{6.29i}$$

$$p_1 = H_1/3 + H_2 \tag{6.29j}$$

$$p_2 = H_2/2 \tag{6.29k}$$

$$p_3 = -H_3/3 \tag{6.29l}$$

Calculation of C.G.:

General expressions for the area A_1, A_2 and A_3 are given by:

$$A_1 = \frac{1}{2} \Delta x_i H_1 \tag{6.30a}$$

$$A_2 = \Delta x_i H_2 \tag{6.30b}$$

$$A_3 = \frac{1}{2} \Delta x_i H_3 \tag{6.30c}$$

The C.G. is obtained by taking the first moment of the areas A_1, A_2, A_3 about the bottom most point of the slice as:

$$Z_Q = \frac{A_1 p_1 + A_2 p_2 + A_3 p_3}{A_1 + A_2 + A_3} \tag{6.31}$$

6.2.4 Presence of Thin Shear Zone in the Foundation

6.2.4.1 General

In situations where there is existence of a thin shear zone in the foundation, the technique of stability analysis needs some special considerations. This is due to the fact that whereas physically the major portion of the failure surface is expected to lie along the thin shear zone, there is evidence that as the search for the critical slip surface continues, the intermediate trial slip surfaces tend to lift off from the thin shear zone. This leads the optimization scheme to converge to a local minimum. The critical slip surface so obtained does not follow the shear zone and has a higher factor of safety. Such an observation has been reported by Satyam Babu (1986) in connection with a study on the stability of the Beas dam in India. As a remedial measure, the author has taken a 'reasonable' thickness of the shear zone which is much higher than its actual thickness. The critical slip surface so obtained has been found to lie along

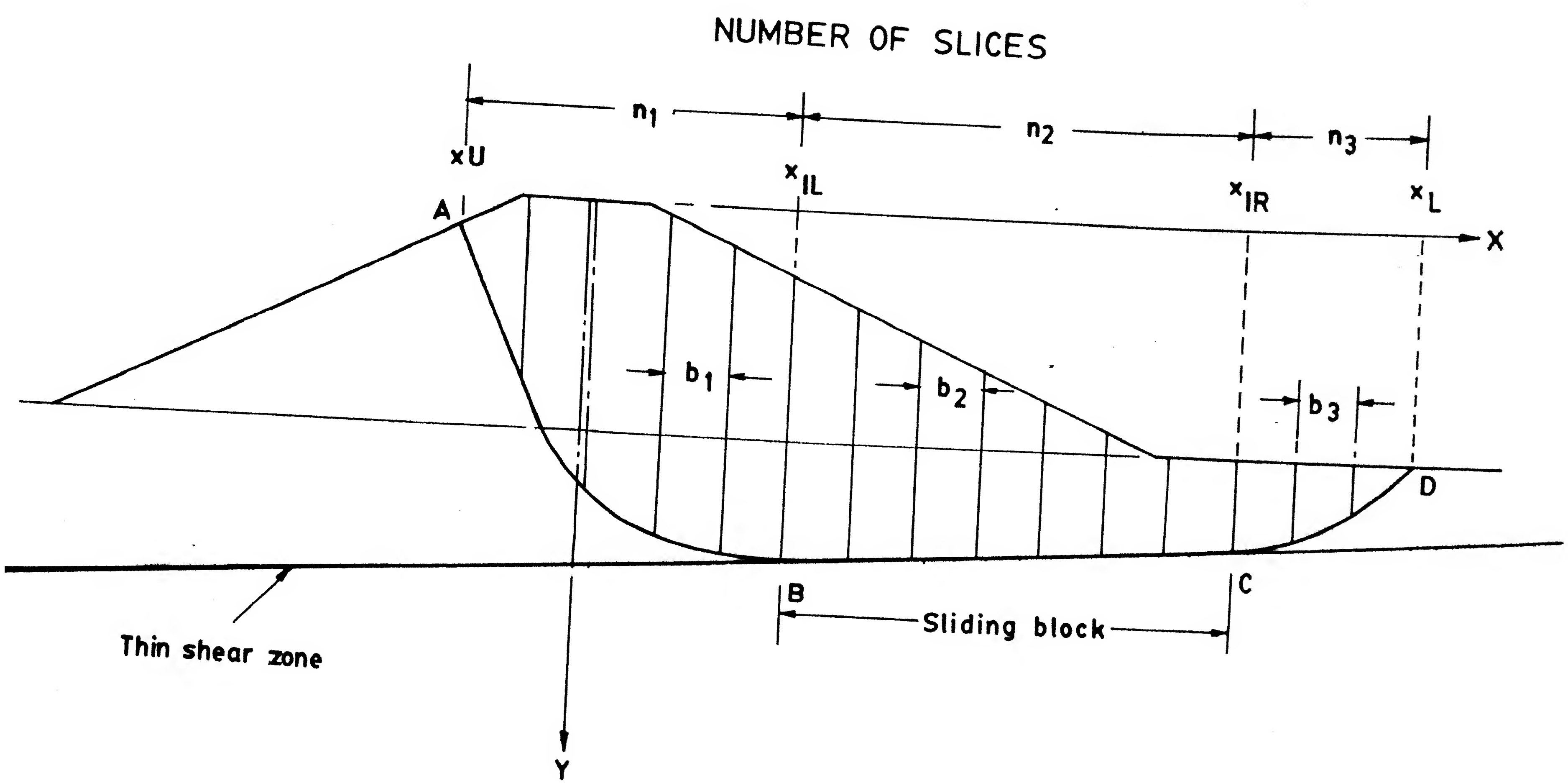


FIG. 6.10 TYPICAL SECTION OF AN EMBANKMENT FOUNDED ON A THIN SHEAR ZONE

1. The y-co-ordinates of the points in the active zone excluding points A and B. These make (n_1-1) variables.
2. The y-co-ordinates of the points in the passive zone excluding points C and D. These make (n_3-1) variables.
3. The x-co-ordinates of the boundary points A,B,C and D. These are denoted as x_U, x_{IL}, x_{IR} and x_L which make 4 variables. Summing up, the total number of design variables,

$$n_{dv} = (n - n_2 + 2)$$

where n = the total number of slices.

However, in the Direct procedure of critical surface determination, two additional design variables are used, namely, the factor of safety, F and the interslice force angle, θ. As such, in this case the total number of design variables,

$$n_{dv} = (n - n_2 + 4)$$

In contrast, as per the general formulation presented in Chapter 3, the total number of design variables in the case of Direct procedure (which is more efficient than the Indirect procedure) is,

$$n_{dv} = n + 3$$

This means, in the special procedure the number of design variables is reduced by (n_2+7) where n_2 is the number of slices in the middle block. This is likely to reduce the computation time substantially.

Constraints

Unlike the general formulation, in this special formulation the curvature constraints need to be imposed only on the points

lying between A and B and those between C and D (Figure 6.10). All other constraints remain unchanged.

Steps to be Followed in the Discretization

The following steps are to be followed in discretizing the mass above the slip surface by subdividing the same into a number of slices in each of the three regions.

1. To draw a trial shear surface of the kind shown in Figure 6.10. The x-coordinates of the points A,B,C,D become immediately known.
2. To select the total number of slices, n, and also the number of slices to be present in the middle block, n_2.
   Both these parameters are kept constant during the search for the critical surface.
3. From the known x-co-ordinates of the points B and C, width of the slice in the middle block (uniform) is given by
$$b_2 = \frac{x_{IR} - x_{IL}}{n_2}$$
4. Taking, as a first trial, $b_1 = b_2$, one gets
   n_1 = the integer part of $\left(\frac{x_{IL} - x_U}{b_2}\right)$
   b_1 is then recalculated as:
$$b_1 = \frac{x_{IL} - x_U}{n_1}$$
5. Finally, the remaining data are obtained as:
$$n_3 = n - (n_1+n_2)$$
   and
$$b_3 = \frac{x_L - x_{IR}}{n_3}$$

6.3. EXAMPLE PROBLEM 6.1

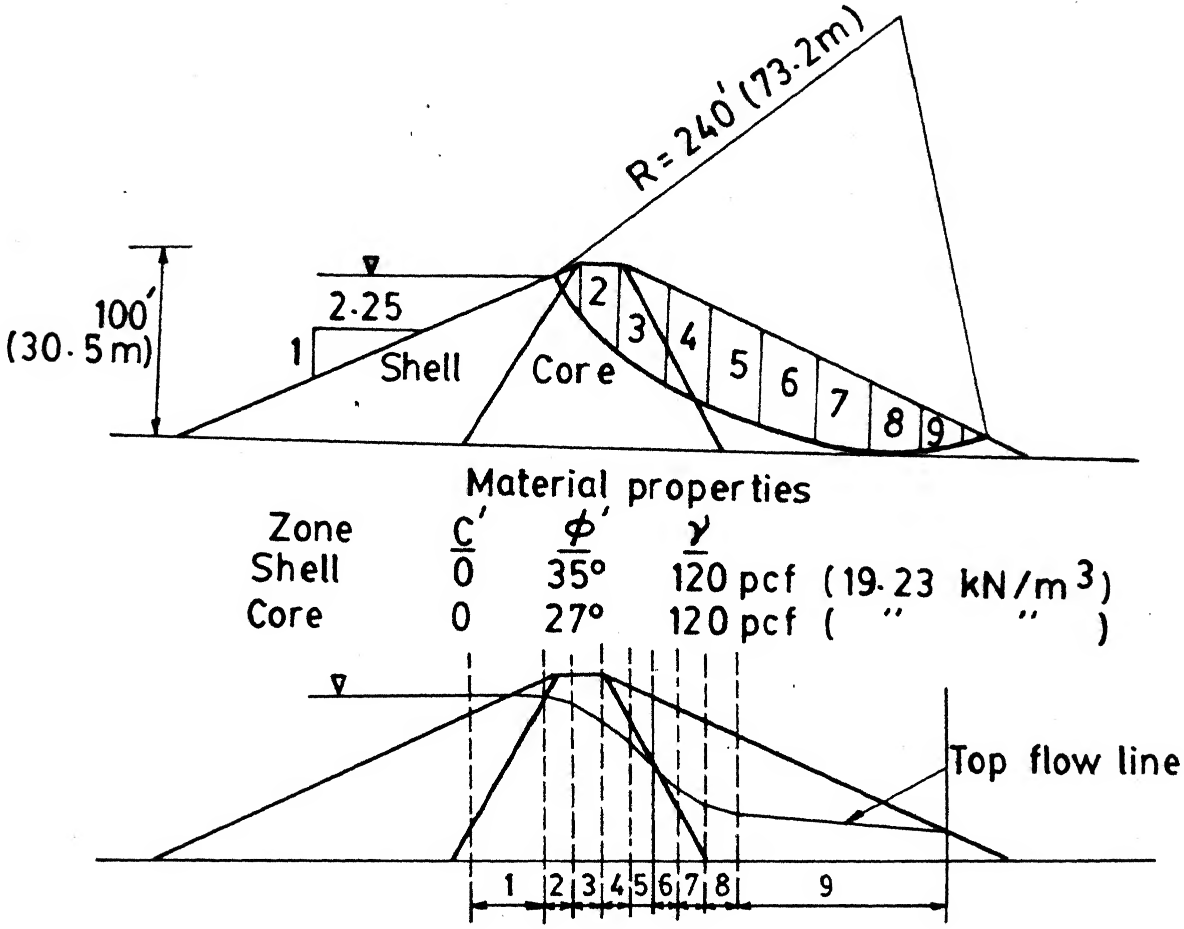


FIG. 6.11 SLOPE SECTION IN EXAMPLE PROBLEM 6.1
(Lambe et.al,1989)

6.3.1 General

Figure 6.11 shows a zoned dam section along with a given slip circle. The mass above the slip circle is divided into 10 slices. The top flow line is also given (and shown in the lower figure) for obtaining the pore water pressures.

Lambe (1989) has taken this example as a sample problem to make a comparative study of some of the commonly used limit equilibrium methods and computer softwares for slope stability analysis. He considered three cases:

(i) zero pore pressure (u=Ø)

(ii) pore pressures (u) calculated by assuming the top flow line as the piezometric surface(as Lambe (1989) points out, this assumption implies existence of vertical equipotentials).

(iii) pore pressures (u) calculated from flow net.

In the present study, the developed software has been used to analyse the given slip surface using Spencer's method, Janbu's GPS, simplified Bishop method and the Ordinary method of slices - for which provisions are there in the developed package. Solutions have been obtained for the cases (i) and (ii) above i.e., (i) u = Ø, and (ii) u obtained from vertical equipotentials.

The choice of this example enables to draw a comparison of the results obtained using the developed computer program and the other existing programs and, in the process, test the performance of the subroutines developed for the calculations of slice weights, pore pressures and shear strength etc. for a heterogeneous section before proceeding with the determinations of critical slip surfaces in subsequent examples.

6.3.2 Results and Discussion:

Table 6.1 shows the factors of safety calculated for the two cases of pore water pressure. For case (ii), the given top flow

line which has been treated as a piezometric surface, has been idealised by breaking it up into nine (9) linear segments as shown in Figure 6.11.

Table 6.1 shows that the results obtained using the developed computer program are in close agreement with those reported by Lambe (1989) and hence it may be concluded that the programs are working satisfactorily.

TABLE 6.1

Factor of Safety of a Given Shear Surface by Various Computer Programs (Example Problem 6.1)

Computer Program	Method	u = Ø	u from vertical Equipotential
LEASE	Ordinary Method of Slice	1.81	1.44
	Simplified Bishop	1.94	1.51
SSTAB	Spencer	1.96	1.54
STABL4	Simpilfied Janbu	1.77	1.44
	Simplified Bishop	1.93	1.54
STABR	Ordinary	1.81	1.46
	Simplified Bishop	1.95	1.58
DEVELOPED PROGRAM			
	Ordinary	1.83	1.41
	Simplified Bishop	1.98	1.54
	Spencer (1973)	2.ØØ	1.59
	Janbu's GPS	2.Ø4	1.61

6.4 EXAMPLE PROBLEM 6.2

6.4.1 General

Chugh (1981a,b) has reported the existence of more than one numerical solution when solving for the factor of safety satisfying both moment and force equilibrium as presented by Spencer (1967, 1973). According to him,

"... in all presentations on slope stability reported in the technical literature, it has tacitly been assumed that there exists only one pair of factor of safety and interslice force inclination (F,δ) defining the solution to a slope stability problem. The possible existence of multiple positive solutions to the slope stability equations has not been addressed thus far....".

Chugh's observations regarding the multiplicity of numerical solutions are based on the stability analysis of the problem shown in Figure 6.12 which illustrates a cross-section of the river bank slope located downstream of the Grand Coulee Dam in the state of Washington, U.S.A. The figure also shows the failure surface whose stability status is of interest. The data for the profile geometry, the material properties and the piezometric line description are converted to SI units and presented in Tables 6.2 (a), (b) and (c) respectively. It is pertinent to quote from Chugh (1981a,b):

"....The objective of this note is to describe by the analysis of a typical slope stability problem, that there exists at least two pair of (F,δ), each satisfying the boundary conditions at the toe and head of a potential slide surface. This is accomplished by solving the slope stability equations in parts numerically and seeking the final solution graphically..... Attempts to solve this problem using the computer code SSTABI (Wright) available at the U.S. Bureau of Reclamation, failed to give a solution. In order to investigate this problem further, it was decided to solve the problem using the numerical-graphical solution procedure....:.".

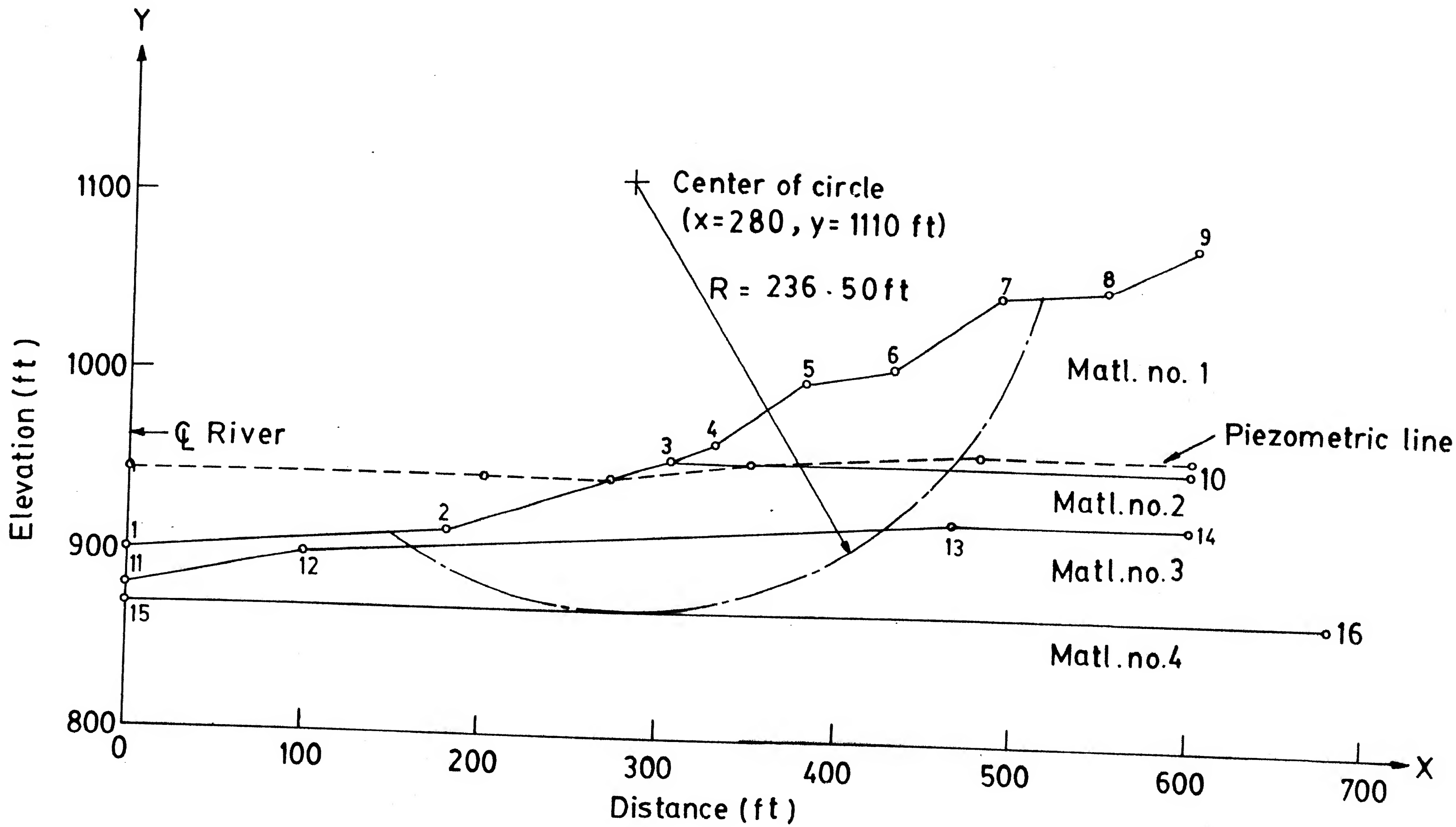


FIG. 6.12 EXAMPLE PROBLEM 6.2 (Chugh, 1981)

TABLE 6.2(a)

Profile Geometry For Example Problem 6.2

Node	X (m)	Y (m)
1	0.00	274.32
2	54.86	278.59
3	92.96	291.08
4	100.58	294.13
5	115.82	304.80
6	131.06	307.24
7	149.35	320.04
8	167.64	321.56
9	182.88	329.18
10	182.88	291.08
11	0.00	268.22
12	30.48	274.32
13	141.73	281.64
14	182.88	281.64
15	0.00	265.18
16	207.26	265.79

TABLE 6.2(b)

Material Properties For Example Problem 6.2

Material type	c' kPa	ϕ' deg	γ kN/m^3
1	0.0	34.0	20.0
2	0.0	34.0	22.1
3	0.0	12.0	20.0
4	48.8	45.0	24.0

TABLE 6.2(c)

Piezometric Line description

For Example Problem 6.2

X (m)	Y (m)
0.00	287.73
60.96	287.73
82.75	287.73
106.68	291.02
146.30	293.22
182.88	293.22

Following the numerical-graphical procedure Chugh obtained two pairs of solution parameters (F,δ) which satisfy the force and moment equilibrium requirements. The numerical values of the solution parameters for the possible solutions 1 and 2 respectively are as follows:

Solution No.	F	δ (=θ)
1	0.7957	9.6700500°
2	0.652386	15.2692238°

The object of the present study is to investigate, with reference to the same problem as described above, the existence of multiple solutions when the proposed equation-solver based on Spencer method in conjunction with the SUMT is used. The formulation of the proposed method has already been presented in Chapter 3.

6.4.2 Results and Discussion

The sliding mass has been divided into 20 slices in the present analysis. In contrast to Chugh's findings, unique solution has been obtained in the present study based on the proposed equation-solver when different starting points have been used. In order to make direct comparison with Chugh's results the two sets of F and θ values obtained by him as two different solutions have been used as initial guess values or starting

points in the two experiments conducted. In both cases the final results have converged to practically the same values of F and θ. Finally, a third experiment has been conducted using a different starting point. This has also converged to practically the same result as is obvious from Table 6.3(a).

Out of the two solutions Chugh points out that the associated line of thrust (for total stress) and other responses are not acceptable for solution 2 and, on the other hand, these are satisfactory for solution 1. These informations "assist the designer in selecting solution 1 over solution 2". From Table 6.3(a), it is also seen that the unique solution obtained in the present study is closer to solution 1 than solution 2.

In all the three solutions obtained in the present study as presented in Table 6.3(a), the associated total stress lines of thrust are satisfactory as they lie virtually within the middle third. Out of these, the line of thrust obtained in the Test no.2 has been shown in Figure 6.13. It is seen that the obtained line of thrust agrees well with that reported by Chugh corresponding to the solution 1, except for a small portion near the upper end. The L/H and L'/H values corresponding to the total and effective stress lines of thrust respectively are also presented in Table 6.3(b). It is seen that although the line of thrust for total stress is satisfactory, that for effective stress crosses the slip surface at its lower end and hence is not satisfactory. However, the values of Z' are all positive as are the values of normal and shear stresses along the slip surface as also seen from the same table. As the object of the present

TABLE 6.3 (a)

Study of Multiplicity of Solution

Results of Present Analysis for the Example Problem 6.2

Test	Initial guess		Optimal Solution					
No.	F	θ (rad.)	F	θ (rad.)	Z_n (kN/m)	$\frac{Z_n}{\gamma b H_t}$	M_n (kn-m/m)	$\frac{M_n}{\gamma b H_t^2}$
1	0.80	0.15 (8.594°)	0.808564	0.175534 (10.05736°)	6.76×10^{-3}	2.25×10^{-6}	-4.98×10^{-1}	-5.53×10^{-6}
2	0.65	0.26 (14.8970)	0.808564	0.175533 (10.05730°)	3.25×10^{-3}	1.08×10^{-6}	5.08×10^{-2}	5.64×10^{-7}
3	1.25	0.10 (5.7296°)	0.808546	0.175531 (10.05720°)	6.31×10^{-2}	2.10×10^{-5}	2.46×10^{0}	2.73×10^{-6}

Note : To non-dimensionalize Z_n and M_n the following average values have been considered : $\gamma = 20.0$ kN/m^3 b = 5.0m H_t = 30.0m

TABLE 6.3 (b)

Calculated Responses for the Solution of Ex. Prob. 6.2

Slice No.	σ (kPa)	σ' (kPa)	τ (kPa)	L/H	L'/H	Z (kN/m)	Z' (kN/m)
1	199.6	80.60	57.35				
				0.59	-52.30	993.0	13.0
2	235.2	85.40	23.53				
				0.59	0.16	1801.0	520.0
3	323.9	154.00	40.46				
				0.50	0.45	3010.0	1390.0
4	379.3	184.30	50.05				
				0.47	0.46	4103.0	2100.0
5	436.5	231.53	60.83				
				0.44	0.44	5289.0	2869.0
6	448.0	233.01	61.24				
				0.42	0.43	5847.0	3202.0
7	479.8	259.80	68.32				
				0.40	0.42	6429.0	3784.0
8	506.2	281.20	73.38				
				0.39	0.40	6837.0	3957.0
9	528.9	295.94	78.87				
				0.38	0.42	6943.0	3942.0
10	543.6	308.63	81.53				
				0.36	0.40	6911.0	4150.0
11	575.1	340.10	90.41				
				0.33	0.36	6878.0	4673.0
12	590.8	365.80	97.21				
				0.31	0.40	6125.0	4224.0
13	609.9	404.91	107.08				
				0.31	0.35	5477.0	3946.0
14	581.7	391.67	104.72				
				0.34	0.40	4468.0	3188.0
15	482.5	327.50	87.24				
				0.45	0.49	2776.0	2371.0
16	357.1	245.10	166.22				
				0.58	0.57	1921.0	1860.0
17	274.7	209.70	182.43				
				0.58	0.56	1281.0	1228.0
18	155.5	163.50	205.91				
				0.48	0.48	1292.0	1292.0
19	159.9	159.90	96.20				
				0.48	0.48	416.0	416.0
20	40.1	40.10	33.46				

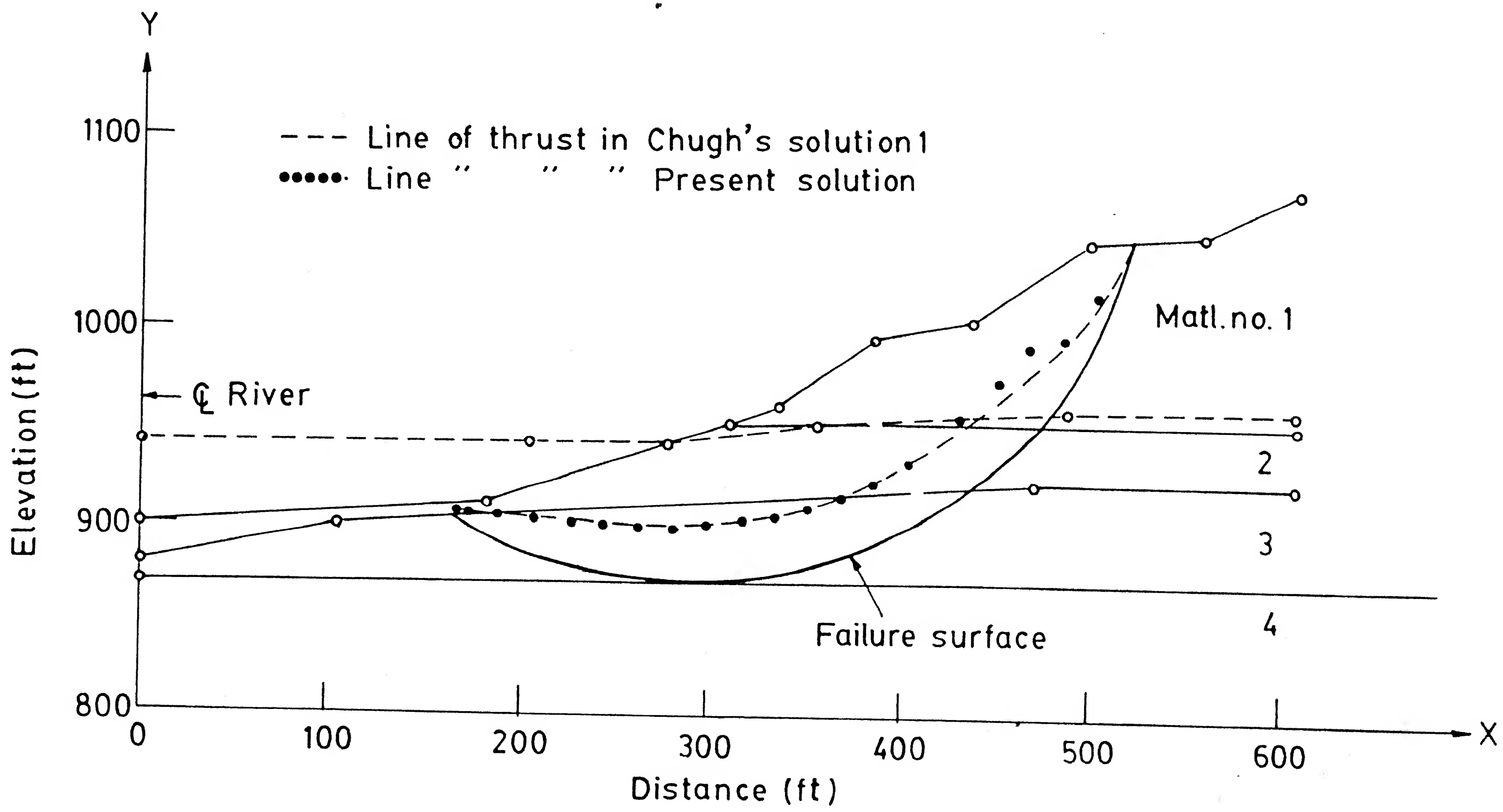


FIG. 6.13 LINES OF THRUST IN VARIOUS SOLUTIONS OF EXAMPLE PROBLEM 6.2

study is restricted to the investigation of whether for a given slip surface and a given interslice force function more than one set of values are obtained as the solution which satisfies complete equilibrium, no effort has been given in the direction of obtaining an acceptable effective stress line of thrust (Chugh, however, has not presented the effective stress line of thrust).

Comments: Contrary to Chugh's findings, the present investigation has brought out that there is no existence of multiple solutions to a slope stability problem satisfying both force and moment equilibrium requirements. Indirectly it has established the proposed method of solution based on SUMT as an effective equation-solver. The method appears to be all the more powerful when one considers the fact that "attempts to solve this problem using the computer code SSTABI, available at the U.S. Bureau of Reclamation, failed to give a solution" and the program STABLTY (Chugh, 1981) could not give a unique solution.

The reason why Chugh found multiple solutions to exist may lie in the possible premature termination i.e., the iterations might have been stopped before Z_n and M_n individually have reduced sufficiently. Due to improper convergence such premature termination may result in a solution which is considerably far off from the correct one (Morgenstern and Price, 1967). Chugh, however, has not reported values of Z_n and M_n corresponding to the multiple solutions obtained by him.

6.5 EXAMPLE PROBLEM 6.3 : THE OKETE DAM U/S SLOPE

6.5.1 General

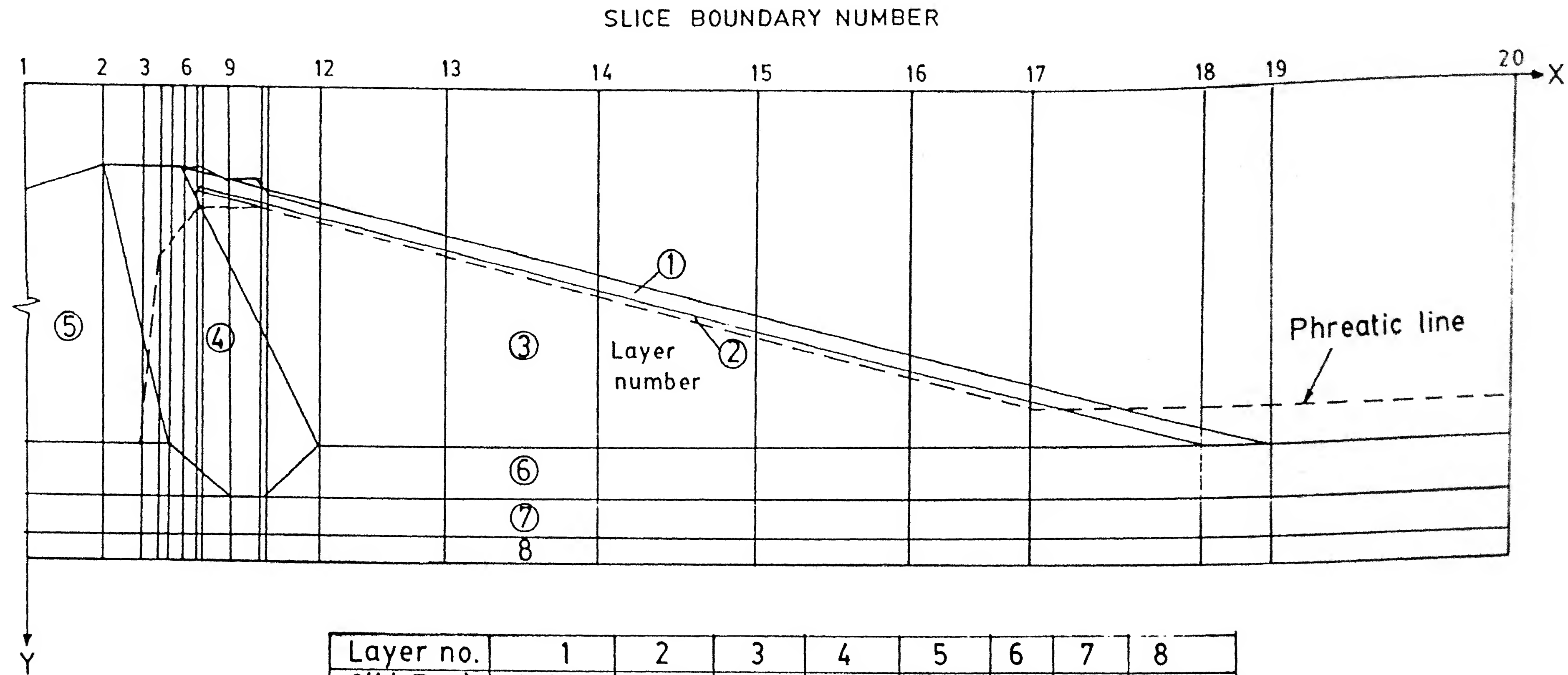
Figure 6.14(a) shows a section of the Okete dam upstream slope along with the layering, soil properties and the phreatic line. The ordinates of the various points on the dam section including the layer boundaries and the phreatic line are as given in Table 6.4. All these data are taken from a manual by Baker (1979) who has presented detailed results of the analysis of the same section with the help of the program SSOPT which is based on the dynamic programming technique in conjunction with the Spencer's method (1967). In the said analysis, pore pressures have been generated using vertical distances below the phreatic line shown in Figure 6.14(a). This phreatic line is reportedly a top flow line and the forces due to water ponding on the face of the slope have been considered in the analysis.

The purpose of studying this example in the present thesis is the following:

(i) to test the effectiveness and efficiency of the proposed numerical scheme in handling a zoned dam with a complex geometry, layering and a steep phreatic line.

(ii) to draw a comparison between the results obtained by using the program SSOPT based on the dynamic programming technique and the developed program based on the SUMT.

Analyses have been carried out to search the critical surface using both Spencer's and Janbu's methods.



Layer no.	1	2	3	4	5	6	7	8
C'(kPa)	0.00	0.00	0.00	10.0	0.00	30.0	10.0	2000.0
ϕ'(deg)	42.0	30.0	30.0	19.0	30.0	30.0	19.0	20.00
γ(kN/m^3)	18.50	18.50	19.80	18.80	18.00	18.50	19.50	20.00

FIG. 6.14 (a) SECTION OF THE OKETE DAM UPSTREAM SLOPE (Baker, 1979)

TABLE 6.4

General Geometry of the Okete Dam Upstream Slope

Slice Boundary	X Co-ord. (m)	Y_s (m)	Y co-ordinates of the layer boundaries(m) Layer 1	Layer 2	Layer 3	Layer 4	Layer 5	Layer 6	Layer 7	Layer 8	Y_{Ph} (m)
1	0.00	8.0	8.00	8.00	8.00	8.00	27.00	31.00	34.00	36.00	27.00
2	6.00	6.0	6.00	6.00	6.00	6.00	27.00	31.00	34.00	36.00	27.00
3	9.00	6.0	6.00	6.00	6.00	18.40	27.00	31.00	34.00	36.00	27.00
4	10.40	6.0	6.00	6.00	6.00	24.10	27.00	31.00	34.00	36.00	13.80
5	11.00	6.0	6.00	6.00	6.00	27.00	27.00	31.00	34.00	36.00	11.90
6	12.00	6.0	6.00	6.00	6.00	27.80	27.80	31.00	34.00	36.00	10.50
7	13.00	6.0	7.70	7.70	7.70	28.60	28.60	31.00	34.00	36.00	9.70
8	13.50	6.0	7.50	7.80	9.00	29.20	29.20	31.00	34.00	36.00	9.00
9	15.60	7.0	8.00	8.30	12.90	31.00	31.00	31.00	34.00	36.00	9.00
10	18.00	7.0	8.60	8.90	17.80	31.00	31.00	31.00	34.00	36.00	9.00
11	18.60	8.0	8.80	9.10	19.00	31.00	31.00	31.00	34.00	36.00	9.10
12	22.60	9.0	9.80	10.10	27.00	27.00	27.00	31.00	34.00	36.00	10.10
13	32.00	11.00	12.20	12.50	27.00	27.00	27.00	31.00	34.00	36.00	12.50
14	44.00	14.00	14.20	14.50	27.00	27.00	27.00	31.00	34.00	36.00	14.50
15	56.00	17.00	18.20	18.50	27.00	27.00	27.00	31.00	34.00	36.00	18.50
16	68.00	20.00	21.20	21.50	27.00	27.00	27.00	31.00	34.00	36.00	21.50
17	79.40	22.00	23.50	23.80	27.00	27.00	27.00	31.00	34.00	36.00	21.50
18	92.20	26.00	27.00	27.00	27.00	27.00	27.00	31.00	34.00	36.00	21.50
19	97.50	27.00	27.00	27.00	27.00	27.00	27.00	31.00	34.00	36.00	21.50
20	115.50	27.00	27.00	27.00	27.00	27.00	27.00	31.00	34.00	36.00	21.50

Note : Y_s = Ordinates of the slope surface

Y_{Ph} = Ordinates of the phreatic surface

6.5.2 Results and Discussion

6.5.2.1 Previous Solution by Baker using the Program SSOPT

As already stated, Baker (1979) solved the problem earlier using the program SSOPT. The critical shear surface given by the program SSOPT based on the dynamic programming technique and the Spencer method as reported by Baker (1979) is presented in Figure 6.14(b) and also in each of the Figures 6.15, 6.16 and 6.17. The search domain and the discretization used by Baker (1979) are also shown in Figure 6.14(b). The values of F_{min} and θ are 1.467 and 8.976 degrees respectively. The calculated values of various slice characteristics associated with the critical surface have also been reported in the form of a computer output of the program SSOPT and the same is presented in Table 6.5. It is seen from the table that the resultant interslice forces (Q), the normal forces at the slice base (N) are all positive as they should be. The positions of the line of thrust for total stress are also reasonable. However, the line of thrust for effective stress has not been reported by Baker.

6.5.2.2 Present Solution by Spencer's Method

Before proceeding with the critical surface determination, the critical surface reported by Baker has been re-analysed using the proposed equation-solver and the obtained values (F = 1.466, $\theta = 8.976^{o}$) are observed to be in agreement with those reported by Baker. The slice configurations used in Baker's analysis have also been used in the present re-analysis of Baker's critical surface.

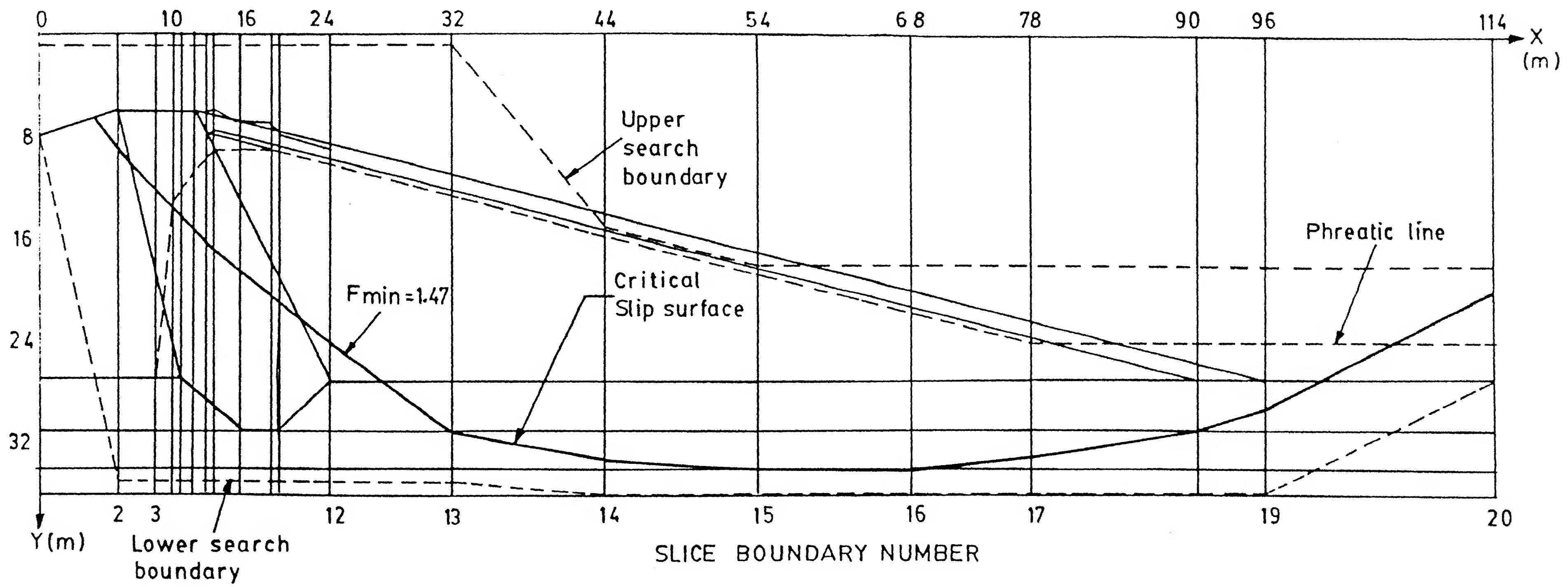


FIG. 6·14 (b) CRITICAL SHEAR SURFACE FOR THE EXAMPLE PROB. 6.3 BY THE PROGRAM SSOPT (Baker, 1979)

TABLE 6.5

A Part of the SSOPT Output for the Example Problem 6.3

THE CRITICAL PATH IS

J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
YC	1	9	12	13	14	15	16	17	19	20	21	24	31	33	34	34	33	31	29	20

SLICES DATA

J	CN	L	WT	CL	U	UTX	UTY	UK	HT	N	PR	SK	Q	Z
1	0.9916	10.00	4.86	0.00	0.00	0.00	0.00	0.00	1.50	4.84	24.70	1		
													2.76	7.94
2	0.9766	4.24	25.27	2.89	0.00	0.00	0.00	0.00	4.50	24.37	26.99	1		
													13.82	10.27
3	0.9994	1.72	17.11	1.72	0.00	0.00	0.00	0.00	6.50	16.38	28.62	1		
													19.33	11.10
4	0.8219	1.17	8.46	1.17	0.89	0.00	0.00	0.00	7.50	9.62	29.45	1		
													26.20	11.81
5	0.9468	1.41	15.98	1.41	4.67	0.00	0.00	0.00	8.50	16.75	30.31	1		
													35.47	12.65
6	0.9468	1.41	17.83	1.41	7.64	0.00	0.00	0.00	9.50	19.12	31.15	1		
													46.54	13.47
7	0.7722	1.12	9.87	1.12	7.99	0.00	0.00	0.00	10.50	13.80	32.02	1		
													58.08	14.14
8	0.9562	2.90	45.91	2.90	26.10	0.00	0.00	0.00	11.50	49.89	33.30	1		
													87.37	15.71
9	1.0271	2.60	57.90	2.60	27.30	0.00	0.00	0.00	12.50	56.75	34.43	1		
													101.82	16.59
10	0.8219	1.17	15.20	1.17	13.35	0.00	0.00	0.00	13.00	20.45	35.18	1		
													117.82	17.22
11	1.0382	5.00	110.15	2.00	64.50	0.00	0.00	0.00	14.00	113.77	36.80	1		
													172.66	19.43
12	1.0683	11.72	322.72	20.09	189.87	0.00	0.00	0.00	17.50	324.94	40.69	1		
													314.87	24.22
13	1.0020	12.17	455.16	12.17	225.06	0.00	0.00	0.00	19.50	449.08	43.47	1		
													328.83	26.41
14	0.9801	12.04	418.98	12.04	204.71	0.00	0.00	0.00	18.00	419.28	43.08	1		
													305.00	27.82
15	0.9511	12.00	358.62	12.00	168.00	0.00	0.00	0.00	15.50	367.30	41.70	1		
													249.37	28.55
16	0.9136	11.44	272.92	11.44	137.33	-0.13	0.71	3.51	12.50	289.40	39.38	1		
													179.79	28.80
17	0.8799	12.96	193.38	12.96	136.03	-10.00	32.00	59.00	8.00	246.74	36.01	1		
													96.28	28.71
18	0.6744	5.66	34.32	16.99	48.15	-5.00	26.50	2.44	3.50	84.83	32.62	1		
													36.26	27.69
19	0.5850	20.12	7.40	13.42	29.07	-0.00	22.00	0.00	1.00	47.38	29.92	1		
													0.01	0.00

TET = 0.8975992E+01 F = 0.1467210E+01

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

END OF RUN

The critical shear surfaces have been determined using the Direct procedure. As regards discretization of the sliding mass, both Model I and Model II have been used to solve this problem.

Figure 6.15 shows the critical surface obtained by using the discretization Model I with 19 slices. The initial trial surface is also shown in the figure. The corresponding F_{min} and θ have been obtained as 1.483 and 8.706 degrees respectively which are quite close to the values of 1.467 and 8.976 corresponding to the critical surface reported by Baker. However, as seen from Figure 6.15, the two critical surfaces are quite different from each other especially towards the toe,with the present critical surface being much deeper. The calculated responses associated with the obtained critical surface are presented in Table 6.6. It can be seen from this table that both the lines of thrust (for total stress and effective stress) lie within the middle-third except at a couple of interslice boundaries near the upper end. The effective interslice forces (Z') as well as the effective normal stress and shear stresses at the slice bases are all positive and hence acceptable.

Figure 6.16 shows the initial trial surface and the critical slip surface when the discretization Model II has been used. The positions of the fixed vertical slice boundaries (19 slices) used in Baker's analysis have been retained. The critical surface has F_{min} and θ values of 1.46 and 8.609 degrees which are again quite close to Baker's values of 1.47 and 8.976 degrees and the values of 1.48 and 8.706 corresponding to the present solution using the discretization Model I. Compared to the critical surface

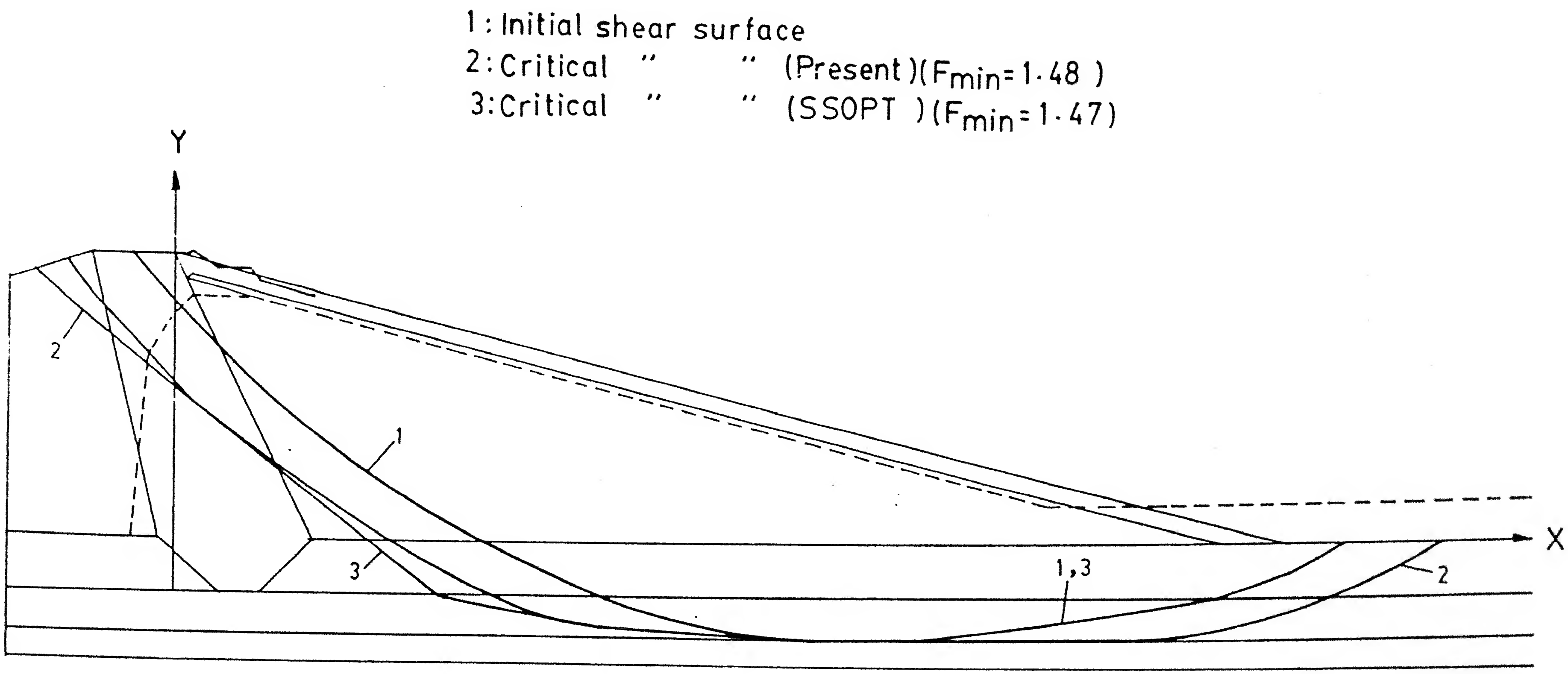


FIG. 6.15 CRITICAL SHEAR SURFACES FOR THE EXAMPLE PROBLEM 6.3 BY SPENCER'S METHOD
(Discretization model I used in the present analysis)

TABLE 6.6

Calculated Responses for the Solution by Spencer's Method and Discretization Model I for the Example Proble 6.3

Slice No.	σ (kPa)	σ' (kPa)	τ (kPa)	L/H	L'/H	Z (kN/m)	Z' (kN/m)
1	111.6	70.7	36.0				
				0.56	0.40	500.0	253.2
2	164.9	101.3	41.1				
				0.48	0.47	1109.0	754.2
3	187.3	110.9	26.2				
				0.45	0.47	1446.0	966.8
4	217.2	127.5	31.8				
				0.42	0.44	1784.o	1257.0
5	240.5	141.5	35.7				
				0.41	0.44	2094.0	1564.0
6	264.1	163.3	39.4				
				0.41	0.45	2388.0	1804.0
7	285.5	182.0	43.1				
				0.39	0.44	2625.0	1837.0
8	310.1	195.8	46.9				
				0.38	0.43	2864.0	1929.0
9	335.9	206.3	48.2				
				0.37	0.42	3096.0	1971.0
10	350.7	209.0	48.1				
				0.36	0.43	3213.0	1692.0
11	361.2	216.4	48.5				
				0.35	0.39	3235.0	1634.0
12	366.3	188.0	48.0				
				0.35	0.40	3150.0	1512.0
13	320.8	146.6	62.3				
				0.34	0.38	2616.0	1178.0
14	287.4	128.1	67.7				
				0.31	0.31	2118.0	968.2
15	262.4	121.0	47.2				
				0.30	0.28	1583.0	693.8
16	227.9	103.9	40.5				
				0.26	0.16	1026.0	362.9
17	182.5	91.4	33.3				
				0.25	0.15	481.8	250.2
18	118.3	65.4	26.1				
				0.31	0.31	97.3	96.2
19	39.7	39.7	15.5				

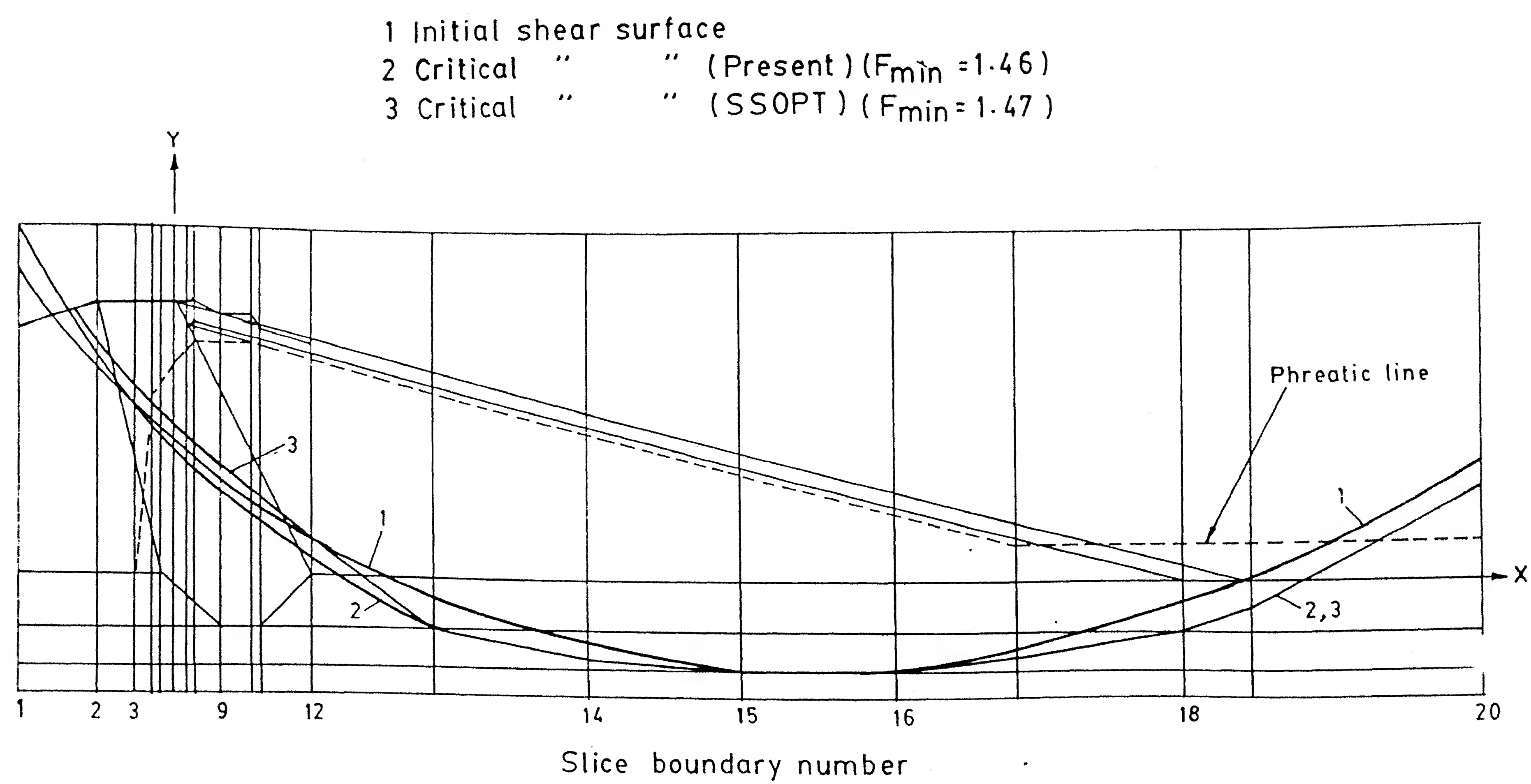


FIG. 6.16 CRITICAL SHEAR SURFACES FOR THE EXAMPLE PROBLEM 6.3 BY SPENCER'S METHOD
(Discretization model II used in the present analysis)

obtained using Model I, Model II gives a surface which is much closer to Baker's surface. The associated values of the internal variables are presented in Table 6.7. As in case of Model I, the lines of thrust, the interslice forces and the stresses at the slice bases are all reasonable.

The reason why solution to this problem has been attempted using Model II when use of Model I has already yielded an acceptable solution is the following:

The phreatic line used for calculation of pore pressures has quite a few sharp discontinuities under the top width of the dam section and also to its left. It has been possible to take these discontinuities into account in the analysis using Model I also. However, it was felt that there might be some difference in the obtained result if, using a finer grid at this region, the y coordinates of the discrete points on the slip surface are forced to vary at these sharp discontinuities (keeping their x-coordinates fixed). However, as already reported, the difference in the F_{min} values is only marginal. Moreover, it has been observed that it becomes difficult for the numerical scheme using Model II to satisfy the curvature constraints at the very closely spaced slice boundaries at certain locations as stated earlier. The analysis carried out with the curvature constraints imposed yielded a F_{min} of 1.49; however, in the obtained solution, quite a few curvature constraints remained violated. In view of this difficulty, a fresh analysis has been tentatively conducted without imposing these constraints. The solution corresponds to what has been already presented in Figure 6.16 and

Table 6.7. It is observed that this surface somehow maintains the upward concavity of shape. However, this cannot be expected to succeed in all situations. In contrast, with Model I no such difficulty has been encountered. In this respect it may be said that the flexibility of the numerical scheme is reduced when Model II is used.

Tables 6.8 and 6.9 present the values of the design vector and the constraints at the starting and the optimal points corresponding to the analyses using Model I and II respectively. It is seen that, as has been observed in the case of analysis of homogeneous slopes in Chapter 4, the proposed Direct procedure is capable of handling the equality constraint as well as infeasible design points quite efficiently.

Figures 6.17 and 6.18 show the variation of the function values f and ψ with the progress of the optimization in terms of number of function evaluation as well as the number of r-minimizations, corresponding to Direct procedures using Model I and II respectively. It is seen that both cases follow the same trend as has been observed in the case of homogeneous slopes using Model I for the analysis by Direct procedure presented in Chapter 4. The function values initially go on decreasing to reach a minimum and then show a tendency to increase to converge to the feasible minimum. It is also observed that the number of function evaluations required in using Model II is almost $1\frac{1}{2}$ times as many as required in using the Model I. As a result the c.p.u. time required is also correspondingly larger in case of Model II. While Model I takes about 2 minutes of c.p.u. time

TABLE 6.7

Calculated Responses for the Solution by Spencer's Method and Discretization Model II for the Example Problem 6.3

Slice No.	σ (kPa)	σ' (kPa)	τ (kPa)	L/H	L'/H	Z (kN/m)	Z' (kN/m)
1	100.2	61.5	44.9				
				0.65	0.52	272.1	144.1
2	152.5	92.3	47.2				
				0.50	0.52	953.4	702.9
3	193.8	113.6	33.4				
				0.41	0.45	1835.1	1416.6
4	253.9	146.7	42.8				
				0.40	0.45	2482.4	1700.3
5	306.1	167.3	45.1				
				0.37	0.41	3040.0	1817.1
6	356.4	190.4	51.8				
				0.34	0.37	3445.9	1848.1
7	369.3	187.7	51.1				
				0.33	0.36	3141.2	1384.8
8	295.4	125.4	41.4				
				0.30	0.30	1992.3	794.9
9	261.7	116.0	30.0				
				0.28	0.26	1519.5	572.4
10	248.8	111.5	35.2				
				0.26	0.21	1437.3	490.4
11	234.2	106.5	40.9				
				0.27	0.24	1109.7	394.7
12	209.9	99.4	46.7				
				0.26	0.26	835.1	319.6
13	189.6	93.9	33.1				
				0.26	0.28	760.8	352.4
14	172.6	93.0	29.1				
				0.27	0.32	608.7	367.7
15	154.0	99.6	30.7				
				0.28	0.25	475.4	389.1
16	127.7	95.7	34.0				
				0.29	0.30	390.7	360.8
17	106.0	94.8	28.3				
				0.31	0.31	238.8	236.3
18	65.9	65.9	30.4				
				0.37	0.37	42.5	41.6
19	19.3	19.3	7.51				

TABLE 6.8

Design Vector and Constraints For the Example Problem 6.3

(Solution by Spencer's method and Model I)

No. of Slices = 19
No. of design variables, n_{dv} = 22

Starting Point

F = 1.2500 θ = 0.1000

Design Variables (not normalized):

-2.6667	-4.4444	-5.7778	-6.4444	-6.9778	-6.9999
-6.9999	-6.9999	-6.9999	-5.9999	-5.1111	-3.7778
-1.9999	0.0000	2.6889	6.1111	10.5778	15.3778
96.0000	-3.0000	1.2500	0.1000		

Constraints (Inequality):

-0.9398	-0.7884	-0.7695	-0.7520	-0.7256	-0.6928
-0.6623	-0.6296	-0.5936	-0.5616	-0.5150	-0.4779
-0.4453	-0.3997	-0.3287	-0.2519	-0.1747	-0.1127
-0.8890	-0.4443	-0.6668	-0.1332	-0.5113	-0.0221
0.0000	0.0000	-1.0000	0.1112	-0.4445	-0.4446
-0.2220	-0.6890	-0.7333	-1.0445	-0.3333	-0.8222
-0.5980	-0.5843	-0.5366	-0.4588	-0.4172	-0.3831
-0.3629	-0.3413	-0.3172	-0.2849	-0.2561	-0.2162
-0.1481	-0.0624	0.0898	0.4220	1.3823	3.6408
-0.4020	-0.4157	-0.4633	-0.5312	-0.5828	-0.6169
-0.6370	-0.6587	-0.6828	-0.7150	-0.7439	-0.7838
-0.8519	-0.9376	-1.0898	-1.4219	-2.3823	-4.6407
-0.2600	-0.1000				

Constraint (Equality) : 0.3456E+00

ε = -0.10 δ_t = 0.001 r_o = 1.0 x 10^{-4}

f = 1.25 ψ = 1.3448 Z_n = 0.1033E+03 M_n = 0.2332E+04

TABLE 6.8 contd.

Design Vector and Constraints at the final (optimal) point

Optimal Point

F = 1.4835 θ = Ø.1519
Design Variables (not normalized):

-2.5382	-4.5821	-5.3997	-6.1Ø77	-6.6261	-6.94Ø9
-6.9999	-6.999Ø	-6.9394	-6.554Ø	-5.9199	-5.ØØ73
-2.3489	Ø.57Ø2	3.5Ø59	6.8329	1Ø.7466	15.Ø999
87.8154	-1Ø.3439	1.4835	Ø.151953		

Constraints (Inequality):

-1.ØØØØ	-1.ØØØØ	-Ø.8635	-Ø.8Ø89	-Ø.7766	-Ø.7463
-Ø.7196	-Ø.6753	-Ø.6323	-Ø.5865	-Ø.5359	-Ø.4897
-Ø.4527	-Ø.4Ø22	-Ø.326Ø	-Ø.2391	-Ø.1665	-Ø.Ø9Ø8
-Ø.ØØ14	-Ø.9768	-Ø.3347	-Ø.2952	-Ø.1567	-Ø.2547
Ø.Ø776	Ø.ØØ27	-Ø.36Ø5	Ø.3432	-Ø.Ø922	-2.Ø383
-Ø.Ø119	-Ø.ØØ37	-Ø.Ø223	-Ø.7552	-1.1883	-Ø.ØØ12
-Ø.6989	-Ø.6462	-Ø.61ØØ	-Ø.5317	-Ø.478Ø	-Ø.4539
-Ø.4414	-Ø.42Ø6	-Ø.3992	-Ø.3838	-Ø.3735	-Ø.3653
-Ø.3533	-Ø.3263	Ø.3Ø98	Ø.2868	-Ø.2442	-Ø.3186
-Ø.3Ø11	-Ø.3538	-Ø.3899	-Ø.4683	-Ø.522Ø	-Ø.5461
-Ø.5586	-Ø.5794	-Ø.6ØØ8	-Ø.6162	-Ø.6265	-Ø.6347
-Ø.6467	-Ø.6737	-Ø.69Ø2	-Ø.7132	-Ø.7558	-Ø.6814
-Ø.4935	-Ø.1519				

Constraint (Equality) : Ø.1268E-Ø9

No. of r-minimizations required = 7

f = 1.4835 ψ = 1.4855 Z_n = Ø.2933E-Ø2 M_n = -Ø.1843E-Ø1

N.B. Out of a total of 74 inequality constraints, First18 are boundary constraints, next 18 are curvature constraints, next 36 are constraints on line of thrust and the last two are side constraints on F and θ respectively.

TABLE 6.9

Design Vector and Constraints For the Example Problem 6.3
(Solution by Spencer Method and Model II)

No of slices = 19
No. of design variables, n_{dv} = 22

Starting Point

F = 1.2500 θ = 0.1000
Design Variables (not normalized):

9.0000	-0.0010	-1.6640	-5.1120	-6.9900	-7.1330
-4.9930	-1.5200	3.0910	5.4700	5.9440	7.2280
8.7970	9.4150	9.9860	10.9370	11.4840	12.6000
15.7640	23.5860	1.2500	0.1000		

Constraints (Inequality):

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
-0.9274	-0.9336	-0.8712	-0.8325	-0.7780	-0.6922	-0.6090
-0.4998	-0.3873	-0.1175	-0.5E-04	0.15E+02	-0.4515	-0.3198
-0.3261	-0.0775	-0.0194	0.1098	0.3824	0.5765	0.5866
0.7336	0.8884	0.9881	1.0861	1.2798	1.4080	1.7178
3.3260	-0.16E+02	-0.5485	-0.6802	-0.6739	-0.9225	-0.9806
-1.1098	-1.3824	-1.5765	-1.5866	-1.7336	-1.8884	-1.9888
-2.0861	-2.2798	-2.4080	-2.7178	-4.3260	-0.2600	-0.1000

Constraint (Equality): 0.3579E+03

ε = -0.01 δt = 0.0001 $r_o = 0.1 \times 10^{-5}$

f = 1.25 ψ = 5.113 z_n = 0.2171E+04 M_n = 0.4002E+05

Optimal Point

F = 1.4586 θ = 0.1503
Design Variables (not normalized) :

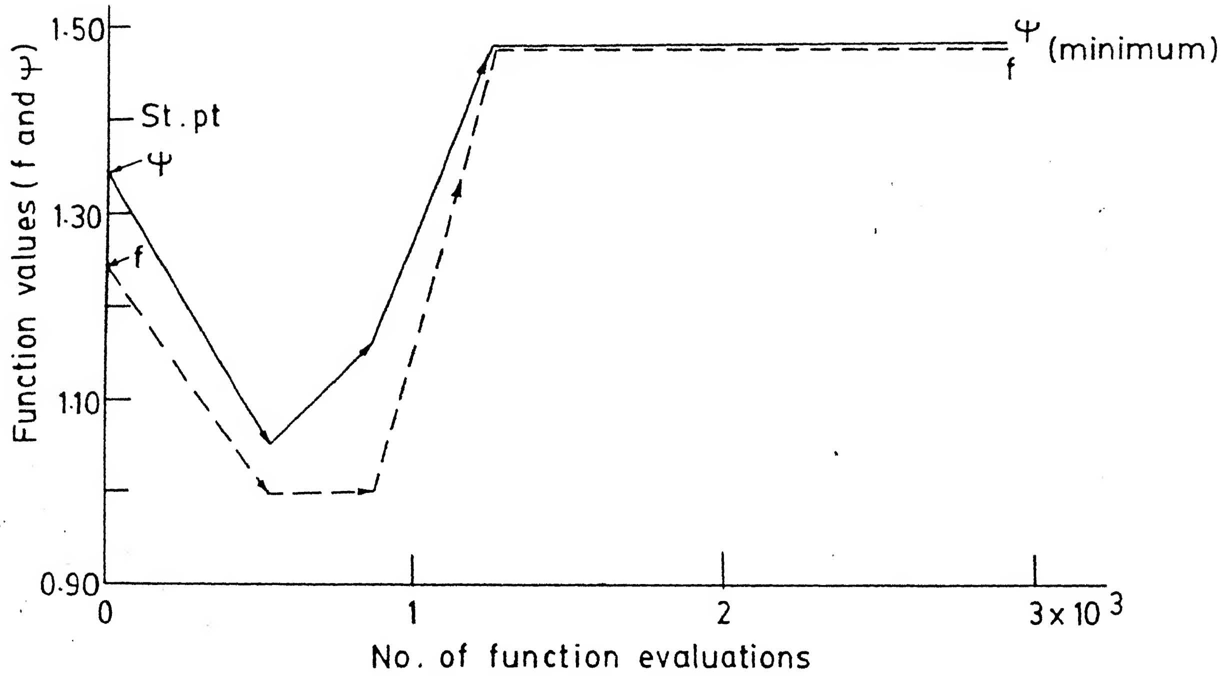
7.5075	-1.5785	-4.0095	-6.1848	-6.9989	-7.0000	-6.4349	
-4.0002	1.9054	4.2947	4.7039	6.4254	8.0360	3.5024	
9.5454	10.5935	11.3947	13.2237	17.3267	26.5553	1.4586	0.1503

Constraints (Inequality):

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9314
-0.9386	-0.8803	-0.8429	-0.8000	-0.7084	-0.6071	-0.4999	-0.4114
-0.2003	-0.0699	-0.6499	-0.5036	-0.4080	-0.3983	-0.3650	-0.3423
-0.3324	-0.2925	-0.2798	-0.2629	-0.2689	-0.2580	-0.2610	-0.2684
-0.2801	-0.2878	-0.3067	-0.3705	-0.3500	-0.4964	-0.5920	-0.6017
-0.6350	-0.6577	-0.6676	-0.7075	-0.7202	-0.7370	-0.7311	-0.7420
-0.7390	-0.7316	-0.7199	-0.7122	-0.6933	-0.6295	-0.4686	-0.1503

Constraint (Equality) : 0.4182E-13
No. of r-minimization required = 8
= 1.4586 ψ = 1.4588 Z_n = 0.411E-05 M_n = 0.5044E-03

.B. Out of a total of 56 inequality constraints, first 18 are boundary constraint, next 36 are constraints on line of thrust and the last two are side constraints on F and θ respectively. It may be noted that this analysis has been carried out without curvature constraints.



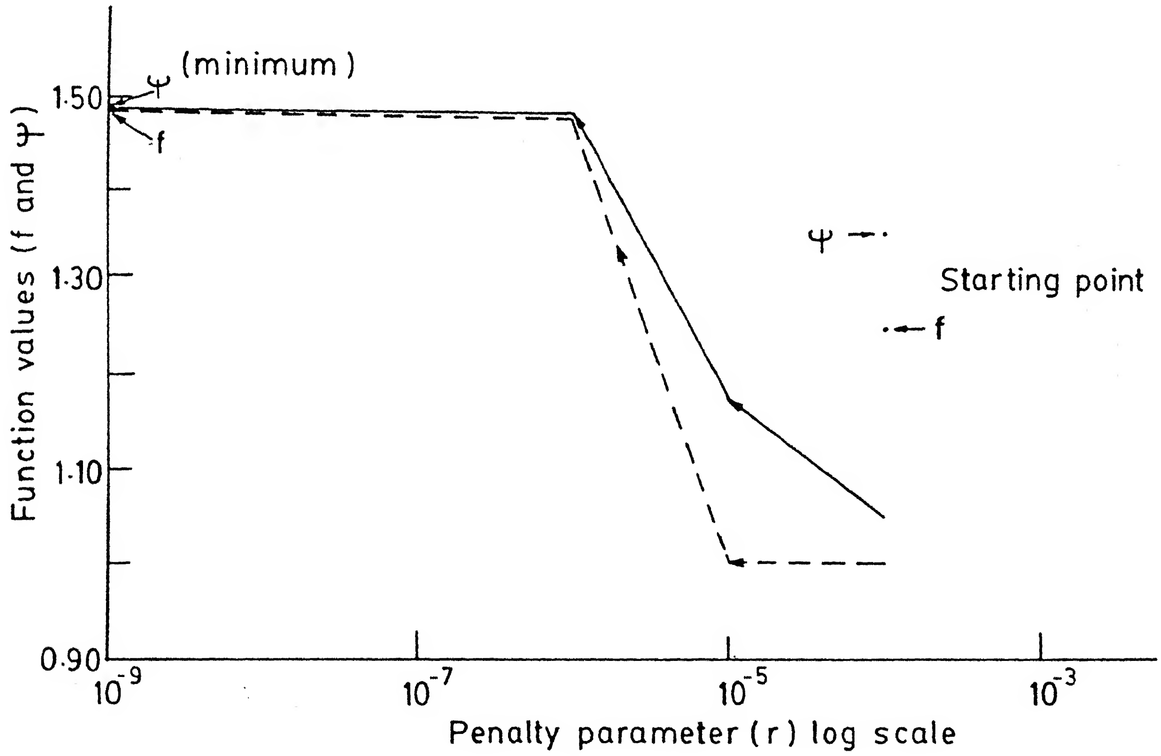


FIG. 6·17 PATH FOLLOWED BY THE MINIMIZATION SCHEME USING MODEL I & SPENCER METHOD (Example problem 6·3)

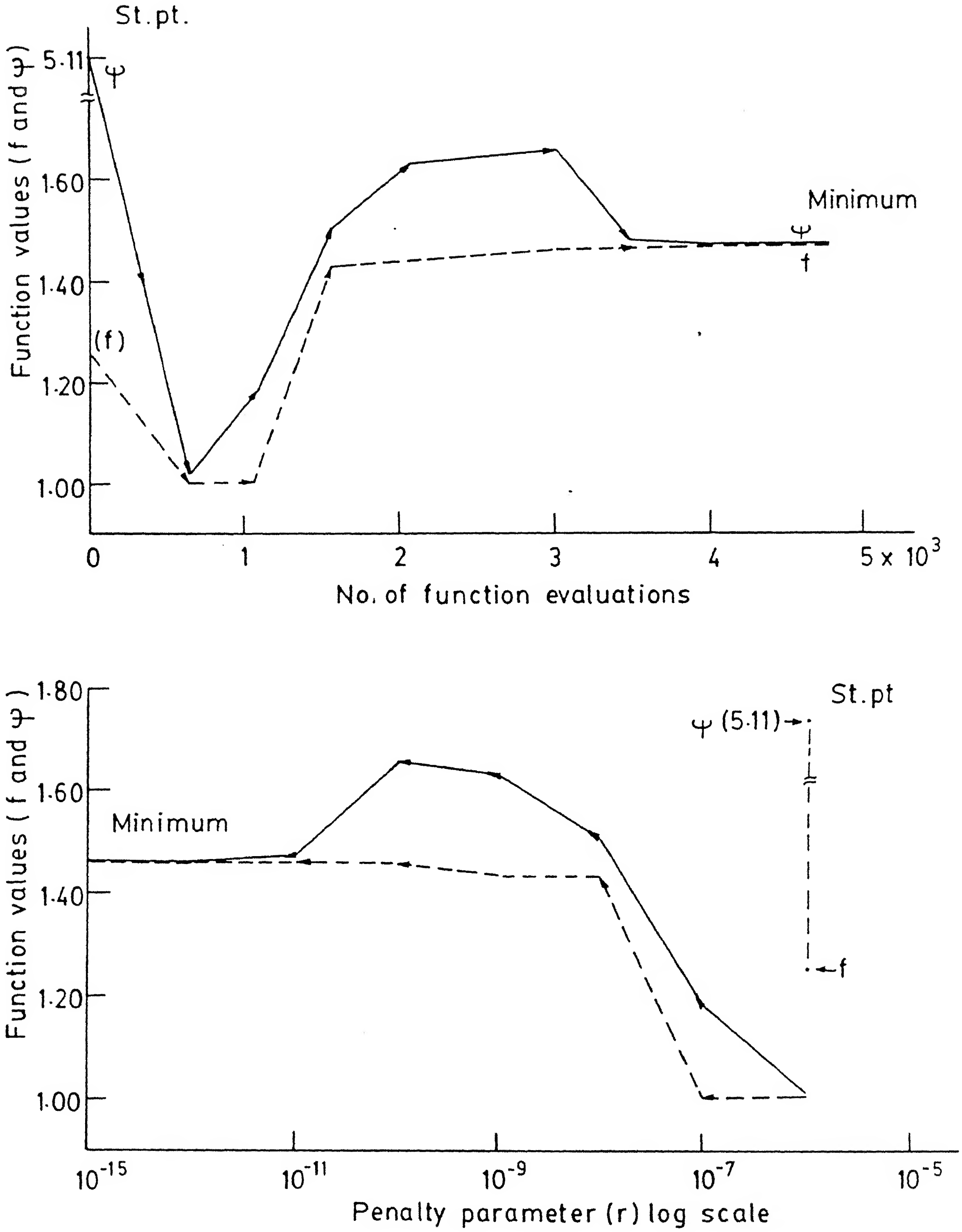


FIG. 6·18 PATH FOLLOWED BY THE MINIMIZATION SCHEME USING MODEL II & SPENCER'S METHOD (EX. PROBLEM 6·3)

(which is comparable to that taken by SSOPT) the Model II takes about 3 minutes, for the solution of this problem. It should be, however, mentioned that a near convergence occurs as early as when only half of the total number of function evaluations have been performed. The rest of the function evaluations are used for the purpose of satisfying the equality constraint until Z_n and M_n become negligible to the extent desired.

6.5.2.3 Present Solution by Janbu's Method

The critical slip surfaces obtained by using the Janbu's GPS using Model I discretization have been presented in Figure 6.19. Solutions have been obtained using two initial surfaces, one of them being the critical shear surfaces reported by Baker and is marked S_1 (and 1) in Figure 6.19. The value of the factor of safety of Baker's critical surface when re-analysed by using Janbu's method has been obtained as 1.665 using 14 slices. The critical shear surface corresponding to this surface as the initial trial surface is the one marked F_1 (and 2) in Figure 6.19. It has a F_{min} of 1.45 and it has not shifted much from the critical surface, S_1 (and 5), given by SSOPT (it is to be noted that 1 and 5 refer to the same surface).

The second initial surface marked S_2 (and 3), a shallower one, has factor of safety of 1.62. It yields a critical surface marked, F_2 (and 4) which has a higher F_{min} of 1.47. This also does not move much from its initial position. This shows, as has been already demonstrated in the case of homogeneous slopes, the starting point dependence of the proposed scheme.

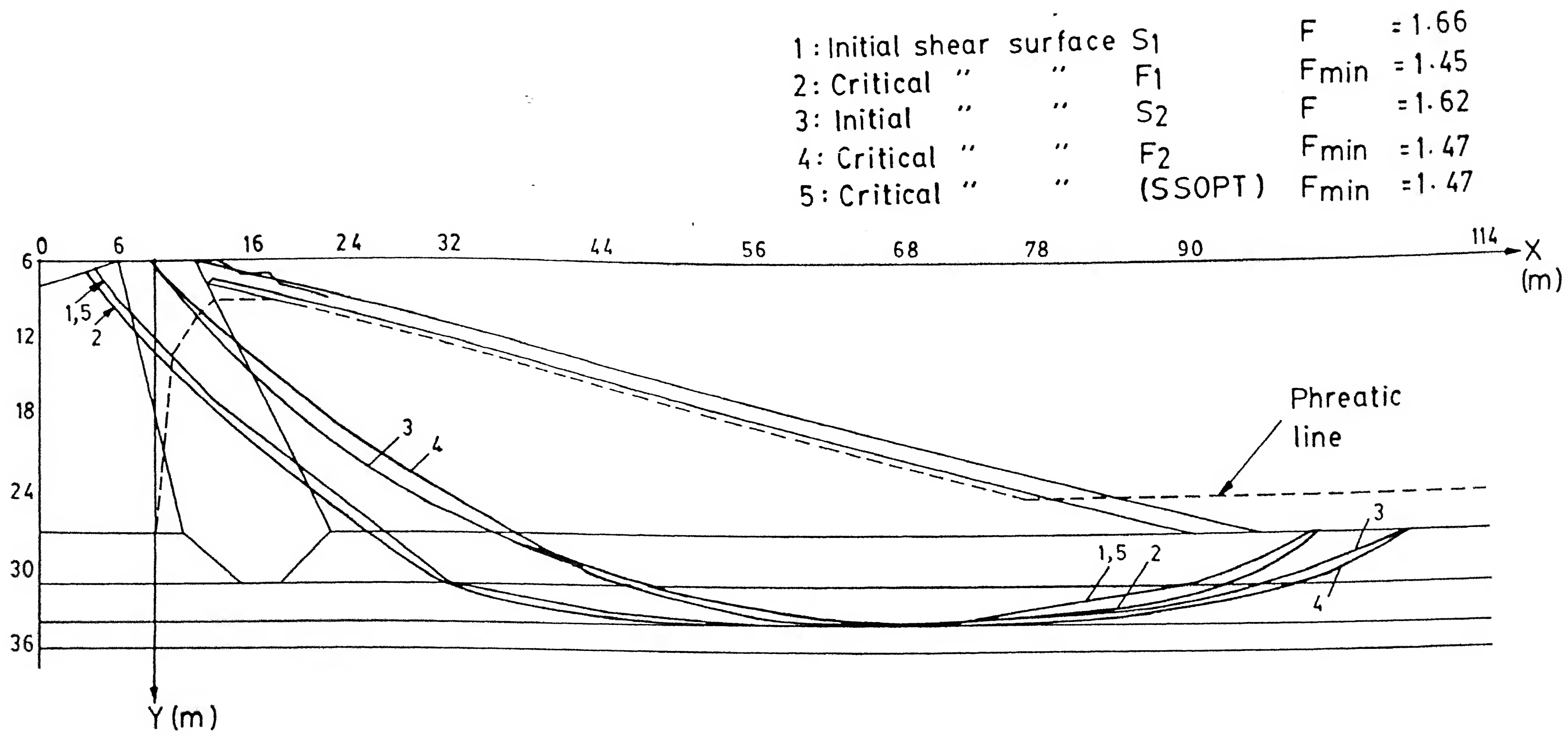


FIG. 6·19 CRITICAL SHEAR SURFACES FOR THE EXAMPLE PROBLEM 6·3 BY JANBU'S METHOD (Discretization model I used in the present analysis)

Table 6.10 presents the assumed line of thrust positions and the calculated stresses and forces. The signs of these responses are found to be admissible. It is worthwhile to mention that initially, however, results were obtained taking 19 slices. The obtained critical surfaces, starting from S_2, had a F_{min} of 1.46. However, corresponding to that solution the vertical shear force 'T' at the last interslice boundary (towards the toe) was found to be positive which is not admissible. In this unacceptable solution, the maximum value of the ratio of the mean height of a slice to its width was 3.30 which is higher than Janbu's recommended maximum value of 3 (Janbu, 1980). Keeping the above recommendation in mind, the number of slices has been reduced to 14 in a fresh analysis with the result that all the 'T' forces are now negative and hence acceptable.

Table 6.11 presents the design vector and constraints at the starting point and the optimal point corresponding to S_2 and F_2. It is seen that although the design vector is infeasible at the starting point as it violates the end constraint, the final design vector is brought back to the feasible region by virtue of the property of the Extended penalty function method.

Figure 6.20 shows the path followed by the function values (f and ψ) on the way to convergence to the minimum. It is noticeable that unlike the Direct procedure, here the decrease in function values is monotonic. The c.p.u. time required in this case is about 1 minute which is almost half of that taken by the solution by the Spencer method discussed earlier.

TABLE 6.10

Assumed Line of Thrust and Calculated Responses for the Solution by Janbu's Method and Discretization Model I (Example Problem 6.3)

Slice No.	Assumed h_t/z	σ' (kPa)	τ (kPa)	E (kN/m)	T (kN/m)
1		42.8	16.6		
	0.34			178.1	-33.7
2		74.5	28.5		
	0.35			636.2	-114.1
3		81.5	31.4		
	0.36			1312.0	-239.9
4		112.8	46.7		
	0.38			1949.0	-352.5
5		95.9	56.6		
	0.38			2409.0	-634.4
6		167.4	45.9		
	0.38			2656.0	-564.6
7		163.8	45.2		
	0.38			2608.0	-566.8
8		164.6	45.5		
	0.38			2376.0	-506.3
9		158.8	44.2		
	0.38			2057.0	-378.8
10		131.3	37.7		
	0.38			1749.0	-279.3
11		99.0	30.2		
	0.38			1358.0	-260.4
12		77.2	25.0		
	0.38			907.0	-187.2
13		66.9	22.6		
	0.43			517.0	-9.0
14		28.4	18.0		

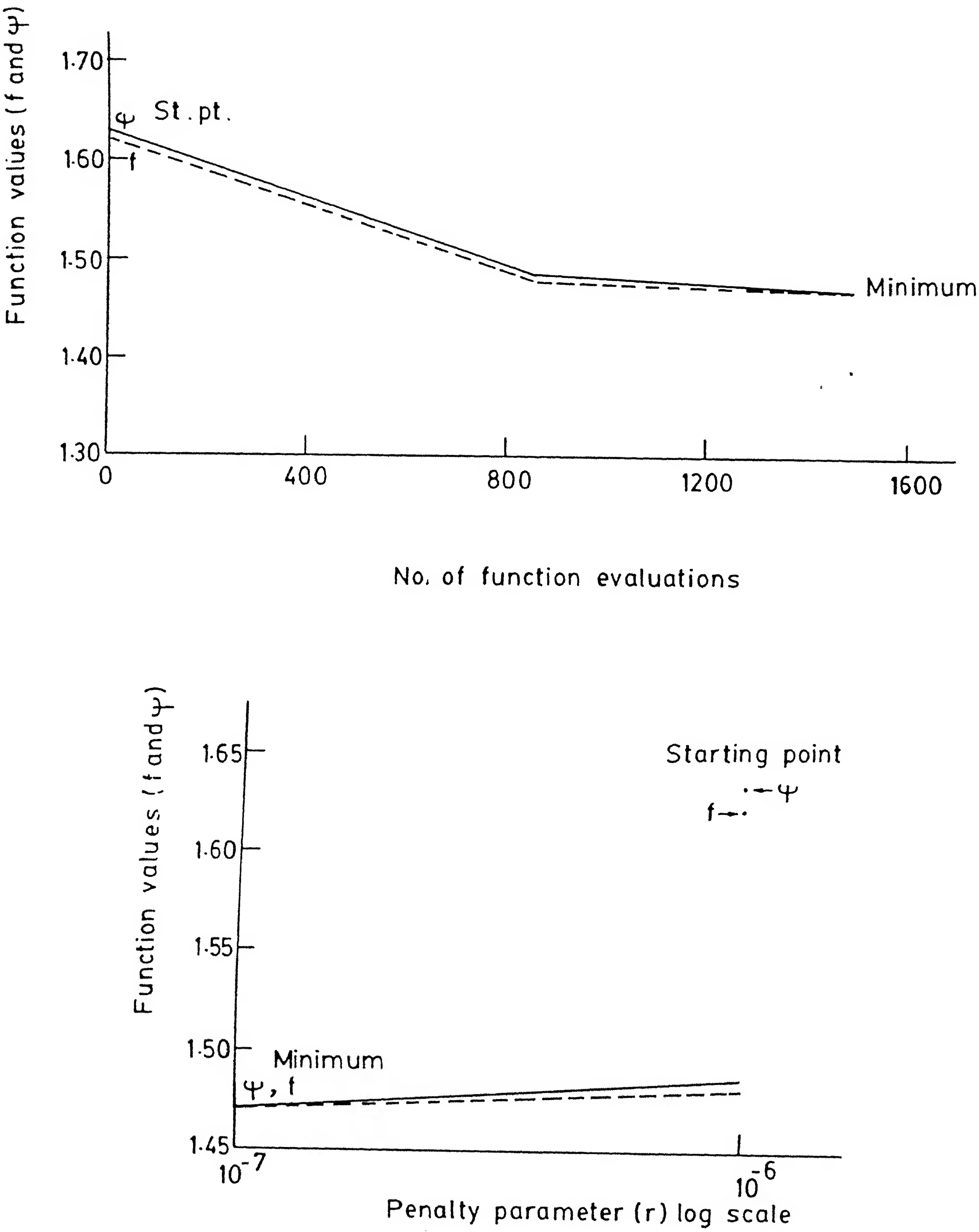


FIG. 6.20 PATH FOLLOWED BY THE MINIMIZATION SCHEME USING MODEL I & JANBU'S METHOD (Ex. Problem 6.3)

6.5.3 Comparison of the Various Solutions

Table 6.12 presents a summary of all the results obtained in the present analysis in the form of the minimum factor of safety (F_{min}) and the corresponding interslice force angle (θ). Results reported by Baker (1979) are also presented in the same table for ready reference. As can be seen from the table, all the results are fairly close. The corresponding critical surfaces are, however, quite different. That a band of differently located critical surfaces have more or less equal factor of safety perhaps indicate the presence of a critical shear zone rather than a strictly defined critical shear surface.

TABLE 6.12
Comparison of Results of Various Analyses of the Okete Dam Section

PRESENT ANALYSIS					BAKER'S ANALYSIS	
SPENCER'S METHOD (DIRECT PROCEDURE)				JANBU'S METHOD		
Model 1		Model II		Model I		
F	θ deg.	F	θ deg.	F	F	θ deg.
1.48	8.7063	1.46	8.6096	1.45	1.47	8.9760

6.6 EXAMPLE PROBLEM 6.4 : THE BIRCH DAM

6.6.1 General

This example is drawn from the stability analysis of Birch Dam in Oklahoma, as described by Celestino and Duncan (1981). The geometrical configuration presented in Figure 6.21 has been scaled from Celestino and Duncan's paper. For this dam section, Celestino and Duncan (1981) used a simplified search scheme proposed by them to find the critical shear surface using the Spencer (1967) method to compute the factor of safety. Later, Nguyen (1985) used the Simplex reflection technique in combination with the simplified Bishop method to search for the critical shear surface for the same section. However, the x co-ordinates of the various points forming the section, as scaled from Nyugen's paper, show some discrepancy with those scaled from Celestino and Duncan's paper. The co-ordinates are shown in Figure 6.22.

The object of the present studies is to find the critical shear surface using the developed technique and compare the same with those reported by Celestino and Duncan (1981) and Nguyen (1985). The proposed Direct procedure has been adopted to search for the critical slip surface. Further, in order to compare the relative efficiency of the SUMT with the dynamic programming technique with reference to this problem, results have also been obtained using the program SSOPT developed by Baker (1979).

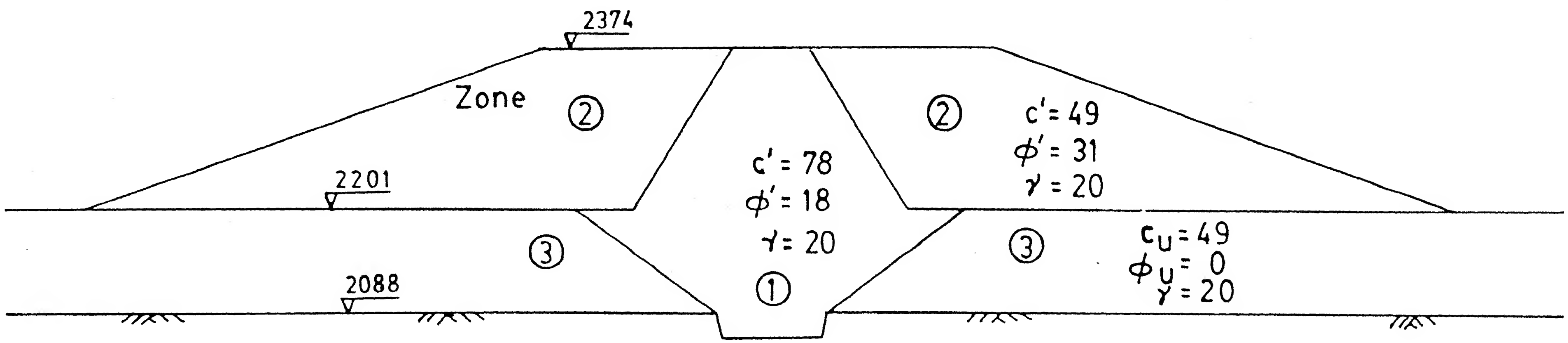
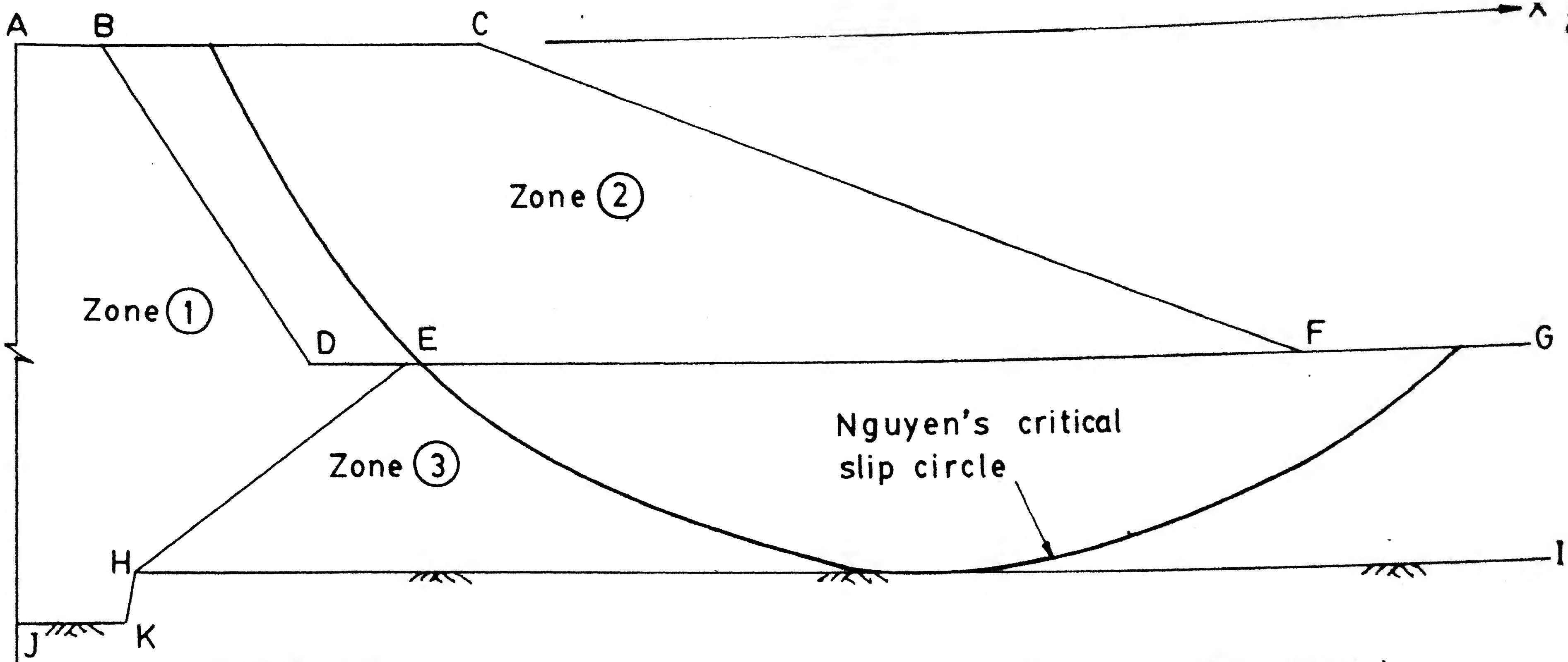


FIG. 6.21 THE BIRCH DAM SECTION FOR EXAMPLE PROBLEM 6.4
(Celestino and Duncan, 1981)

N.B :- In the above c is in kPa , ϕ is in degree and γ is in kN / m^3



POINTS	A	B	C	D	E	F	G	H	I	J	K
x (m)	0.0 (0.0)	4.35 (7.1)	24.50 (25.9)	15.0 (16.9)	21.50 (22.0)	68.25 (69.82)	—	6.0 (8.6)	—	0.0 (0.0)	5.60 (8.20)
y (m)	0.0	0.0	0.0	17.30	17.30	17.30	17.30	28.60	28.60	31.40	31.40

FIG. 6.22 CO-ORDINATES OF VARIOUS POINTS OF THE BIRCH DAM SECTION ALONG-WITH NGUYEN'S CRITICAL SLIP CIRCLE (As scaled from Celestino and Duncan's paper)

N.B :- Figures within the parenthesis indicate those scaled from Nguyen's paper

6.6.2 RESULTS AND DISCUSSION

6.6.2.1 Previous Solutions

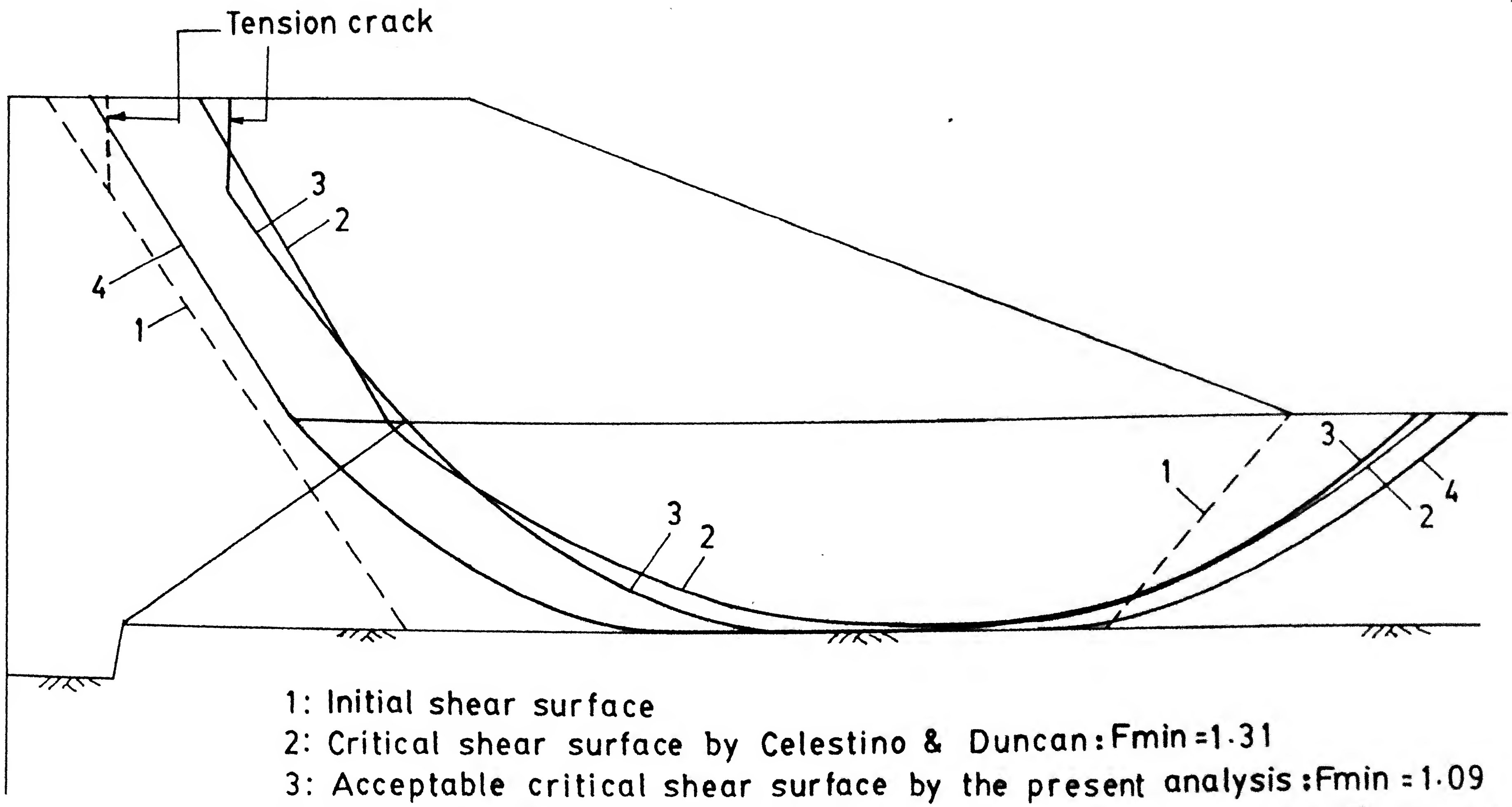
Among the previously reported solutions of the problem, the critical slip surface of general shape obtained by Celestino and Duncan using the simplified search procedure coupled with the Spencer's method (Spencer, 1967) of analysis is as shown in Figure 6.23. The corresponding minimum factor of safety has been reported as 1.31. The critical slip circle reported by Nguyen (1985) using the Simplex reflection technique in conjunction with the simplified Bishop method is shown in Figure 6.22. The corresponding minimum factor of safety is 1.117. In the above solutions, however, the associated lines of thrust and the internal stresses have not been reported.

6.6.2.2 Solution using the Proposed Procedure

Analysis of a given shear surface:

In their paper Celestino and Duncan (1981) have also presented, along with the critical slip surface, the initial trial slip surface chosen by them to start the iterations involved in the search scheme. The factor of safety of this surface (also shown in Figure 6.23) has been reported to be 1.81. It is intended, in order to check the relative efficiency of the schemes, to use the same initial trial surface in the search for the critical slip surface by the proposed Direct procedure also.

Before proceeding with the search for the critical surface, the proposed equation-solver has been used to compute the factor



1: Initial shear surface
2: Critical shear surface by Celestino & Duncan: Fmin = 1.31
3: Acceptable critical shear surface by the present analysis: Fmin = 1.09
4: Unacceptable critical shear surface by the present analysis: Fmin = 1.22

FIG. 6.23 CRITICAL SHEAR SURFACES FOR EXAMPLE PROBLEM 6.4

of safety of this surface. Taking 14 slices for the analysis, the factor of safety and the interslice force angle have been obtained as 1.58 and 0.057 rad respectively. The factor of safety value reported by Celestino and Duncan as already stated is, however, 1.81. Furthermore, it is also seen from a detailed computer output (not presented here) that the positions of the line of thrust at some interslice boundaries as well as signs of some of the interslice forces and normal stresses at the slice bases are inadmissible. An acceptable solution may probably be obtained by considering tension crack and/or a different k-distribution. However, since this is only a trial shear surface no effort has been put in that direction.

It has been stated (Chapter 3) that the constraints on the line of thrust have not generally been included in the formulation of the proposed equation-solver in anticipation that this would not make any significant difference to the solution. Since the obtained line of thrust for the trial shear surface in question is inadmissible, this case has been utilized to study the effect of inclusion of the said constraints in the analysis. For the purpose of this study a fresh analysis has been performed including the constraints on the line of thrust. As expected, the results now obtained are practically the same as the earlier results. This serves as an example to justify the non-inclusions of such constraints in the formulation of the proposed equation solving technique for a complete equilibrium method.

Determination of Critical Shear Surface:

In obtaining the critical shear surface the following studies and observations have been made.

1. Results obtained by using the present solution technique, starting from the chosen initial shear surface as discussed earlier, has revealed that no convergent solution is possible. The kind of ill-conditioning that was encountered in solving Example Problem 4.3 has also been observed to occur in this case; this leads to the conclusion that under the assumed conditions there is no existence of any solution which satisfies the constraints imposed on the line of thrust. As no convergent solution has been achieved, the obtained numerical results are not presented herein for the sake of brevity. The probable reasons for the nonconvergence may be explained as follows:

The Extended penalty function method, by virtue of its adaptability to infeasible design points, brings back the solution to the feasible region by gradually satisfying the inequality constraints which are violated during the initial stages of the iteration. However, in the process of doing so it fails to effectively handle the equality constraints. The equality constraint function value does not diminish as it normally does and, as such, remains violated in the sense that this value is greater than ε, the tolerance at that stage of r-minimization. This violation causes the value of the penalty term shoot up resulting in ψ function becoming too large compared to f, the objective function. As a consequence, the subsequent minimizations of ψ function are not significantly affected by the

changes in the objective function-cum-design variable, f, which eventually leads to non-convergence.

2. The above observations are verified by removing the constraints on the line of thrust. Removal of the constraints helped the convergence. The factor of safety is found out to be 1.22, the corresponding Z_n = Ø.3338E-Ø4 and M_n = Ø.6714E-Ø2. However, as expected, the result is not acceptable as it associates with a meaningless line of thrust as presented in Table 6.13(a). The obtained surface is shown in Figure 6.23.

3. In order to find an acceptable solution, in addition to the constraints on the line of thrust, water-filled tension crack of constant depth has been introduced. Several trials have been performed varying the depth of the crack (z_t), incrementing it by 1m at a time. It has been observed that for z_t = 1m, 2m and 3m, there is no convergence. The first convergence has been noted at z_t = 4m. As far as the associated line of thrust is concerned, the constraints are satisfied but the last L/H ratio is obtained as Ø.96 which is rather high. Details are, however, not presented.

In an effort to obtain a better solution, z_t = 5m has been considered; the obtained solution is better than the previous one. The said L/H value now is Ø.72 which is quite close to the desirable value of Ø.67. The minimum factor of safety is 1.Ø9. The critical surface so obtained is shown in Figure 6.23. Table 6.14 presents all the calculated responses corresponding to this solution. It is seen that the line of thrust is reasonable

TABLE 6.13
Influence of the Number of Slices on the Acceptability of the Solution (Example Problem 6.4)

(a) Unacceptable Solution

No. of slices = 14
F_{min} = 1.22

Slice No.	σ kPa	τ kPa	L/H	$\frac{Z}{\gamma b H_t}$
1	69.7	40.30		
			0.51	0.26
2	148.3	40.27		
			0.43	0.69
3	226.3	40.27		
			0.33	1.09
4	290.7	40.27		
			0.28	1.37
5	338.7	40.27		
			0.23	1.53
6	381.0	40.27		
			0.20	1.65
7	423.1	40.27		
			0.17	1.76
8	464.7	40.27		
			0.15	1.87
9	495.4	40.27		
			0.13	1.83
10	479.0	40.27		
			0.06	1.17
11	422.6	40.27		
			-0.17	0.35
12	288.9	104.30		
			0.60	-0.02
13	108.9	93.21		
			0.49	-0.23
14	-11.2	61.10		

(b) Apparently Acceptable Solution

No. of slices = 5
F_{min} = 1.12

Slice No.	σ kPa	τ kPa	L/H	$\frac{Z}{\gamma b H_t}$
1	170.7	43.9		
			0.44	0.54
2	329.5	43.9		
			0.29	0.66
3	420.8	43.9		
			0.22	0.69
4	368.9	43.9		
			0.48	0.02
5	69.3	81.2		

TABLE 6.14

Calculated Responses Associated with the Acceptable Critical Surface of the Birch Dam Section

Slice No.	σ kPa	τ kPa	L/H	$\frac{Z}{\gamma b H_t}$
1	65.2	44.9		
			0.55	0.33
2	161.5	44.9		
			0.44	0.81
3	234.1	44.9		
			0.37	1.25
4	285.0	44.9		
			0.33	1.55
5	329.7	44.9		
			0.30	1.80
6	375.1	44.9		
			0.28	2.04
7	411.7	44.9		
			0.26	2.21
8	434.8	44.9		
			0.24	2.24
9	432.9	44.9		
			0.22	1.98
10	413.4	44.9		
			0.22	1.47
11	374.3	44.9		
			0.22	0.78
12	276.0	76.0		
			0.40	0.19
13	124.6	113.5		
			0.72	0.07
14	58.0	76.9		

although not strictly within the middle-third. The interslice forces and the normal and shear stresses at the slice bases are also admissible.

Table 6.15 presents the design vector and the constraints before and after minimization. From the table it is evident that like the previous example problems the proposed numerical scheme has effectively and efficiently handled the equality constraint as well as the infeasible design points in the present case also.

4. A number of efforts have been made to explore the possibility of a still better solution and are discussed as follows:

i) z_t is treated as a design variable and at the same time, to take advantage of this flexibility, a more stringent version of the line of thrust constraints has been applied to force the line of thrust to lie within the middle third of the heights of the interslice boundaries. However, a convergent solution could not be obtained.

ii) A different k-distribution, shown as 'input' in Figure 4.10 has been assumed in combination with the changes mentioned in(i). The assumed k-distribution has been tentatively chosen from Spencer's paper (1973). However, convergent solution could not be obtained in this case also.

iii) The stringent version of the line of thrust constraints has been retained but z_t has been kept constant. Both the k-distributions namely, k=1 throughout and the one mentioned above have been tried successively taking z_t=4m and 5m. However,

TABLE 6.15

Design Vector and Constraints For The Example Problem 6.4
(corresponding to the acceptable solution)

Total number of slices = 14
Total number of design variables = 17

Starting Point

$F = 1.5000 \quad \theta = 0.3000$

Design Variables (Not normalized) :

-6.85	-11.29	-11.29	-11.29	-11.29	-11.29	-11.29	-11.29	-11.29	-11.29
-8.711	-0.04	8.63	43.75	-19.33	1.50	0.30			

Constraints (Inequality):

-1.0000	-1.0000	-1.0000	-1.0000	-0.9558	-0.8934	-0.8311	-0.7688
-0.7065	-0.6442	-0.5819	-0.5195	-0.3574	-0.3518	-0.3933	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2284	-0.6993
0.0250	0.5794	-0.4775	-0.4405	-0.4614	-0.4767	-0.4880	-0.4963
-0.5024	-0.5069	-0.5100	-0.5216	-0.6660	-1.0963	-1.7602	-0.5225
-0.5595	-0.5386	-0.5233	-0.5120	-0.5037	-0.4976	-0.4931	-0.4899
-0.4784	-0.3340	0.0963	0.7602	-0.5000	-0.3000		

Constraint (Equality): 0.3232E+01

$\varepsilon = -0.01 \quad \delta_t = 0.001 \quad r_o = 0.10 \times 10^{-3}$

$f = 1.5000 \quad \psi = 1.5803 \quad Z_n = 2.226E+03 \quad M_n = 0.2803E+04$

Optimal Point

$F = 1.0901 \quad \theta = 0.0947$

Design Variables (Normalized):

-0.5440	-0.6254	-0.8103	-0.8965	-0.9447	-0.9870	-1.0000	-0.9664
-0.8403	-0.6294	-0.4212	23.6165	0.7655	1.1526	-0.6717	0.7269
0.3156							

Constraints (Inequality):

-1.0000	-1.0000	-0.9886	-0.9168	-0.8574	-0.8011	-0.7412	-0.6768
-0.6072	-0.5341	-0.4492	-0.3285	-0.1772	-0.1051	-0.1767	-0.1218
-0.0423	-0.0063	-0.0287	-0.0484	-0.0947	-0.1010	-0.1485	-0.3208
-1.1089	-0.0049	-0.5577	-0.4393	-0.3655	-0.3244	-0.2952	-0.2736
-0.2559	-0.2395	-0.2210	-0.2016	-0.2004	-0.3907	-0.7379	-0.4423
-0.5607	-0.6345	-0.6756	-0.7048	-0.7264	-0.7441	-0.7605	-0.7790
-0.7984	-0.7996	-0.6093	-0.2821	-0.1001	-0.0947		

Constraint (Equality) : 0.9842E-12

No. of r-minimization required = 6

$f = 1.0901 \quad \psi = 1.092 \quad Z_n = 1.549E-03 \quad M_n = 1.893E-03$

N.B. Out of a total of 55 inequality constraints, first 13 are boundary constraint, next 13 are curvature constraints, next 26 are the constraints on the line of thrust and the last two are side constraints on F and θ respectively. Lower boundary constraints are not imposed; instead an additional layer has been considered.

such attempts to obtain a convergent solution were also unsuccessful.

iv) Finally, reverting back to the liberal thrust line constraints (for which a convergent solution has been obtained already), it was investigated whether the other k-distribution shown in Figure 4.10 makes any improvement or not. It was observed that the variation in k-distribution has practically no effect on the final results. Analysis using several other k-distributions may show some change. However, these have not been tried .

5. It has been already reported that by removing the constraints on the line of thrust it has been possible to get a convergent solution using the Direct procedure. The surface so obtained has a F_{min} of 1.22 but the solution is not an acceptable one as the associated line of thrust and the interslice forces are not satisfactory. Having obtained such an unacceptable (critical) surface, a study has been undertaken here to investigate whether by applying "corrections" to this surface in the form of introduction of tension crack and/or assumption of other k-distributions, an acceptable solution can be obtained. The idea is that, if this attempt is successful, one can perhaps always ignore the acceptability constraints during the minimization process and then apply corrections as mentioned above to obtain an acceptable solution which corresponds to the true critical surface or very close to the true critical surface. However, such an attempt has failed here even after repeated trials taking water-filled tension crack of constant depths 2.0m,

2.5m, 3.0m and 4.0m although with z_t=4.0m, the value of the factor of safety has reduced to 1.12 which is quite close to the value of 1.09 corresponding to the acceptable solution as reported earlier. Trials with various k-distributions have revealed that it has very little effect on the solution in comparison to z_t and, as such, these results are not presented .

6. In all the studies reported so far a total of 14 slices have been used. Now, in order to study the effect of the number of slices and also, in view of the fact that in their study Celestino and Duncan have considered six co-ordinate points which means five slices, an analysis has been performed taking only five slices. Discretization Model I has been used. No constraint on the line of thrust has been included in the analysis so that there is no problem of convergence. The solution has converged to F_{min}= 1.12 and θ = 0.0584 rad. Moreover, the calculated line of thrust and the interslice forces have been found to be satisfactory as shown in Table 6.13(b). Considering that the analysis using 14 slices has resulted in a grossly unacceptable line of thrust etc., it may be inferred from this study that analysis with too few slices may lead to suppression of the true (with respect to the method of analysis) stress conditions.

Comparison with Celestino and Duncan's Solution

In Figure 6.23 the critical surface reported by Celestino and Duncan has also been presented by plotting representative co-ordinates scaled from the authors' paper. The said surface is

not different by a great degree from the critical surface obtained in the present study, except towards the crest. The critical surface reported by Celestino and Duncan has been re-evaluated and the factor of safety obtained is 1.16. However, Celestino and Duncan's reported value is 1.31. Further, it has been found in the present evaluation that the line of thrust associated with the critical surface reported by Celestino and Duncan is not reasonable. In addition, the last two of the resultant interslice forces are found to be negative as can be seen from the Table 6.16. With regard to the above, Duncan (1989) has expressed the following views:

> "Although analyses using Spencer's method sometimes result in unreasonable positions of the line of thrust (or unreasonable internal stress distributions) this is not important with regard to the magnitude of the factor of safety. If, for example, Morgenstern and Prices' method is used to find a solution with a more reasonable line of thrust, the factor of safety will change by no more than 10%.
>
> We have found that factors of safety calculated using any method that satisfies all conditions of equilibrium differ by no more than ±5% from what would be considered the "correct" or best calculable value. Because the consequence of changing the solution to eliminate unreasonable positions of the line of thrust is at most a 5% change in the estimate of the factor of safety, we do not consider this necessary. Eliminating the requirement that the internal stress distribution should be reasonable reduces the effort required very considerably, and has no significant influence on the calculated factor of safety."

In this connection it may be noted that surprises do occur and one may often come across a situation for which a solution satisfying all the constraints differs to a large extent from the solution obtained through an analysis in which the acceptability constraints are not imposed to reduce the computational effort. With the advent of super computers and availability of very

TABLE 6.16

Calculated Responses associated with the Critical Slip Surfaces Reported by Celestino and Duncan, Nguyen (Example Problem 6.4)

CELESTINO AND DUNCAN'S CRITICAL SURFACE					NGUYEN'S CRITICAL SURFACE			
Slice No.	σ kPa	τ kPa	L/H	$\frac{Z}{\gamma b H_t}$	σ kPa	τ kPa	L/H	$\frac{Z}{\gamma b H_t}$
1	79.5	42.44			71.4	41.08		
			0.59	0.28			0.57	0.20
2	140.6	42.44			143.2	41.08		
			0.49	0.63			0.40	0.38
3	212.1	42.44			220.1	41.08		
			0.40	1.00			0.34	0.52
4	276.8	42.44			288.6	41.08		
			0.36	1.33			0.30	0.62
5	328.5	42.44			339.4	41.08		
			0.34	1.59			0.28	0.62
6	368.2	42.44			364.4	41.08		
			0.32	1.73			0.26	0.47
7	396.3	42.44			392.5	41.08		
			0.31	1.74			0.25	0.30
8	412.7	42.44			384.2	41.08		
			0.32	1.60			0.24	-0.04
9	416.7	42.44			399.4	41.08		
			0.34	1.31			0.24	-0.31
10	406.8	42.44			384.4	41.08		
			0.43	0.86			0.32	-0.69
11	380.2	42.44			345.7	41.08		
			1.27	0.26			-2.22	-1.06
12	275.0	89.35			180.2	117.73		
			-1.68	-0.25			-0.79	-0.90
13	124.2	106.90			98.6	90.66		
			-1.72	-0.50			-1.58	-0.70
14	56.1	45.35			70.5	44.62		

efficient optimization algorithms, it is only desirable that efforts continue to be directed towards finding a feasible solution satisfying all the constraints. Such an acceptable solution can possibly be obtained by including the k-distribution as well as the depth of tension crack in the design vector in addition to the design variables normally present. However, it will increase the dimension of the problem enormously and such studies have not been undertaken in the present thesis.

Comparison with Nguyen's Solution:

As already stated, Nguyen (1985) also analysed the Birch dam section using the Simplex reflection technique of minimization in conjunction with the simplified Bishop method for factor of safety computations. The minimum factor of safety reported by the author is 1.117. It has been also reported that the obtained solution (i.e., the co-ordinates of the critical circle) are in excellent agreement with that obtained by Celestino and Duncan. As such, it is surprising that the value of the factor of safety obtained by Nguyen is significantly different from the value of 1.31 obtained by Celestino and Duncan. In the present analysis, Nguyen's circular surface has been re-evaluated and the obtained results are presented in Table 6.17. The table shows that the magnitudes of the factors of safety computed are quite close to the value reported by Nguyen and hence it is verified that scaling of the geometrical configuration of the Birch dam done from Nguyen's paper is correct. The table also demonstrates the effect of number of slices in case of circular surfaces.

TABLE 6.17

Results of Re-evaluation of Nguyen's Critical Slip Circle (Example Problem 6.4)

Sl. No.	No. of Slices	Method Used	Factor of Safety	Remarks
1	14	Spencer	1.193	Line of thrust and internal forces not satisfactory
2	14	Simplified Bishop	1.196	-
3	14	Ordinary Method of Slices	1.171	-
4	40	Spencer	1.119	Line of thrust and internal forces not satisfactory
5	40	Simplified Bishop	1.121	-
6	40	Ordinary Method of Slices	1.111	-

However, it should be noted that re-analysis of the critical circle reported by Nguyen using Spencer's method has demonstrated that the slip circle is unacceptable as it violates the constraints on the line of thrust (Table 6.16).

6.6.2.3 Present Solution Using the Program SSOPT

In order to compare the present results with that obtained by using the dynamic programing technique, the computer program SSOPT has been executed with the input data as read from Nguyen's paper. The configuration used is as shown in Figure 6.24. The result (Table 6.18) shows a minimum factor of safety of Ø.88. However, as can be seen from the table, the interslice forces (Q) and the line of thrust positions (Z) are not acceptable.

6.6.3 SUMMARY OF RESULTS

Table 6.19 gives a summary of the results obtained by the various analyses all of which are not acceptable solutions. All these unacceptable solutions excepting that given by SSOPT overestimates the calculated minimum factor of safety or, in other words, they are in error on the unsafe side. If the factor of safety of Celestino and Duncan's critical surface is taken as 1.31 the overestimation in this case is 2Ø.2% (in case the value of 1.16 is taken the overestimation is about 7%). The wide range of the unacceptable solutions, from a maximum of 1.31 (or 1.22) to a minimum of Ø.88 is of special interest. This highlights the importance of obtaining a feasible solution satisfying all the constraints. The factor of safety value obtained after

TABLE 6.18

A Part of the SSOPT Output for the Example Problem 6.4

```
  THE CRITICAL PATH IS
J   1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18
YC  0  0  0  0 14 22 26 30 32  33  33  33  33  33  33  33  33  33
```

SLICES DATA

J	CN	L	UT	CL	U	UTX	UTY	UM	HT	N	PR	SK	Q	Z
1	0.9943	7.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	9.19	1		
													0.00	0.00
2	0.9943	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.76	1		
													0.00	0.00
3	0.9943	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.68	1		
													0.00	0.00
4	1.1418	16.28	48.02	51.27	0.00	0.00	0.00	0.00	4.50	1.22	17.50	1		
													-29.04	9.79
5	1.1540	9.49	132.60	46.49	0.00	0.00	0.00	0.00	13.00	78.73	25.40	1		
													-19.50	6.45
6	0.8027	5.59	148.20	27.37	0.00	0.00	0.00	0.00	19.00	159.03	30.89	1		
													67.64	29.21
7	0.8470	6.47	223.52	31.68	0.00	0.00	0.00	0.00	22.00	239.96	34.39	1		
													188.68	28.80
8	0.9642	5.47	233.68	26.75	0.00	0.00	0.00	0.00	23.00	232.58	36.83	1		
													246.16	29.80
9	0.9961	5.18	228.60	25.37	0.00	0.00	0.00	0.00	22.50	225.66	37.78	1		
													261.74	30.48
10	0.9943	5.08	213.36	24.89	0.00	0.00	0.00	0.00	21.00	216.86	37.74	1		
													233.48	30.76
11	0.9943	5.08	193.04	24.89	0.00	0.00	0.00	0.00	19.00	196.04	37.20	1		
													205.23	31.03
12	0.9943	5.08	172.72	24.89	0.00	0.00	0.00	0.00	17.00	175.72	36.66	1		
													176.97	31.30
13	0.9943	5.08	152.40	24.89	0.00	0.00	0.00	0.00	15.00	155.40	36.12	1		
													148.72	31.57
14	0.9943	5.08	132.08	24.89	0.00	0.00	0.00	0.00	13.00	135.08	35.68	1		
													120.47	31.84
15	0.9943	3.28	75.44	16.07	0.00	0.00	0.00	0.00	11.50	77.38	35.13	1		
													102.22	32.02
16	0.9943	10.00	220.00	49.00	0.00	0.00	0.00	0.00	11.00	225.91	34.43	1		
													46.61	32.55
17	0.9943	8.88	184.36	41.06	0.00	0.00	0.00	0.00	11.00	189.32	33.45	1		
													-0.00	-37.68

```
TET = 0.610414E+01      F = 0.886043E+00
x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x
     END OF RUN
```

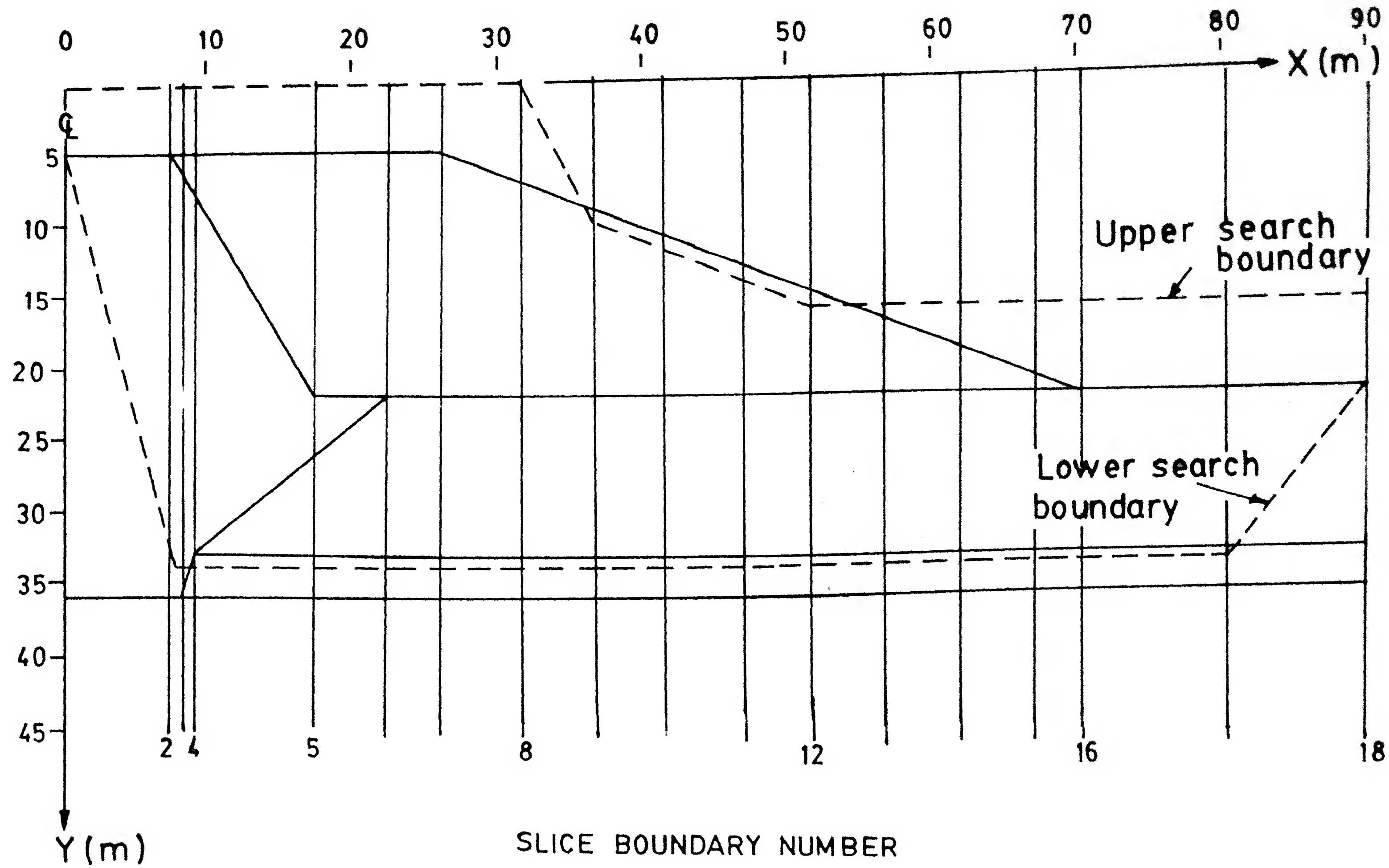


FIG. 6·24 BIRCH DAM SECTION AS ADOPTED FOR ANALYSIS BY SSOPT

TABLE 6.19

Summary of the Results of various Analysis of the Birch Dam

Sl. No.	Technique of critical surface determination	Method of analysis	Minimum factor of safety	Remarks
1	Simplified Search procedure - Celestio and Duncan (1981)	Spencer	1.31(1.16)	Unacceptable solution
2	Simplex Reflection technique - Nguyen (1985)	Simplified Bishop	1.117(1.121)	- do -
3	Dynamic programming technique (SSOPT) - Present studies	Spencer	0.88	- do -
4	SUMT (without incorporating acceptability constraints) - Present studies	Spencer	1.22	- do -
5	SUMT (incorporating acceptability constraints and tension crack) - Present studies	Spencer	1.09	Acceptable solution

Note : Values in the paranthesis indicate those obtained in the present studies.

satisfying all the acceptability constraints by using the present formulation is 1.09. The unacceptable (F_{min} = 1.22) and the acceptable (F_{min} = 1.09) critical shear surfaces obtained by using the present formulation are also widely different in location (Figure 6.23).

6.7 EXAMPLE PROBLEM 6.5 : THE BEAS DAM DOWNSTREAM SLOPE

6.7.1 GENERAL

Figure 6.25 shows a typical section of the Beas dam in Punjab, India. The dam is founded on folded, stratified, sedimentary, Siwalik formations. The Siwalik rocks in this region have rather complicated geologic history. In the process of folding where the shear stresses have exceeded the shear strength, shear zones have developed in the weaker clay shale members (Yudhbir and Varadarajan, 1975). Henkel and Yudhbir (1966) have demonstrated that the topography at the site is controlled by the low shear strength of these zones and that the field shearing resistance in the shear zones is at residual state.

To avoid unnecessary complications in the calculation of the weights and other characteristics of slices, the dam section shown in Figure 6.25 has been idealised without any major deviation from its original. The idealised section along with the co-ordinates of the various intersection points with respect to the chosen axes and the material properties are as shown in Figure 6.26. However, as the present study deals with the stability analysis of the downstream slope only, a truncated section as shown in Figure 6.27 has been adopted for the computations. The hard stratum underlying the shear zone has been represented by an additional layer with unusually high values of c' and ϕ'. The co-ordinates of the slope surface and the layer boundaries along with the soil and pore pressure

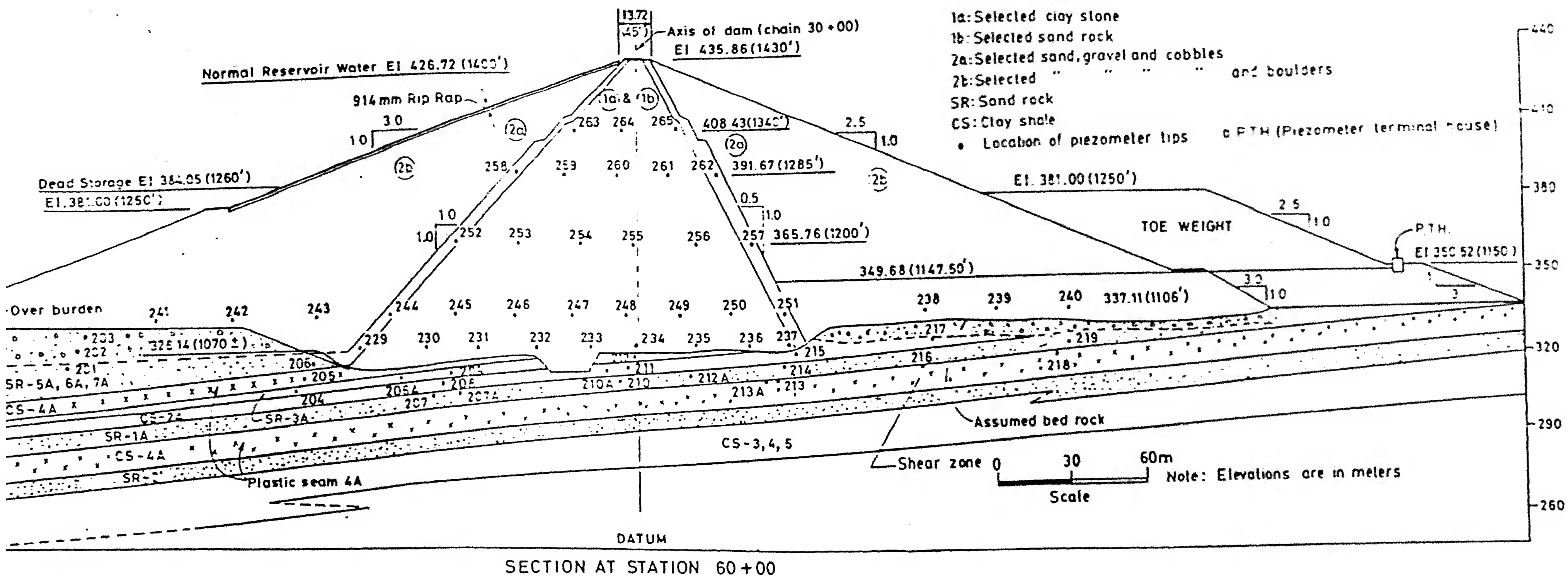
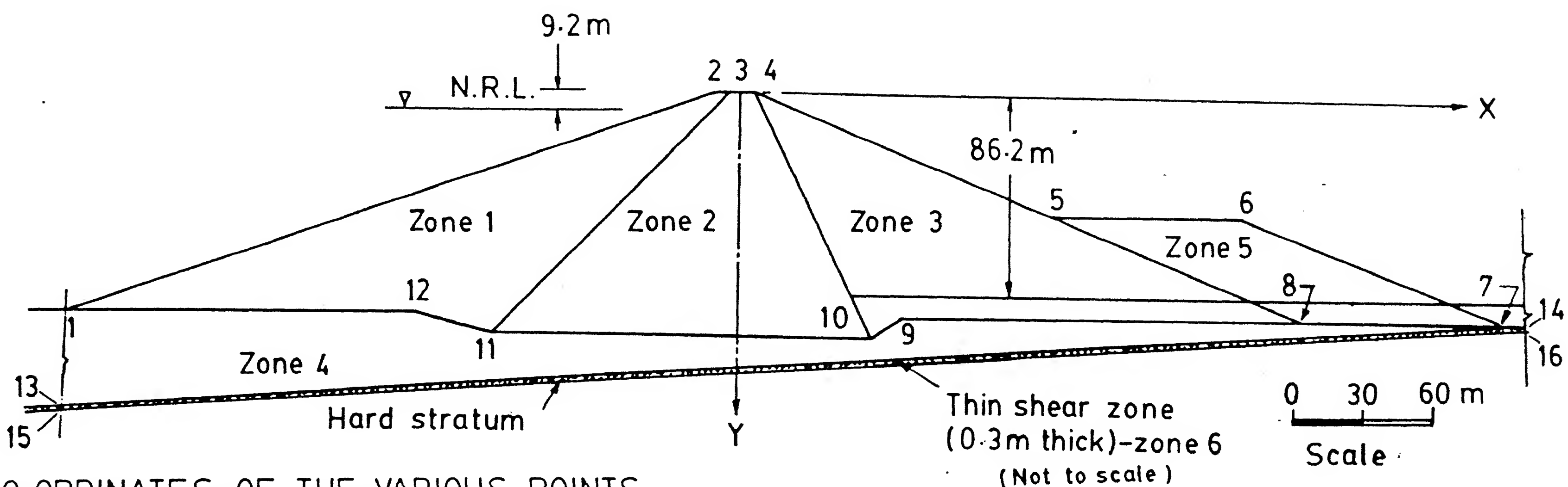


FIG. 6.25 THE ORIGINAL TYPICAL SECTION OF BEAS DAM



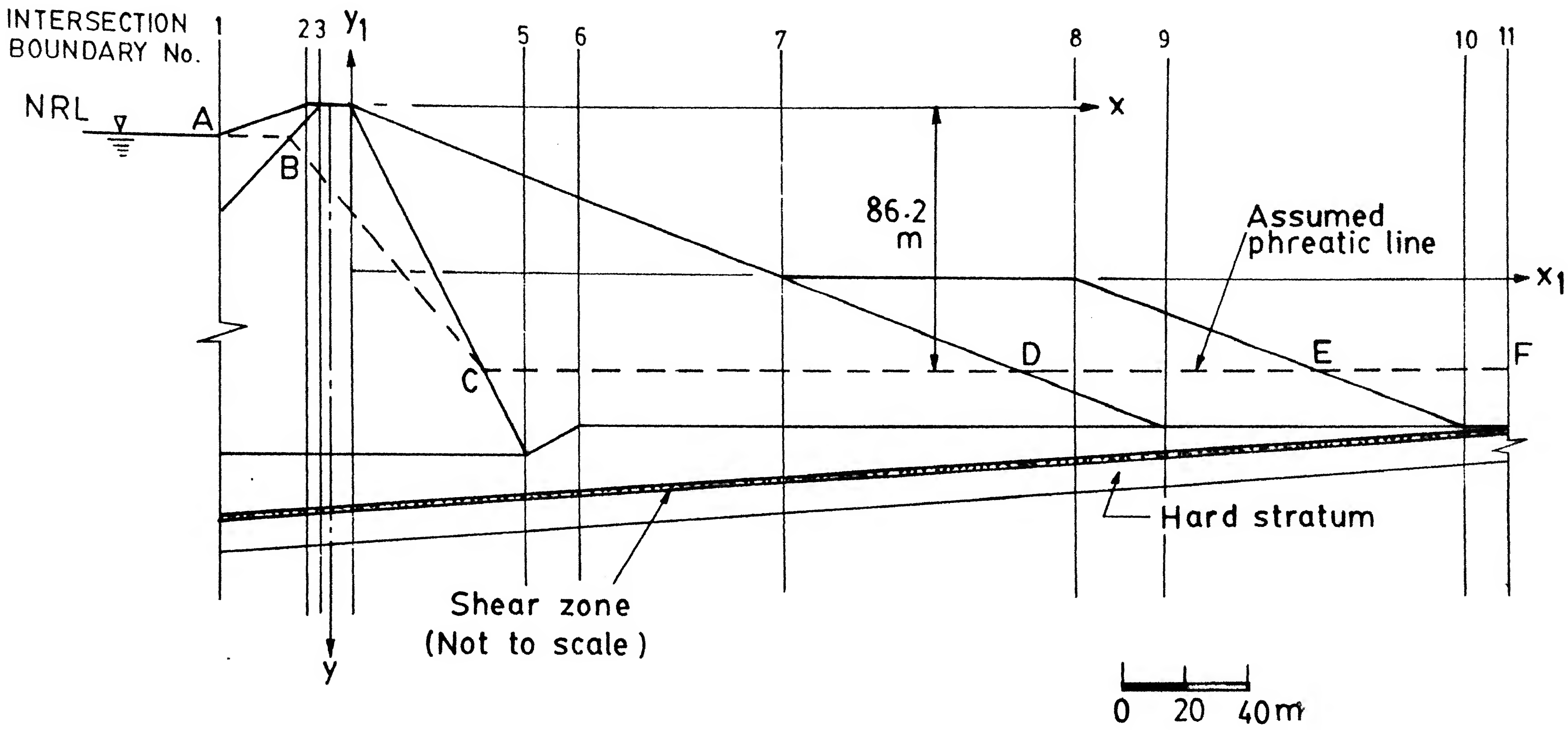
CO-ORDINATES OF THE VARIOUS POINTS

POINT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
X	-319.4	-6.9	-3.0	6.9	144.9	239.9	364.8	265.7	78.6	63.3	-115.8	-152.0	-319.4	374.8	-319.4	374.8
Y	104.2	0.0	0.0	0.0	55.2	55.2	104.2	104.2	104.2	112.8	112.8	104.2	150.4	104.2	150.7	104.7

MATL. PROPERTIES

ZONE	C′ kPa	ϕ'°	γ kN/m^3	r_u
1	0.0	35	24.0	0.35
2	0.0	26.5	24.0	0.35
3	0.0	35	24.0	0.35
4	0.0	35	24.0	0.35
5	0.0	35	24.0	0.35
6	0.0	18	24.0	0.35

FIG. 6.26 IDEALISED SECTION FOR THE ORIGINAL BEAS DAM SECTION



x_1, y_1 represent the axis system chosen in the analysis by Spencer's method whereas x, y represent that in the analysis by Janbu's method

FIG. 6.27 IDEALISED SECTION ADOPTED FOR THE ANALYSIS OF THE D/S SLOPE OF THE BEAS DAM

properties as adopted in the analysis are presented in Table 6.2Ø.

The downstream slope of this section has earlier been analysed by Satyam Babu (1986) using the program SUMSTAB based on Janbu's GPS and the Interior Penalty function method in conjunction with the Powell's method and the Quadratic fit. The soil and pore pressure properties of the various zones of the dam section as presented in Figure 6.26 have been taken from the above reference.

The special feature of the Beas dam section is that it is founded on a thin shear zone. The numerical difficulties due to the presence of thin shear zone as reported by Satyam Babu (1986) in connection with the analysis of the same section and the Special procedure proposed in this thesis have already been presented earlier in this chapter. The present study is comprised of the following:

A. Analysis by the procedure formulated in Chapter 3 and its extension in this chapter. This will henceforth be referred to as the General procedure.

B. Analysis by the Special procedure for dams founded on thin shear zone formulated in this Chapter.

C. Comparison of the results obtained by using the General and the Special procedures as well as that reported by Satyam Babu (1986).

Both Janbu's and Spencer's methods have been used to obtain the results. While using Spencer's method, the Direct procedure has been adopted. Most of the results have been obtained using

TABLE 6.20

Geometry and Material Properties for the Beas Dam D/S Slope Section

Inter-section -> boundary / Layer boundary	1	2	3	4	Y - C O O R D I N A T E S 5	6	7	8	9	10	11	Material c' kPa	Material Ø' deg	Properties kN/m^3	Properties r_u
0	9.20	0.00	0.00	0.00	22.71	28.87	55.20	55.20	65.32	104.2	104.2	0.0	35.0	24.0	0.35
1	9.20	0.00	0.00	0.00	22.71	28.87	55.20	93.81	104.2	104.2	104.2	0.0	35.0	24.0	0.35
2	9.20	0.00	0.00	0.00	112.8	104.2	104.2	104.2	104.2	104.2	104.2	0.0	35.0	24.0	0.35
3	32.7	3.90	0.00	0.00	112.8	104.2	104.2	104.2	104.2	104.2	104.2	0.0	26.5	20.4	0.35
4	112.8	112.8	112.8	112.8	112.8	104.2	104.2	104.2	104.2	104.2	104.2	0.0	35.0	24.9	0.35
5	131.9186	129.974	129.7106	129.0421	125.2338	124.2006	119.7238	113.309	111.5669	104.8752	104.2	0.0	18.0	24.0	0.35
6	132.2186	130.274	130.0106	129.3421	125.5338	124.5006	120.0238	113.609	111.8669	105.1752	104.5	2000.0	40.0	25.0	0.35
7	134.2186	132.274	132.0106	131.3421	127.5338	126.5006	122.0238	115.609	113.8669	107.1752	106.5				
x-coordinate	-35.70	-6.90	-3.00	63.30	78.60	144.90	239.90	265.70	364.80	374.80					

N.B. Co-ordinates refer to the axis system x-y in Figure 6.27
Layer boundary 0 means the slope surface

the average r_u-values in the calculation of pore pressure. However, in a few cases minimum factor of safety has also been computed using an assumed phreatic line. For this purpose, the phreatic line ABCDEF shown in Figure 6.27 has been considered. This phreatic line has been obtained by joining the normal reservoir level and the tail water level by a straight line. Pore pressures are generated using vertical distances below this phreatic line and the forces due to water ponding on the face of the slope have been considered. In order to make a sound comparison with Satyam Babu's results, the water ponding forces have not been considered in the case of calculation of pore pressures using the average r_u-values, noting that the author did not consider such forces in his analysis. However, he considered pseudo-static earthquake forces and as such, for the sake of comparison, pseudo-static earthquake forces have been included in some selected cases as will be described in the following sections.

6.7.2 RESULTS AND DISCUSSION

6.7.2.1 Analysis Using the General Procedure

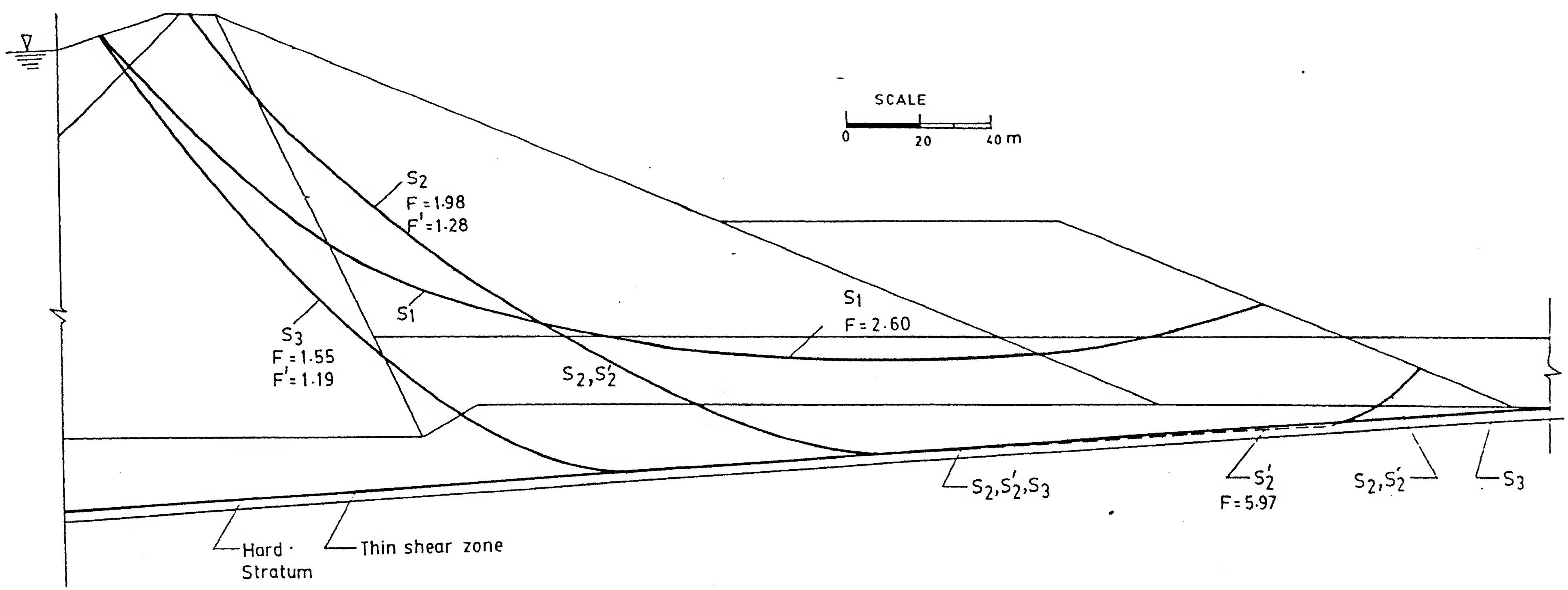
Unlike Satyam Babu (1986) who resorted to an approximate analysis, an attempt has here been made to perform the analysis using the General procedure thereby treating the thin shear zone as any other layer. In doing so the actual thickness of the thin shear zone has been used in the computations whereas a hypothetical shear zone of large thickness is to be necessarily considered in the Satyam Babu's solution approach.

Solution by Janbu's Method

For all the analyses using Janbu's GPS, the line of thrust has been assumed to lie at 1/3rd heights of the interslice boundaries from the shear surface unless otherwise specified. The Indirect procedure of critical surface determination, as discussed earlier, allows one to use the Interior penalty function method provided an initial feasible design point is available. In this case an initial feasible design vector may be found without much difficulty. However, for shear surfaces lying partly along the shear zone, the curvature constraints are likely to be violated at one or two points. If the Extended penalty function method is used, no adjustment of the design variables needs to be made to satisfy the curvature constraints. As such, the Extended penalty method has been employed here.

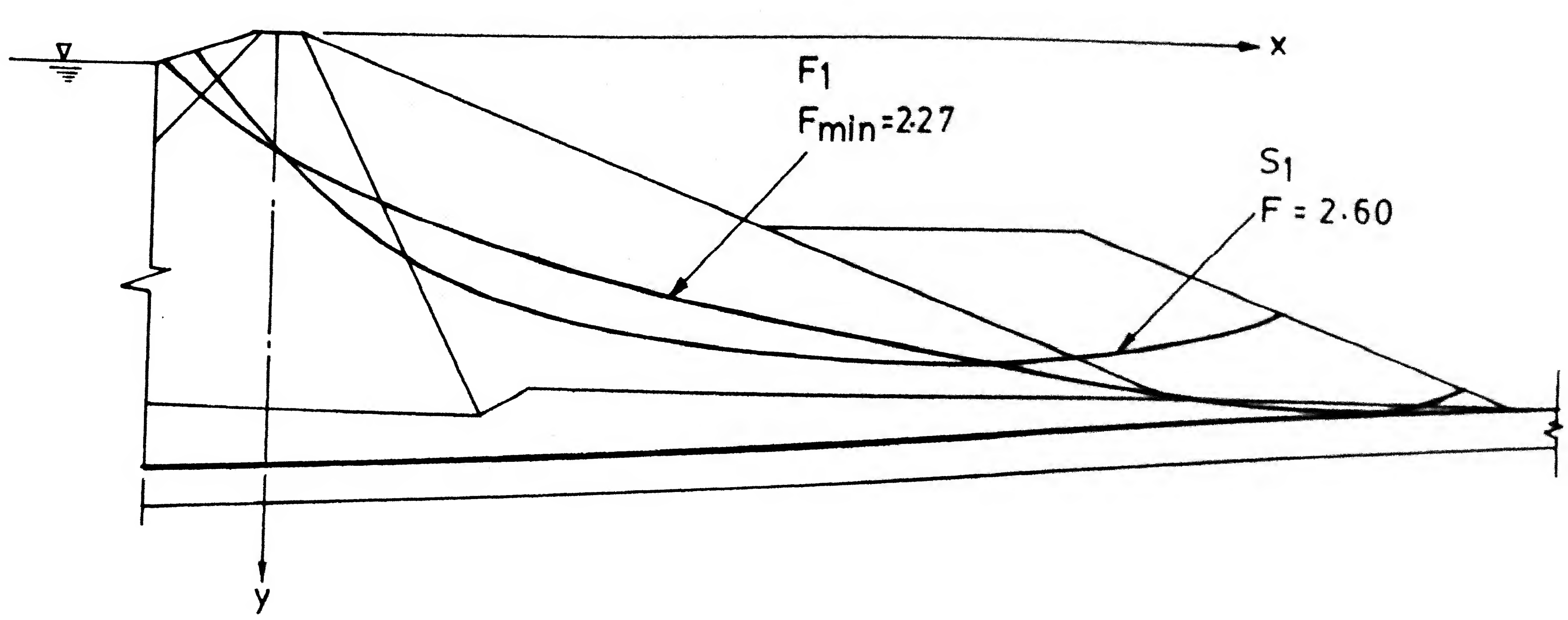
Although the critical slip surface in this problem is expected to lie along the shear zone, results have been obtained using a number of starting point surfaces (marked S_1, S_2, S_2', S_3 in Figure 6.28; some of these, however, do not lie along the shear zone) with a view to study the starting point dependence.

(i) Initially, the shallow input surface, S_1 (Figures 6.28 and 6.29), with initial factor of safety of 2.60, has been tried. As shown in Figure 6.29, the obtained final surface, F_1, does not lie along the shear zone contrary to what is expected from an engineering sense. The corresponding factor of safety is 2.27. The value has been obtained by using the average r_u-method of pore pressure calculation without any water ponding force or earthquake force.



N.B: F' values correspond to the pseudo-static analysis with $\alpha_h = 0.1$

FIG 6.28 VARIOUS INITIAL SHEAR SURFACES CONSIDERED IN THE PRESENT ANALYSIS OF EXAMPLE PROBLEM 6.5 TO STUDY STARTING POINT DEPENDENCE



Note : General procedure coupled with Janbu's method has been used

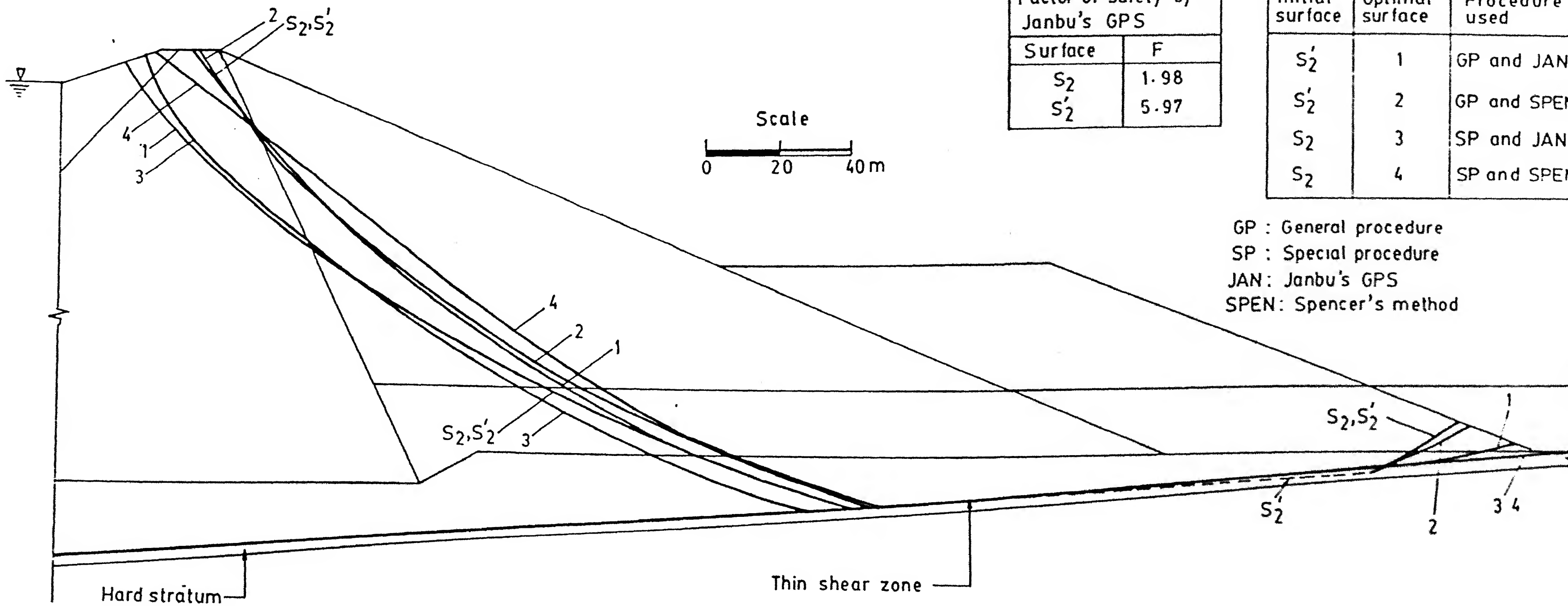
FIG. 6.29 OPTIMAL SHEAR SURFACE CORRESPONDING TO INITIAL SURFACE S_1 FOR THE BEAS DAM SECTION (Study of starting point dependence)

(ii) Subsequently the input surface S'_2 (Figures 6.28 and 6.30) lying partly along the shear zone and partly along the high strength zone below the shear zone (hard stratum) has been tried. It has an initial factor of safety of 5.97. The high value of the initial factor of safety can be attributed to the fact that part of the surface S'_2 lies in the hard stratum for which exceptionally large values of c' and ϕ' have been assumed in the analysis. The corresponding final surface (marked 1 in Figure 6.30) has a factor of safety of 1.73 which is much lower than the earlier one of 2.27 obtained using S_1 as the initial surface. Further, it follows the shear zone as expected.

(iii) Finally, the input surface S_3 (Figure 6.28 and 6.31) lying mostly along the shear zone and for the rest part passing through the zone II (which is weaker than zone III through which the surfaces S_2 and S'_2 pass) has been tried. This is very close to the initial surface considered by Satyam Babu (1986). This initial surface has a factor of safety of 1.55. The corresponding final surface has hardly shifted from S_3 and hence it has not been separately shown in Figure 6.31. The corresponding factor of safety is obtained as 1.49.

The three trials described above demonstrate that the developed numerical scheme is starting point dependent as also observed in the case of Example Problem 6.3. As such, engineering judgment needs to be applied to choose a suitable starting point surface.

For a seismic coefficient of α_h equal to 0.1, Satyam Babu (1986) used the program SUMSTAB to calculate a factor of safety



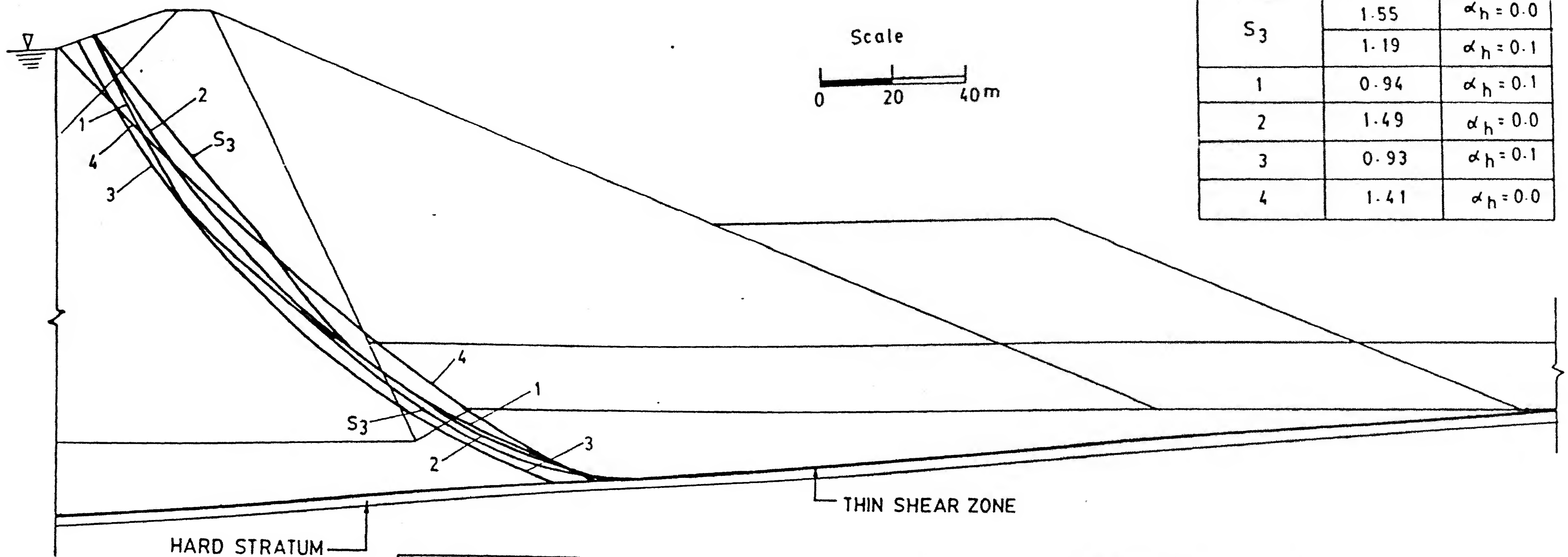
INITIAL SURFACES	
Factor of safety by Janbu's GPS	
Surface	F
S_2	1.98
S_2'	5.97

OPTIMAL SHEAR SURFACES			
Initial surface	Optimal surface	Procedure used	F_{min}
S_2'	1	GP and JAN	1.73
S_2'	2	GP and SPEN	1.85
S_2	3	SP and JAN	1.68
S_2	4	SP and SPEN	1.76

GP : General procedure
SP : Special procedure
JAN : Janbu's GPS
SPEN : Spencer's method

FIG. 6.30 OPTIMAL SHEAR SURFACES CORRESPONDING TO THE INITIAL SURFACE S_2 AND S_2' FOR THE BEAS DAM SECTION (study of starting point dependence)

SURFACE MARK	FACTOR OF SAFETY	REMARKS
S_3	1.55	$\alpha_h = 0.0$
	1.19	$\alpha_h = 0.1$
1	0.94	$\alpha_h = 0.1$
2	1.49	$\alpha_h = 0.0$
3	0.93	$\alpha_h = 0.1$
4	1.41	$\alpha_h = 0.0$



	THE STARTING POINT SURFACE S_3
1	THE OPTIMAL POINT SURFACE BY JANBU'S GPS IN GENERAL PROCEDURE CORRESPONDING TO $\alpha_h = 0.1$
2	THE " " " " " " IN SPECIAL " " " $\alpha_h = 0.0$
3	THE " " " " " " " " " " $\alpha_h = 0.1$
4	THE " " " " SPENCER METHOD " " " " $\alpha_h = 0.0$

FIG. 6.31 VARIOUS OPTIMAL POINT SURFACES CORRESPONDING TO THE INITIAL SURFACE S_3

zone. However, due to the extremely small thickness of the shear zone together with the restriction of maintaining upward concavity, the freedom of movement of the shear surface is so severely curbed that it cannot assume all of the positions which the minimization scheme would require to try in following the path of minimization,i.e., in order to search for the critical shape and location yielding the minimum factor of safety. As such, it is hardly possible for the shear surface to lie within this zone and at the same time maintain the upward concavity of shape. So unless the starting point surface itself corresponds to the global minimum, the solution, in all probability, would converge to a local minimum.

Satyam Babu (1986) has solved the problem by providing hypothetically increased shear zone thickness thereby imparting more freedom or flexibility to the shear surface while keeping the curvature constraints satisfied all through the search. An equivalent numerical flexibility is provided by the Extended penalty technique by virtue of its adaptability to violated constraints in addition to the satisfied ones.

(ii) *Initial value of r is so chosen that the value of the composite function ψ becomes much larger than the objective function f:*

Satyam Babu has also appreciated this factor. He has reported that if the initial value of ψ is unreasonably high compared to f the results might be unusual to the extent that the value of the objective function F after minimization could be

greater than the corresponding value at the initial point. This can be explained as follows:

The unconstrained minimization scheme searches directly for a feasible minimum value of the ψ-function and hence it always finds and ensures a solution which corresponds to a ψ-function value lower than its initial value (provided the initial design vector is in the feasible domain). However, it is not binding on the objective function f, which is only a component of the ψ-function, to diminish in order that ψ-function diminishes. Now, if initially ψ is much larger than f, the latter enjoys the freedom to assume, as the search goes on, values even higher than its initial value as long as the value of ψ is not exceeded beyond the corresponding initial value. Such a possibility arises when at an intermediate stage the shear surface lifts up from its initial position intelligently chosen such that the surface lies along the shear zone for the most part. At this lifted up position, the shear surface assumes a higher factor of safety because of increased shear resistance of the material above the shear zone. For reasons explained in (i) above, it is not possible for the shear surface to hit back the shear zone and eventually the solution terminates at a local minimum. Had there been small difference between f and ψ it would not have been possible for the shear surface to take a position which corresponds to a significantly increased value of $\bar{F}$. Thus the lifting up of the shear surface from the shear zone would have been indirectly guarded against.

(iii) *Design variables are not normalized:*

The Beas dam falls under the category of high dams. With the axis system chosen by Satyam Babu, the ratio of the largest and the smallest co-ordinates of the discrete points on a shear surface is of the order of 10^2. In such a situation it is wiser to normalize the design variables for a better numerical behaviour of the problem. In the present analysis reported already, better results have been obtained through such normalization. However, details of this particular study are not presented here to save space.

Table 6.21 presents the design variables and constraints at the starting and the optimal points corresponding to the initial shear surface S_2'. It is seen that a few curvature constraints at the optimal stage are actively satisfied or even (actively) violated.

Solution by Spencer's Method

The Direct procedure based on Spencer's method has been applied to solve this problem. The working of the Direct procedure has been tested using only one starting point surface, namely, the surface S_2'. The obtained final surface is shown marked 2 in Figure 6.30 and it yields a factor of safety of 1.85 as against 1.73 obtained earlier by using Janbu's method. This corresponds to r_u-method of pore pressure calculation without water ponding force and earthquake effect.

Table 6.22 presents the design variables and constraints at the starting and optimal points. It is seen that although at the

TABLE 6.21

Typical Design Vector and Constraints For The Example Problem 6.5 (Solution by General procedure using Janbu's method)

Total number of slices : 15

Total number of design variables : 16

Starting Point

F = 5.9684
Design Variables (Not normalized) :

27.5Ø 48.75 66.25 81.25 93.ØØ 1Ø5.ØØ 112.ØØØ 117.5Ø 115.5Ø 114.5Ø 113.75
112.5Ø 11Ø.5Ø 11Ø.Ø Ø.ØØ 343.75

Constraints (all inequality):

-Ø.7655	-Ø.6784	-Ø.6241	-Ø.58Ø8	-Ø.5359	-Ø.5Ø23	-Ø.5Ø71	-Ø.53Ø2
-Ø.5221	-Ø.5179	-Ø.4727	-Ø.3869	-Ø.2945	-Ø.2Ø95	-1.1364	-Ø.3846
-Ø.1887	-Ø.2ØØØ	Ø.Ø134	-Ø.2381	-Ø.Ø669	-Ø.3191	Ø.Ø433	Ø.Ø1Ø9
-Ø.Ø219	-Ø.Ø333	Ø.Ø679	-Ø.6163	-4.9684			

ϵ_o = -Ø.Ø1; δ_t = Ø.ØØØ1; r_o = Ø.1 x $1Ø^{-5}$; f = 5.9684; ψ = 5.97Ø7

Optimal Point

F = 1.7315
Design Variables (normalized)

1.1972	1.Ø554	1.ØØØ8	Ø.9915	Ø.9876	Ø.96Ø4	Ø.9775	Ø.9968
1.ØØØØ	Ø.9949	Ø.9868	Ø.983Ø	Ø.9859	Ø.975Ø	-17.6164	1.Ø449

Constraints (all inequality)

-Ø.9926	-Ø.7987	-Ø.6912	-Ø.62Ø6	-Ø.5586	-Ø.4991	-Ø.4958	-Ø.6287
-Ø.6224	-Ø.5154	-Ø.4425	-Ø.345Ø	-Ø.2446	-Ø.14Ø8	-1.6663	-Ø.357Ø
-Ø.Ø442	-Ø.1855	-Ø.124Ø	-Ø.Ø184	-Ø.Ø449	-Ø.3925	-Ø.47E-Ø2	-Ø.68E-Ø3
-Ø.48E-Ø3	Ø.93E-Ø3	-Ø.23E-Ø2	-Ø.1657	-Ø.7315			

f = 1.7315 ; ψ = 1.8726

No. of r minimizations required = 5

N.B. Out of a total 29 constraints, first 14 are boundary constraints, next 14 are curvature constraints and the last one is the side constraint on the factor of safety. No end constraints has been considered here.

TABLE 6.22

Typical Design Vector and Constraints For The Example Problem 6.5 (Solution by General procedure using Spencer's method)

Total number of slices : 15
Total number of design variables : 18

Starting Point

F = 2.5000 θ = 0.3000
Design Variables (Not normalized):

54.9	-55.4	-57.4	-58.65	-59.4	-60.4	-62.4	-56.9	-49.9	-37.9	-25.95
-10.95	6.55	27.8	336.85	-6.9	2.5	0.30				

Constraints (Inequality):

-0.7638	-0.6771	-0.6229	-0.5798	-0.5359	-0.5023	-0.5071	-0.5302
-0.5221	-0.5179	-0.4727	-0.3869	-0.2945	-0.2095	-0.2470	0.0271
-0.0131	-0.0085	0.0042	0.0166	-0.1202	-0.0263	-0.1002	0.0013
-0.1175	-0.2283	-0.5725	-0.2167	-0.4587	-0.2879	-0.2897	-0.2968
-0.3239	-0.4127	-0.5033	-0.5873	-0.6493	-0.6260	-0.5963	-0.5202
-0.3634	0.0930	-0.5413	-0.7121	-0.7103	-0.7032	-0.6761	-0.5873
-0.4967	-0.4127	-0.3507	-0.3740	-0.4037	-0.4798	-0.6366	-1.0930
-1.5100	-0.3000	-0.0839					

Constraint (Equality): 0.2008E+02
$\varepsilon = -0.1$ $\delta_t = 0.0001$ $r_o = 0.1 \times 10^{-4}$
f = 2.500 ψ = 2.5242 Z_n = 0.1383E+05 M_n = 0.1397E+06

Optimal Point

F = 1.8580 θ = 0.2367
Design Variables (normalized):

-0.9622	-0.9816	-0.9746	-0.9804	-0.9930	-1.0000	-0.9916	-0.9420
-0.9082	-0.9734	-0.9753	-0.8543	1.1855	0.9308	1.0101	-0.8563
0.7436	0.7890						

Constraints (Inequality):

-0.7634	-0.6576	-0.6044	-0.5684	-0.5231	-0.4718	-0.4922	-0.5281
-0.5222	-0.5161	-0.4577	-0.3686	-0.2769	-0.1826	-0.1720	-0.43E-04
-0.13E-04	-0.13E-02	-0.62E-03	-0.21E-04	-0.1571	-0.49E-05	-0.33E-02	-0.0855
-0.1727	-0.1246	-0.1279	-0.4293	-0.4115	-0.3811	-0.3419	-0.3120
-0.3076	-0.3409	-0.3772	-0.4319	-0.4816	-0.4702	-0.4987	-0.4951
-0.5006	-0.5302	-0.5885	-0.6189	-0.6581	-0.6879	-0.6924	-0.6591
-0.6228	-0.5681	-0.5184	-0.5298	-0.5013	-0.5048	-0.4994	-0.4698
-0.8680	-0.2367	-0.1473					

Constraint (Equality): 0.8180E-14

No. of r-minimization required = 9

f = 1.8580 ψ = 1.8592 Z_n = 0.1526E-03 M_n = 0.9543E-02

N.B. Out of a total of 59 inequality constraints, first 14 are boundary constraints, next 14 are curvature constraints, next 28 are the constraints on the line of thrust, the next one is the side constraint on F and the last two are the side constraints on θ

starting point quite a few curvature constraints and constraints on the line of thrust are violated, all of them are satisfied at the optimal point. However, like the solution by Janbu's method, most of the curvature constraints are only actively satisfied.

6.7.2.2 Analysis Using the Proposed Special Procedure

Earlier in this Chapter detailed discussion has been presented for the proposed Special procedure formulated such that a stretch of the thin shear zone boundary is forced to form a part of the shear surface and no curvature constraints are considered for the part adhering to the shear zone. The application of the said procedure has been demonstrated with reference to both the Janbu's and the Spencer's methods.

Solution by Janbu's GPS

As with the analyses by the General procedure already carried out, results have been obtained using two starting points S_2 and S_3 and are discussed as follows:

(i) Corresponding to the starting surface S_2, the final surface is marked 3 in Figure 6.30 and the associated factor of safety has been obtained as 1.68 against 1.73 yielded by the General Procedure. For a total of 15 slices considered in both the analyses, the number of design variables have been reduced from 15 in the case of the General procedure to 11 in the case of the Special procedure and the corresponding c.p.u. time has reduced from about 3.5 minutes to about 1 minute. When horizontal seismic forces (α_h = 0.1) have been included, F_{min} has been obtained as 1.08.

(ii) Starting from the surface S_3, the obtained final surface marked 2 in Figure 6.31 has a factor of safety of 1.487. When earthquake forces are considered, the obtained final surface is marked 3 in Figure 6.31 and the corresponding F_{min} is obtained as 0.932. These analyses are based on the r_u-method of pore pressure calculation without water ponding forces.

When the phreatic line is used for pore pressure calculation, the factor of safety of the obtained final surface is 1.95 in place of 1.487 by the r_u-method. On inclusion of α_h = 0.1, the value drops from 1.95 to 1.15.

Table 6.23 presents the interslice forces and the normal and shear stresses at the slice bases corresponding to the critical slip surface marked 3 in Figure 6.31. The assumed line of thrust positions (h_t/z) are also presented. It is seen that all the forces and stresses have proper signs signifying that the solution is acceptable.

Solution by Spencer's Method

The Special procedure in conjunction with the Direct formulation has also been used to solve the problem. Once again results have been obtained starting from the input surfaces S_2 and S_3 specified earlier.

(i) Figure 6.30 shows the final surface marked 4 corresponding to the starting point surface S_2. The factor of safety of this surface has been obtained as 1.76 as against 1.68 given by the Janbu's method.

TABLE 6.23

Calculated Responses and Assumed Line of Thrust In Analysis by Special Procedure using Janbu's Method (Example Problem 6.5)

Slice No.	σ' (kPa)	τ (kPa)	$\frac{h_t}{z}$ (assumed)	E kN/m	T kN/m
1	134.1	80.7			
			0.34	9398.0	-986.0
2	381.9	203.1			
			0.35	26550.0	-3689.0
3	507.2	271.0			
			0.36	43661.0	-9825.0
4	780.3	417.5			
			0.38	60273.0	-12068.0
5	564.4	423.3			
			0.38	70719.0	-22915.0
6	545.5	409.0			
			0.38	76922.0	-36387.0
7	1924.1	672.1			
			0.38	61783.0	-21748.0
8	1369.8	480.5			
			0.38	51450.0	-15670.0
9	1378.2	480.4			
			0.38	41034.0	-7594.0
10	1070.3	374.7			
			0.38	33456.0	-5838.0
11	981.0	342.3			
			0.38	26664.0	-5529.0
12	972.6	339.4			
			0.38	19914.0	-4864.0
13	828.4	289.2			
			0.38	14296.0	-5477.0
14	784.2	274.1			
			0.38	8638.0	-3279.0
15	564.1	197.7			
			0.38	4515.0	-2232.0
16	435.5	152.2			
			0.43	1398.0	-331.0
17	103.3	36.4			

(ii) Corresponding to the starting surface S_3, the final surface is marked 4 in Figure 6.31. The associated F_{min} is obtained as 1.41 as against 1.49 by the Janbu's method.

When the pore pressure calculations are based on the assumed phreatic line and the water ponding forces are also taken into account, the obtained F_{min} is 2.01.

Table 6.24 presents the calculated lines of thrust for total and effective stresses, the effective horizontal interslice force (Z') and also the normal and shear stresses at the slice bases. It is seen that although the L/H values lie roughly between 1/3 and 2/3, the L'/H values do not. In any case, the σ', τ and Z' values are satisfactory.

Tables 6.25 and 6.26 present the design vector and the constraints at the start and end of minimization for the case (i) above and solved by Janbu's method and Spencer's method respectively. It is noticeable how the design variables and constraints have reduced in number from those in the solution obtained by using General procedure. Figure 6.32 shows typically the paths followed by the f and ψ functions towards convergence in the special formulation coupled with the Direct procedure with particular reference to the case (i) corresponding to the input and output surfaces marked S_3 and 4 respectively in Figure 6.31. It is observed that the trend is the same as in the General procedure i.e., initially both f and ψ decrease to a minima and then gradually increase to converge to a feasible minima satisfying the equality constraint.

TABLE 6.24

Calculated Responses for the Analysis by Special Procedure using Spencer Method (Example Proble 6.5)

Slice No.	σ' (kPa)	τ (kPa)	L/H	L'/H	Z' kN/m
1	9.9	3.70			
			0.70	0.98	45.3
2	102.7	23.60			
			0.30	0.19	129.6
3	259.3	56.9			
			0.26	0.11	876.4
4	465.2	100.0			
			0.25	0.14	2696.0
5	674.6	143.2			
			0.24	0.21	5754.0
6	880.5	186.4			
			0.24	0.24	9955.0
7	1073.0	227.7			
			0.25	0.25	16179.0
8	1294.0	236.5			
			0.29	0.30	23386.0
9	1145.4	242.8			
			0.33	0.36	31082.0
10	1362.8	249.2			
			0.36	0.41	38914.0
11	1217.7	257.3			
			0.38	0.42	45880.0
12	1408.0	298.3			
			0.37	0.42	52939.0
13	975.0	426.7			
			0.34	0.38	46580.0
14	980.9	371.4			
			0.30	0.29	23543.0
15	645.0	229.9			
			0.27	0.19	8841.0
16	406.0	148.3			
			0.28	0.11	1451.0
17	154.5	66.80			

TABLE 6.25

Typical Design Vector and Constraints For The Example Problem 6.5 (Solution by Special procedure using Janbu's method)

Total number of slices : 14
Total number of design variables : 11

Starting Point

No of slices in the three blocks: $n_1 = 8$ $n_2 = 5$ $n_3 = 1$

Design Variables (Not normalized) :

27.50 48.75 66.25 81.25 93.00 105.00 112.000 0.000 200.00 320.00 343.75

Constraints (all Ineqaulity):

-0.7350 -0.6440 -0.5861 -0.5401 -0.4914 -0.4743 -0.5071 -0.1136 -0.0385
-0.0189 -0.0200 0.0013 -0.0236 -0.0134 -0.9846

$\varepsilon_o = -0.01$ $\delta_t = 0.0001$ $r_o = 0.1 \times 10^{-5}$
$f = 1.9846$ $\psi = 1.9850$

Optimal Point

No of slices in the three blocks: $n_1 = 8$ $n_2 = 5$ $n_3 = 1$

Design Variables (Not normalized)

32.100 49.62 64.54 79.28 91.99 102.21 110.81 -13.18 166.38 344.33 374.80

Constraints (all inequality):

-0.9703 -0.7987 -0.7051 -0.6460 -0.5979 -0.5509 -0.5053 -0.1958 -0.0264
-0.0013 -0.0128 -0.0136 -0.0078 -0.0052 -0.6789

$f = 1.6789$; $\psi = 1.6789$

No. of r - minimizations required = 5

N.B. Out of a total 15 constraints, first 7 are boundary constraints, next 7 are curvature constraints and the last one is the side constraint on the factor of safety.

TABLE 6.26

Typical Design Vector and Constraints For The Example Problem 6.5
(Solution by Special procedure using Spencer's method)

Total number of slices : 14
Total number of design variables : 13

Starting Point

No of slices in
the three blocks: $n_1 = 8$ $n_2 = 5$ $n_3 = 1$

Design Variables (Not normalized):

-57.0 -50.0 -38.0 -25.25 -11.25 -6.25 27.5 336.85 313.15 193.1 -6.9 2.0 0.30

Constraints (Inequality):

-0.7359	-0.6447	-0.5867	-0.5406	-0.4920	-0.4748	-0.5076	-0.0543
-0.1000	0.0066	-0.1238	-0.2222	-0.6000	-0.2309	-0.4537	-0.4571
-0.4242	-0.3922	0.4080	-0.4521	-0.5147	-0.6347	0.7583	0.9163
-1.1364	-1.7550	-4.6194	-0.5463	-0.5429	-0.5758	-0.6078	-0.5920
-0.5479	0.4853	-0.3653	-0.2417	-0.0337	0.1364	0.7550	0.0107
-1.0100	-0.3000	-0.0840					

Constraint (Equality): 0.3619E+01

$\varepsilon_o = -0.01$ $\delta_t = 0.0001$ $r_o = 0.1 \times 10^{-4}$

$f = 2.0000$ $\psi = 2.0069$ $Z_n = 0.1288E+04$ $M_n = -0.4183E+06$;

Optimal Point

No of slices in
the three blocks: $n_1 = 8$ $n_2 = 5$ $n_3 = 1$

$F = 1.7665$ $\theta = 0.2524$

Design Vector (normalized)

-1.0994 -1.0086 -1.0126 -0.9038 -0.4493 2.3219 1.2484 1.0922
1.0860 1.0255 -2.3513 0.8833 0.8415

Constraints (Inequality):

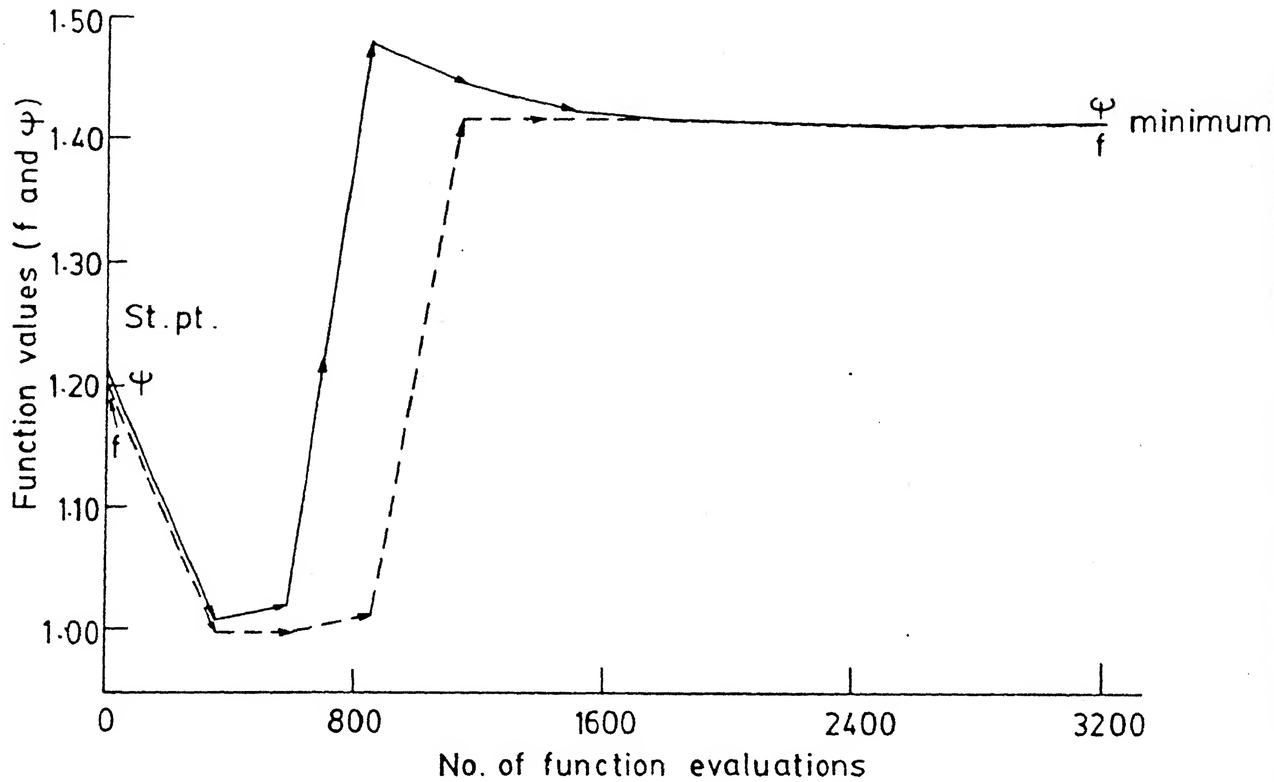
-0.7953	-0.6296	-0.5708	-0.5370	-0.4964	-0.4769	-0.5301	-0.2205
-0.826E-05	-0.7275E-01	-0.1652	-0.1771	-0.0174	-0.506E-02	-0.5100	-0.2730
-0.2499	-0.2433	-0.2509	-0.3019	-0.3523	-0.4224	-0.4449	-0.4898
-0.4844	-0.4731	-0.3961	-0.4899	-0.7269	-0.7500	-0.7567	-0.7490
-0.5981	-0.6477	-0.5776	-0.5551	-0.5101	-0.5156	-0.5269	-0.6039
-0.7765	-0.2524	-0.1315					

Constraint (Equality): 0.1412E-15

$f = 1.7665$ $\psi = 1.7669$ $Z_n = -0.4196E-04$ $M_n = 0.3967E-03$

No. of r minimizations required = 9

N.B. Out of a total 43 inequality constraints, first 7 are boundary constraints, next 7 are curvature constraints, next 26 are constraints on the line of thrust, the next one is the side constraint on F and the last two are the side constraints on θ.



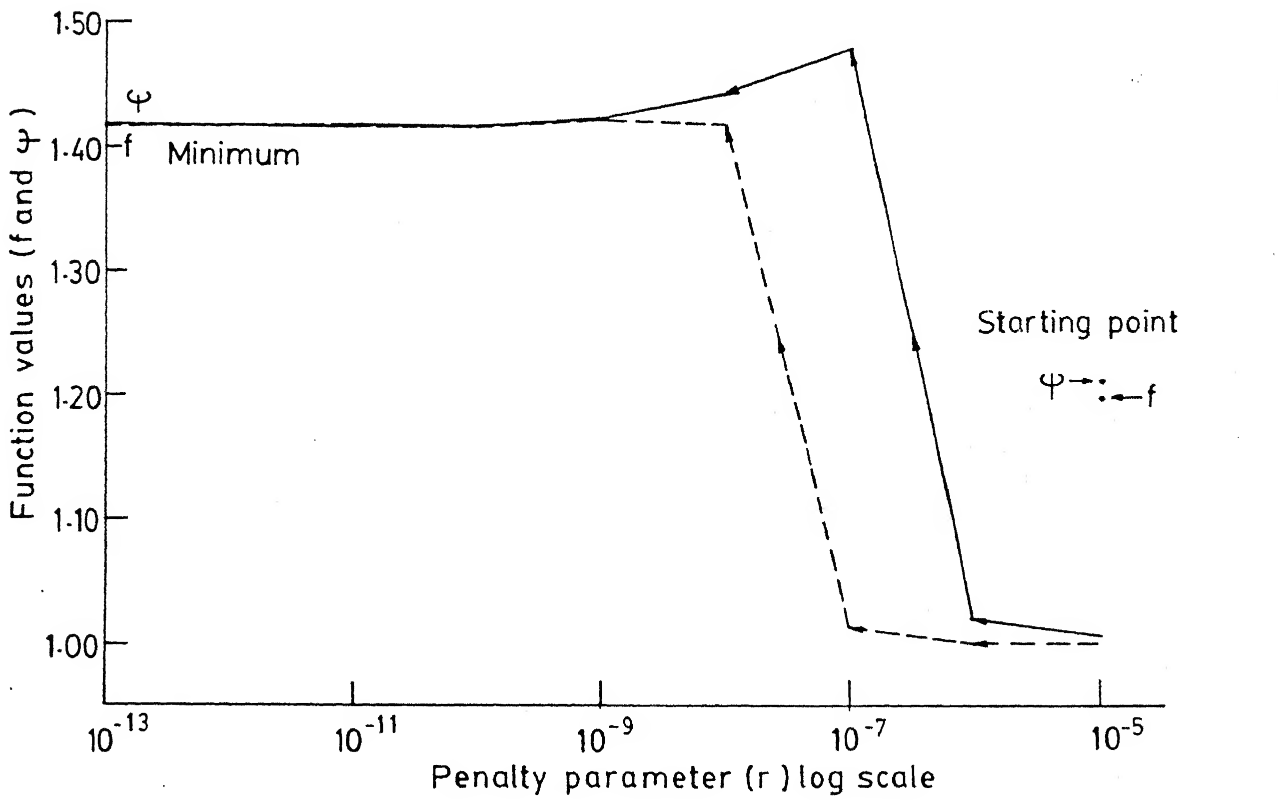


FIG.6·32 PROGRESS OF MINIMIZATION IN SPECIAL FORMULATION IN EX. PROB. 6·5

6.7.2.3 Comparison of the Various Solutions:

Table 6.27 presents a summary of a number of solutions corresponding to the General and the Special procedures using both Janbu's and Spencer's methods to compute the factor of safety.

It can be seen that in predicting the factor of safety value both the General and the Special procedures are equally effective and the obtained results agree excellently with previous solution (Satyam Babu, 1986). It can also be seen from the table that the solution obtained by Spencer's and Janbu's methods are in close agreement. Referring to Figures 6.30 and 6.31 it may be concluded that even though the factor of safety values corresponding to the critical slip surfaces predicted by Janbu's and Spencer's methods used in conjunction with either the General or the Special procedures are in close agreement, the critical surfaces themselves are quite different. Similar observations can also be made comparing the factor of safety values and the critical shear surfaces obtained by General and the Special formulations using either Janbu's or Spencer's method.

From the limited study of pseudo-static stability analysis of the Beas dam d/s slope under earthquake force (α_h = 0.1) it can be seen that the slope becomes unstable (F=0.93) and the percentage drop in the factor of safety is 36.9% compared to the static analysis (α_h = 0). The corresponding critical shear surface (failure surface) is observed to be deeper than that obtained in the static analysis (Figure 6.31). However, consideration of water ponding force and the pore pressure

TABLE 6.27

Comparison of Various Solutions

Starting Point			Optimal Solution								
			Proposed General Procedure				Proposed Special Procedure				F_{min} from Babu's Solution using Janbu's method and $\alpha_h = 0.1$
Trial Surface Marked	F by Janbu's Method		Janbu's Method		Spencer,s Method $\alpha_h=0.0$		Janbu's Method		Spencer's Method $\alpha_h=0.0$		
	$\alpha_h=0.0$	$\alpha_h=0.1$	F_{min} $\alpha_h=0.0$	F_{min} $\alpha_h=0.1$	F_{min}	θ (rad.)	F_{min} $\alpha_h=0.0$	F_{min} $\alpha_h=0.1$	F_{min}	θ (rad.)	
S_1	2.60	-	2.27	-			-	-	-	-	-
S_2	1.98	1.28					1.68	1.08	1.76	0.2524	
S_2'	5.97	-	1.73	1.14	1.85	0.2367					-
S_3	1.55	1.19	1.49 (1.95)	0.94	-	-	1.49	0.93 (1.15)	1.41 (2.01)	0.2355	0.94

Note : Figures in the parentheses are those obtained by using the assumed phreatic line for pore pressure calculation and considering water - ponding forces.

estimated on the basis of the assumed position of the phreatic line yields a minimum factor of safety of 1.15. Although this does not indicate a failed slope, it does show that the dam is not having adequate safety factor that is normally adopted in design practice.

6.7.3 PRESENT ANALYSIS USING THE PROGRAM SSOPT

An attempt was made to solve the Beas dam problem by using the program SSOPT based on the dynamic programming technique in conjunction with the Spencer's method. However, the exercise could not be carried out because of the inherent limitations of the program which are discussed as follows:

There is a restriction in the program SSOPT that the total number of vertical non-dimensional units (DY) should not exceed 35 as, otherwise, the c.p.u. time would increase exponentially. Now, the Beas dam section under study has a maximum vertical dimension of about 110.0 meters which makes it (110/35) i.e., approximately 3.0m of vertical unit thickness (DY). This is a bit too high for accurate representation of the geometry of the section considering Baker's recommendations (Baker, 1979) that DY should be between 0.5m and 1.0m. If the least recommended value of 0.5m is used the total number of such units will be about 220 which is likely to increase the execution time tremendously. However, even this unit thickness is not sufficiently small to represent the actual thickness of the shear zone which is only 0.3m. In addition, the slope of the shear zone cannot be represented accurately because the drop in its elevation between

two vertical boundaries are mostly smaller than Ø.5m and hence if a value of Ø.5 is chosen for DY the inclined shear zone will be practically represented by a horizontal one and the whole geometry will be different from what it is.

Under such serious restrictions it has been contemplated that there is hardly any point in carrying out an analysis using the program SSOPT and hence the same has not been pursued.

6.8 CONCLUSIONS

Based on the studies undertaken in this chapter the following conclusions are drawn :

1. The proposed methodology based on SUMT has proved to be as effective and efficient in locating the critical shear surface and solving for the factor of safety of a given shear surface in the case of heterogeneous slopes as in the case of homogeneous slopes.

2. For a given shear surface the developed program gives factor of safety values which agree well with the reported values obtained by using other currently available programs. This has been checked with respect to methods using both circular and non-circular shear surfaces and various pore pressure conditions. The performances of the subroutines developed for calculation of slice characteristics are also tested in the process.

3. Using the proposed technique for solving Spencer's factor of safety equations it has been conclusively demonstrated that the solution is unique and is independent of starting points.

4. The minimum factors of safety obtained in the present solution agrees very closely with those yielded by other techniques. The corresponding critical surfaces are, in some cases, quite apart indicating the possibility of the presence of a critical zone.

5. The determination of critical shear surfaces by the proposed technique is starting point dependent and hence require the use of few trial starting points to ensure a global minimum; however a single good initial guess based on engineering judgement is likely to be sufficient.

6. The Special formulation proposed herein for analysis of dams or embankments founded on thin shear zone has proved to be very effective and thus made it possible to avoid the difficulties in handling this shear zones reported earlier. In addition to being very effective the analysis procedure based on the Special formulation has reduced the c.p.u. time to almost one-third of that required by the General procedure.

7. The acceptability criteria should be checked; otherwise, the obtained minimum factor of safety may be in error by as much as 20 % and on the unsafe side. It has been reported, in the evaluation of factor of safety of a given shear surface, that non-checking of such criteria might show the existence of multiple solutions.

8. Too few slices used in the analysis might suppress the true stress conditions and let an unacceptable solution appear as an acceptable one.

CHAPTER 7

STABILITY ANALYSIS OF EMBANKMENTS ON CLAY FOUNDATIONS

7.1 INTRODUCTION

In Chapters 4, 5 and 6 the applications of the proposed numerical scheme to case-studies of some slopes in homogeneous as well as nonhomogeneous soil conditions have been presented. Alongside these studies, a series of comparisons between the results obtained by using the proposed procedure and some of the other currently available methods have also been made. In this chapter the application of the developed technique to the analysis of embankments on clay foundations are illustrated through various case-studies.

Embankments on soft ground are traditionally analysed based on the assumption of circular slip surface. There are cases in which a slip circle analysis gives solutions which are not much different from those obtained through analysis of non-circular or general slip surface. Example Problem 7.1 presents such a situation. However, making such a simplifying assumption in general, disregarding the possibility of a non-circular failure path, might be fraught with danger. The object of the Example Problem 7.2 and 7.3 is to highlight this point, as discussed below.

Recently, Low (1989) has proposed a semi-analytical procedure to calculate the factor of safety of embankments constructed on soft clay. Design charts have also been presented

by the author and the application of the proposed method has been illustrated through two example problems. The method proposed by Low is based on the assumption of circular failure surface. However, for the nonhomogeneous soil conditions present in the two example problems, a non-circular failure surface appears to be the most probable. To investigate the criticality of the non-circular failure surface, the numerical scheme proposed in this thesis has been applied to these two example problems (Example Problems 7.2 and 7.3).

The power of the proposed procedure in respect of the prediction of the critical slip surface has also been critically examined by re-analysing two well documented test embankment failures and comparing the predicted critical surfaces with the observed failure surfaces in the respective cases. Pilot (1972), Pilot et al. (1982) presented a detailed description of the soil conditions at four test embankments sites. Three of these test embankments were located in Narbonne, Lanester and Cubzac in France and the fourth one was in Saint-Alban, Quebec. All these test embankments were brought to failure as part of a research programme. The authors presented observed failure surfaces for these embankments. At one of the sites (Saint-Alban in Quebec) stakes driven into the ground were used in order to obtain information with respect to the shape and location of the actual slip surface; but for the other three sites no information is available about the technique used to trace the observed failure surfaces. In every case, however, the observed slip surface was assumed to be circular.

Pilot et al. (1982) also performed an effective stress analysis of these failures using the simplified Bishop method. Total stress analysis of these failures was earlier performed by La Rochella et al. (1974), Pilot et al. (1973a, 1973b) and Blondeau et al. (1977).

Talesnick and Baker (1984) re-analysed all the four embankments mentioned above on the basis of total stress using the program SSOPT which couples Spencer's method with the dynamic programming technique of minimization. However, of them, complete solutions have been reported for only two, namely, the Saint-Alban and the Lanester embankments i.e., in addition to the critical slip surfaces and the corresponding minimum factors of safety, other calculated responses such as the lines of thrust and the internal stresses have also been presented. The same two cases (Example Problems 7.4 and 7.5) have been chosen for re-analysis in the present study, the principal aim of which is the comparison of the observed and the calculated failure surfaces.

7.2 ANALYSIS

7.2.1 General

The objective function, the design variables and the design constraints are the same as defined in Chapter 3 and 6. The analysis has essentially been carried out by using the Direct procedure of searching the critical slip surface formulated in Chapters 3 and 6. In some cases, however, the Indirect procedure coupling the simplified Bishop method or the Ordinary method of slices have also been used. All the analyses are based on total stresses.

7.2.2 Additional Considerations

In those situations in which either the undrained shear strength or the unit weight of the foundation material or both vary with depth, some additional considerations are necessary. These are discussed as follows:

(a) Average Unit Weight:

From the given profile of unit weight (γ), an average value is to be obtained for use in the calculation of the weight of a slice contributed by certain foundation layers through which a given slip surface passes. In this study, weighted average values have been used. These are calculated as,

$$\gamma_{av} = \frac{\Sigma\, \gamma_k \cdot \Delta h_k}{h_m}$$

where, h_m is the mean-depth of the slice from the ground surface, Δh_k is any small depth increment over which γ may be assumed constant and the summation is taken over the depth h_m.

(b) Appropriate Strength for a Slice:

Considering that the actual depthwise variation of the undrained shear strength of the clay foundation material are, in most cases, irregular, the cohesion value to be used for an individual slice has to be done judiciously. The following steps are involved:

Step 1. *Selection of the Strength Profile:*

Generally, two strength profiles are obtained from the vane shear tests, one corresponding to the maximum vane strength and the other to the minimum vane strength. A third profile is

derived by averaging the maximum and the minimum strength profiles. In the present study, as in most of the earlier studies referred, the computations have been carried out on the basis of the average profiles.

Step 2. *Idealisation of the Strength Profile:*

The strength profile selected in Step 1 is then idealised as a continuous or a piecewise continuous linear or nonlinear function of depth. For example, the strength profiles used by Talesnick and Baker in analysing the Saint Alban and the Lanester Embankments are piecewise linear functions. Among the nonlinear idealisations, the following may be adopted in the present analysis:

(i) James et al.'s model

(ii) Piecewise polynomial approximation in the form of cubic splines.

James et al.'s model has already been discussed in Chapter 5. A short note on the spline-fit will be presented in Appendix-C.

7.3 EXAMPLE PROBLEM 7.1

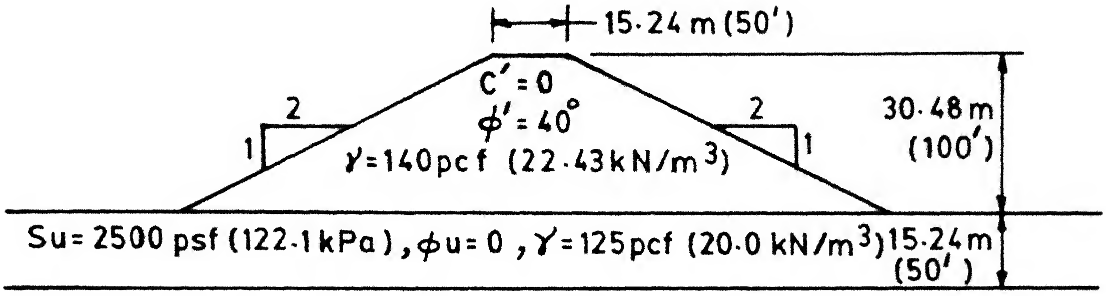


FIG. 7.1 SLOPE SECTION IN EXAMPLE PROBLEM 7.1 (Wright, 1969)

7.3.1 General

As the first illustrative example of the application of the proposed technique to the stability analysis of embankments on soft ground, the simple case shown in Figure 7.1 has been considered. It consists of a cohesionless fill ($c'=0$) on a saturated clay foundation ($\phi_u=0$). The same slope was previously analysed by Wright (1969). Both circular and non-circular analyses were carried out using various methods including Spencer's method and Janbu's GPS. However, for minimization grid-search technique was used in the above study.

In the present study, the problem has been re-analysed using the following techniques:

(i) proposed technique based on SUMT.

(ii) the program SSOPT (Baker, 1979) based on the dynamic programming technique.

(iii)the program PCSTABL5 (Carpenter, 1986) based on random search technique.

Results obtained using the various techniques have been compared.

7.3.2. Results and Discussion

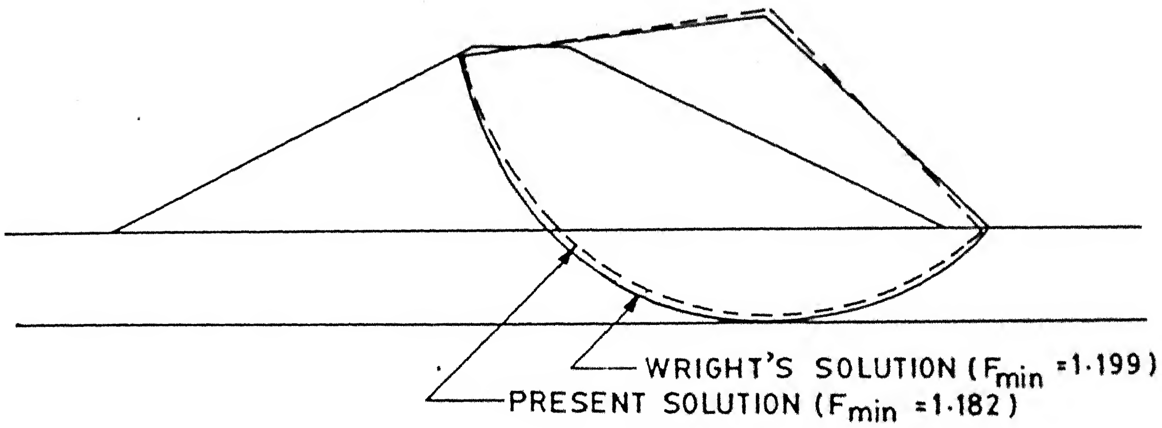
7.3.2.1 Analysis Using the Proposed Technique

This problem has been solved by using the proposed Indirect procedure in conjunction with both Spencer's and Janbu's methods. In the case of Spencer's method, solutions have been obtained for circular as well as non-circular shear surfaces.

(i) Solution by Spencer's method:

Critical Slip Circle: Taking Wright's critical slip circle (Wright, 1969) as the starting surface, the Indirect Procedure making use of the Spencer's method has been used to determine the critical slip circle. Because, in this case, a feasible starting point is available, the Interior penalty function method has been used.

The starting surface itself being a near critical one (obtained by the grid search technique), the critical slip circle obtained in the present solution (Figure 7.2) shows only a marginal difference from the starting surface. The corresponding minimum factor of safety is 1.182 and the associated interslice force angle θ is obtained as 8.9862°. 40 slices have been used in the analysis; an equal number of slices were used by Wright (1969) also. Table 7.1 presents the calculated responses - the line of thrust, the interslice forces and also the normal and the



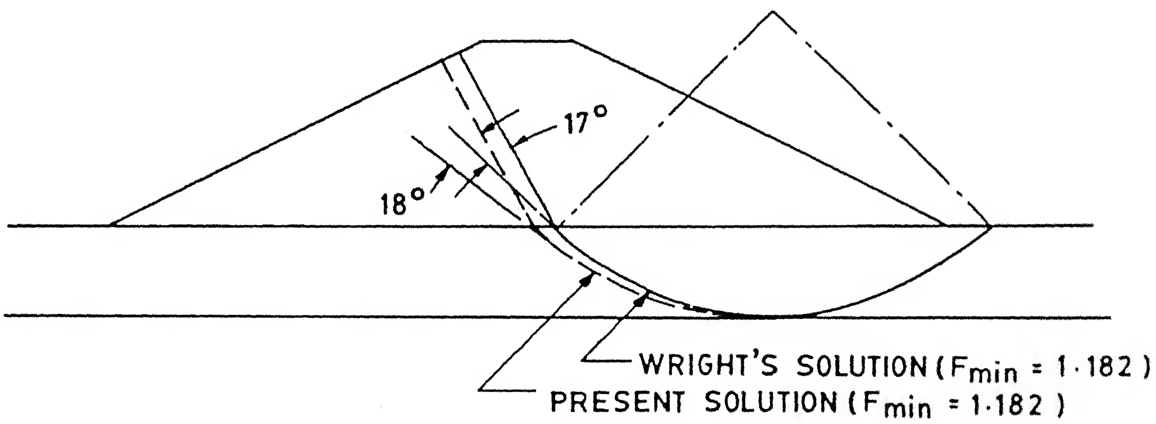


FIG. 7·2 CRITICAL CIRCULAR AND NON-CIRCULAR (COMPOSITE) SHEAR SURFACES FOR SPENCER'S ANALYSIS PROCEDURE

TABLE 7.1

Calculated Responses Associated with the Critical Slip Circle for the Example Problem 7.1

L/H

0.58	0.57	0.53	0.46	0.42	0.39	0.37	0.36	0.34	0.33	0.32	0.31
0.31	0.30	0.29	0.29	0.28	0.27	0.27	0.26	0.26	0.25	0.25	0.24
0.23	0.23	0.22	0.22	0.21	0.21	0.20	0.20	0.22	0.28	0.27	0.27
0.27	0.28	0.39									

$Z/\gamma bH_t$

0.43	0.86	1.31	1.77	2.24	2.72	3.19	3.66	4.11	4.54	4.94	5.31
5.65	5.95	6.22	6.44	6.62	6.71	6.84	6.68	6.57	6.51	6.48	6.47
6.42	6.14	5.79	5.38	4.92	4.39	3.81	3.17	2.47	1.63	1.23	0.97
0.69	0.40	0.12									

σ(kPa)

161.71	190.33	215.39	257.36	302.13	343.68	382.30	418.22
451.58	482.59	511.25	537.71	562.07	584.35	604.58	622.81
639.02	653.23	665.47	675.64	683.79	689.80	693.61	695.17
451.53	691.02	685.06	676.28	664.36	649.08	629.99	606.55
578.19	548.89	331.72	278.06	222.20	163.42	100.73	23.62

τ(kPa)

103.32	103.32	103.32	103.32	103.32	103.32	103.32	103.32
103.32	103.32	103.32	103.32	103.32	103.32	103.32	103.32
103.32	103.32	103.32	103.32	103.32	103.32	103.32	103.32
103.32	103.32	103.32	103.32	103.32	460.78	447.24	430.59
410.46	389.66	235.49	197.39	157.74	116.01	71.51	16.77

shear stresses at the bases of the slices. It is seen that the L/H values are reasonable and the forces and the stresses are all positive. Table 7.2 presents the design variables (x_o,y_o,R) and the constraints at the beginning and end of minimization. The table shows that the design variables have changed very little from their initial values.

Critical Non-circular Slip Surface: Taking again the critical slip circle (F=1.199) reported by Wright as the starting surface, the critical surface of general shape has been determined by using the Indirect formulation. Fifteen slices have been considered. The minimum factor of safety has been obtained as 1.182 while the initial factor of safety was 1.242. The factor of safety of the critical non-circular slip surface reported by Wright is also 1.182. As shown in Figure 7.2, the two critical surfaces are also close, as expected, in shape as well as in location.

As far as the shape of the critical surface is concerned, it may be noted that whereas in the Wright's analysis, it was necessary to apply engineering judgment to make a priori assumption regarding the shape of the shear surface, in the present analysis the developed autosearch technique has successfully brought out such a shape even though the starting surface was chosen as circular. Wright explained that non-circular surfaces of such types as shown in Figure 7.2 were analysed because it seemed reasonable that the critical surface might be broken at the point where it passes from the cohesionless fill into the cohesive foundation. If the principal stresses close to this point are oriented in identical

TABLE 7.2

Design Variables and Constraints in the Slip Circle Analysis by Spencer Method for the Example Problem 7.1

No.of Slices = 4Ø
No. of design variables = 3

Starting Point

F = 1.1939 θ = Ø.152Ø Z_n = Ø.2436E-Ø3 M_n = Ø.2592E-Ø2

Design Variables (Not normalized): 31.5468 33.68Ø4 48.92Ø4

Constraints:

-Ø.5974	-2.199Ø	-1.Ø35Ø	-Ø.ØØ31	-Ø.5773	-Ø.5629	-Ø.5439	-Ø.4696
-Ø.4262	-Ø.3969	-Ø.3754	-Ø.3315	-Ø.3447	-Ø.3332	-Ø.3231	-Ø.3142
-Ø.3Ø62	-Ø.2988	-Ø.2919	-Ø.2855	-Ø.2794	-Ø.2735	-Ø.2678	-Ø.2622
-Ø.2567	-Ø.2514	-Ø.2461	-Ø.24Ø8	-Ø.23Ø5	-Ø.225Ø	-Ø.2196	-Ø.2143
-Ø.2Ø93	-Ø.2Ø52	-Ø.2Ø28	-Ø.2Ø52	-Ø.2229	-Ø.2765	-Ø.27Ø9	-Ø.2669
-Ø.2653	-Ø.2726	-Ø.3837	-Ø.4227	-Ø.4371	-Ø.4561	-Ø.53Ø4	-Ø.5738
-Ø.6Ø31	-Ø.6246	-Ø.6685	-Ø.6553	-Ø.6668	-Ø.6769	-Ø.6858	-Ø.6938
-Ø.7Ø12	-Ø.7Ø81	-Ø.7145	-Ø.72Ø6	-Ø.7265	-Ø.7322	-Ø.7378	-Ø.7433
-Ø.7486	-Ø.7539	-Ø.7592	-Ø.7695	-Ø.775Ø	-Ø.78Ø4	-Ø.7857	-Ø.79Ø7
-Ø.7948	-Ø.7972	-Ø.7948	-Ø.7771	-Ø.7235	-Ø.7291	-Ø.7331	-Ø.7347
-Ø.7274	-Ø.6163	-Ø.2Ø39	-Ø.152Ø				

$r_o = Ø.1 \times 1Ø^{-4}$ f = 1.1939 ψ = 1.194Ø

Optimal Point

F = 1.1817 θ = Ø.1559 Z_n = Ø.3456E-Ø1 M_n = Ø.3111E-Ø2

Design Variables (Not normalized) : 31.5886 34.Ø852 48.935Ø

Constraints:

-Ø.6454	-2.1883	-1.Ø364	-Ø.ØØ79	-Ø.5811	-Ø.5672	-Ø.534Ø	-Ø.463Ø
-Ø.4215	-Ø.3934	-Ø.3727	-Ø.3564	-Ø.3432	-Ø.3319	-Ø.3222	-Ø.3135
-Ø.3Ø57	-Ø.2985	-Ø.2918	-Ø.2854	-Ø.2794	-Ø.2736	-Ø.268Ø	-Ø.2625
-Ø.2574	-Ø.2518	-Ø.2465	-Ø.2412	-Ø.23Ø9	-Ø.2254	-Ø.2199	-Ø.2146
-Ø.2Ø95	-Ø.2Ø51	-Ø.2Ø22	-Ø.2Ø39	-Ø.2199	-Ø.2798	-Ø.2747	-Ø.2714
-Ø.27Ø9	-Ø.28Ø3	-Ø.3879	-Ø.4189	-Ø.4328	-Ø.466Ø	-Ø.537Ø	-Ø.5785
-Ø.6Ø66	-Ø.6273	-Ø.6436	-Ø.6568	-Ø.6681	-Ø.6778	-Ø.6865	-Ø.6943
-Ø.7Ø15	-Ø.7Ø82	-Ø.7146	-Ø.72Ø6	-Ø.7264	-Ø.732Ø	-Ø.7375	-Ø.7426
-Ø.7482	-Ø.7535	-Ø.7588	-Ø.7691	-Ø.7446	-Ø.78Ø1	-Ø.7854	-Ø.79Ø5
-Ø.7949	-Ø.7978	-Ø.7961	-Ø.78Ø1	-Ø.72Ø2	-Ø.7253	-Ø.7286	-Ø.7291
-Ø.7197	-Ø.6121	-Ø.1917	-Ø.1559				

No. of r-minimizations = 2

$r = Ø.1 \times 1Ø^{-5}$ f = 1.18173 ψ = 1.18174

Note: Out of the total number of 84 constraints, first 4 are boundary constraints, the next 78 are the constraints on line of thrust and the last two are the side constraints on F and θ respectively.

directions, the failure surface in the cohesionless fill ($\phi' = 40^{\circ}$) should be inclined more steeply than the failure surface in the cohesive foundation ($\phi_u = 0$). The amount by which the failure surface is more steeply inclined should be equal to one-half the value of the mobilized friction angle for the cohesionless fill ($\phi'_m/2$). In accordance with the above, the amount by which the failure surface obtained in the present solution is expected to be more steeply inclined is equal to:

$$\frac{1}{2} \tan^{-1} \left[\frac{\tan 40^{\circ}}{1.182} \right] = 17\frac{1}{2}^{\circ}$$

This value is the same as in Wright's analysis. The measured angle in Figure 7.2 is about 18° and hence the shape of the obtained slip surface is reasonable.

Table 7.3 presents the associated line of thrust, the interslice forces and normal and shear stress values along the critical surface. It is seen that these values are also reasonable. Table 7.4 shows the design vector and constraints at the starting and optimal points. Interior penalty function method has been applied in the case also as a feasible starting point is available.

(ii) Solution by Janbu's Method

Results have also been obtained by using the proposed technique in conjunction with Janbu's GPS . A starting surface in the vicinity of that used in the solution by Spencer's method already described, has been used. A total of 6 slices have been considered in the analysis. The assumed line of thrust (h_t ...) has been presented in Table 7.5(a). The factor of safety of the

TABLE 7.3

Calculated Responses Associated With the Critical Non-Circular Slip Surface by Spencer's Method (Example Problem 7.1)

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma bH_t$
1	164.31	103.32		
			0.60	0.38
2	255.12	103.32		
			0.48	0.84
3	369.64	103.32		
			0.39	1.32
4	465.19	103.32		
			0.35	1.75
5	540.93	103.32		
			0.32	2.09
6	589.55	103.32		
			0.30	2.29
7	652.95	103.32		
			0.29	2.45
8	685.71	103.32		
			0.27	2.48
9	708.60	103.32		
			0.26	2.38
10	707.57	103.32		
			0.24	2.10
11	699.66	103.32		
			0.22	1.71
12	672.32	103.32		
			0.24	1.19
13	605.02	168.81		
			0.34	0.67
14	200.19	141.98		
			0.42	0.21
15	65.58	46.50		

TABLE 7.4

Design Variables and Constraints in the Non-circular Analysis by Spencer method for the Example Problem 7.1

No.of Slices = 15
No. of design variables = 16

Starting Point

F = 1.2416 θ = 0.1542 rad. Z_n = 0.9308E-02 M_n = 0.1343E-02

Design Variables :(Not normalized)

-5.0609	-8.8004	-11.5462	-13.4744	-14.6738	-15.2066	-15.0916	-14.3242
-12.8722	-10.6688	-7.5943	-3.4339	2.2238	10.4965	65.1914	-19.1863

Constraints:(all inequality) :

-1.0000	-1.0000	-1.0000	-0.9565	-0.8914	-0.8320	-0.7747	-0.7168	
-0.6559	-0.5896	-0.5144	-0.4252	-0.3134	-0.1620	-1.0000	-0.6680	
-0.4225	-0.2424	-0.1155	-0.0372	-0.0022	-0.0098	-0.0601	-0.1554	
-0.2999	-0.5017	-0.7747	-1.3214	-0.9937	-0.8176	-0.7288	-0.6666	
-0.6478	-0.6524	-0.6846	-0.7514	-0.8712	-1.0858	-1.4974	-2.6150	
-9.7376	-0.5154	-0.3886	-0.3419	-0.3146	-0.2949	-0.2789	-0.2649	-0.2520
-0.2402	-0.2304	-0.2271	-0.2619	-0.3241	-0.4287	-0.4846	-0.6114	-0.6581
-0.6854	-0.7051	-0.7211	-0.7351	-0.7480	-0.7598	-0.7696	-0.7729	-0.7381
-0.6759	-0.5713	-0.2516	-0.1542					

Interior Penalty method

r_o = 0.1 x 10^{-4} f = 1.2416 ψ = 1.2426

Optimal Point

F = 1.1822 θ = o.1519 rad. Z_n = 0.1110E-01 M_n = 0.6714E-03

Design Variables (Not normalizsed) :

-4.7181	-8.7862	-11.8666	-13.9115	-15.0801	-15.2326	-15.2295	-14.4671
-13.0826	-10.8081	-7.8754	-4.1462	-0.2190E-02	12.6062	70.0851	-19.4103

Constraints (all inequality) :

-1.0000	-1.0000	-1.0000	-0.9419	-0.8738	-0.8119	-0.7513	-0.6902
-0.6249	-0.5582	-0.4794	-0.3838	-0.2595	-0.1340	-1.0000	-0.6903
-0.4235	-0.2214	-0.0872	-0.0105	-0.49E-03	-0.69E-03	-0.0507	-0.1416
-0.2908	-0.4832	-0.7280	-0.9999	-0.6499	-0.9877	-1.0355	-0.8763
-1.0162	-0.1555	-0.7594	-0.6220	-0.8901	-0.6580	-0.7965	-0.4147
-8.4644	-3.1803	-0.5968	-0.4823	-0.3926	-0.3480	-0.3186	-0.2953
-0.2769	-0.2605	-0.2457	-0.2320	-0.2228	-0.2350	-0.3274	-0.4161
-0.4032	-0.5177	-0.6074	-0.6520	-0.6814	-0.7047	-0.7231	-0.7395
-0.7543	-0.7680	-0.7772	-0.7650	-0.6726	-0.5839	-0.1822	-0.1519

No. of r-minimizations required : 2

r = 0.1 x 10^{-5} f = 1.1822 ψ = 1.1831

Note: Out of the total number of 72 constraints, first 28 are boundary constraints, next 14 are curvature constraints, next 28 are constraints on line of thrust, and rest two are the side constraints on F and θ. At the starting point one lower boundary constraint is less thus making it 71.

TABLE 7.5(a)

Calculated Responses and Assumed Line of Thrust for Solution by Janbu's GPS with 6 Slices (Example Problem 7.1)

Slice No.	σ kPa	τ kPa	h_t/z	E kN/m	T kN/m
1	148.4	98.0			
			0.33	2571	-538
2	638.0	96.2			
			0.34	7292	-1304
3	648.0	96.2			
			0.36	9076	-2337
4	580.0	96.2			
			0.38	7949	-2903
5	588.2	96.2			
			0.43	5390	-918
6	295.4	96.2			

TABLE 7.5(b)

calculated Responses and Assumed Line of Thrust for Solution by Janbu's GPS with 15 Slices (Example Problem 7.1)

Slice No.	σ kPa	τ kPa	h_t/z	E kN/m	T kN/m
1	53.0	35.0			
			0.34	251	67
2	139.7	92.7			
			0.35	1190	36
3	103.7	69.0			
			0.36	2413	-1365
4	173.2	115.8			
			0.38	2380	-3021
5	2106.0	104.0			
			0.38	1203	4798
6	592.7	104.0			
			0.38	7304	4731
7	-930.5	104.0			
			0.38	9501	-3339
8	758.9	104.0			
			0.38	9172	-3045
9	641.4	104.0			
			0.38	8665	-3102
10	640.5	104.0			
			0.38	7826	-2902
11	606.0	104.0			
			0.38	6670	-2561
12	574.7	104.0			
			0.38	5177	-2009
13	514.7	104.0			
			0.38	3441	-1323
14	496.0	104.0			
			0.43	1611	-202
15	222.3	104.0			

starting surface has been computed as 1.51. As the starting point is a feasible one, interior penalty function method has been applied. The obtained critical surface is shown in Figure 7.3 and has a factor of safety of 1.268. The distributions of normal and shear stress at slice bases and the interslice forces are also shown in Table 7.5(a). These values show that the solution is an acceptable one.

The reasons behind taking only six slices in the analysis are explained as follows:

Initially, analysis has been attempted taking 15 slices (the same number of slices was used in obtaining solution by Spencer method, already described). However, no acceptable solution has been obtained as some of the interslice shear forces 'T' have been found to be positive and one normal stress 'σ' has been found to be negative (Table 7.5(b)). It is pertinent to note the experience reported by Wright (1969) in an attempt to solve the same problem by Janbu's GPS. For ready reference, the following is quoted from Wright's Ph.D. thesis (1969).

> "..... A number of analyses were attempted for the example slope using Janbu's GPS procedure, and for any reasonable configuration of the critical shear surface a distinct divergence was encountered in the solution. Although shear surfaces were tried having statically correct entrance and exit angles and having several non-circular shapes, both with and without assumed tension cracks, a reasonably convergent solution could not be obtained".

It has been discussed in Chapter 3 that according to Janbu (1980), one of the factors responsible for non-convergence of Janbu's method is the adoption of too many slices in the computations. Keeping this in mind, a series of attempts have been made to solve the problem taking 6, 8, 10, 12 and 15 slices and

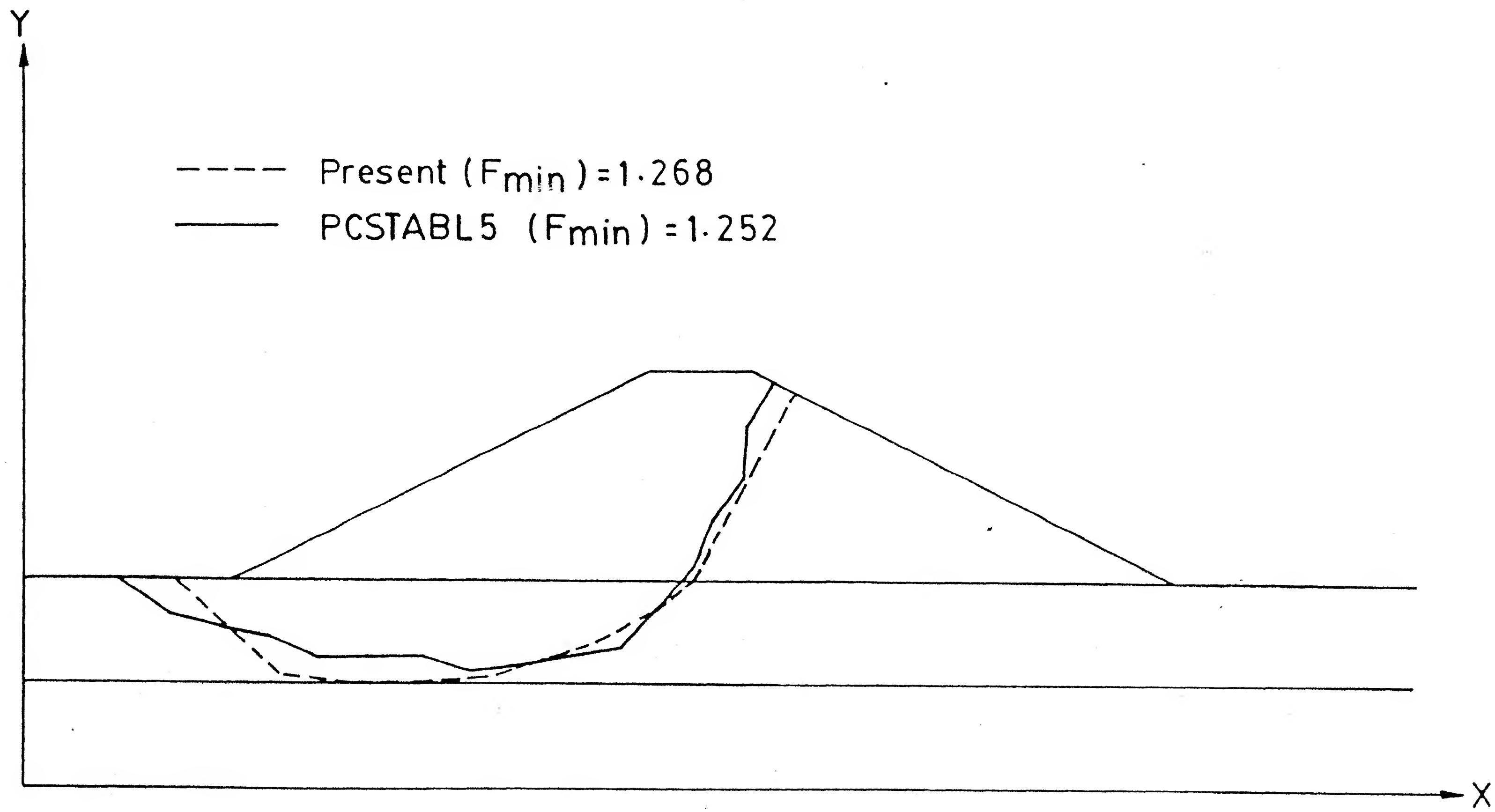


FIG. 7·3 CRITICAL SHEAR SURFACES FOR JANBU'S ANALYSIS PROCEDURE

using the same starting surface. It has been observed that acceptable solutions are obtained in the analyses using 6,8 and 10 slices out of which the one with 6 slices give the smallest F_{min}, namely, 1.27. The other cases give higher F_{min} values; these values are, however, approximately equal for all cases and are very close to 1.33. It has been further observed that in such cases as yielding acceptable solutions, the imposition of end constraint does not make significant difference in the results. In other cases, for instance, the one with 12 slices, F_{min} is obtained as 1.27 or 1.0 (the lower limit on F set in the program is 1.0) in analyses with and without end constraint respectively. The corresponding values in the case of 15 slices are 1.19 and 1.0. In both these cases the solutions are unacceptable as the signs of the interslice forces are inadmissible (the analysis which does not include end constraints, however, give results which are worse than those in which these constraints are included).

In the light of Janbu's recommendations mentioned above, the effect of number of slices on the evaluation of the factor of safety of a given shear surface has also been studied. For this purpose, the starting surface used in the above analysis has been chosen. The following observations have been made.

For analysis using 6,8 and 10 slices which, after minimization, led to acceptable critical surfaces, the iterations involved in the computation of factor of safety of the first trial shear surface converge, satisfying a strict convergence limit of ε = 0.001. When 12 slices are used, it does not converge to ε = 0.001; it converges only when the limit has been

Figure 7.4(a) shows the variation of the value of F discussed above with the discrete number of slices 6,8,10,12 and 15. It is seen that there is no monotonic increase or decrease of the factor of safety value as the number of slices increases. In the given range of number of slices, the maximum difference in the factor of safety value is observed to be about 1%.

Figure 7.4(b) shows the variation of F_{min} with the same discrete number of slices as discussed above. These, however, correspond to both acceptable and unacceptable solutions. It is observed that if the solution is unacceptable it can be quite misleading. For instance, the cases with 12 and 15 slices corresponding to the analysis without the end constraint, result in a F_{min} of unity, indicating that the slope is a failed one, whereas acceptable solutions do indicate it to be a stable one. For acceptable solutions, it is seen that the imposition of such constraints does not make any significant difference in the obtained F_{min} values.

Table 7.6 presents the design variables and constraints at the starting and end points. The table shows the effectiveness and the efficiency of the proposed procedure in arriving at the optimal solution. The identical values of f and ψ at the optimal design point demonstrate that the minimization of the ψ function has led to the minimization of f, the objective function, signifying that there has been no ill-conditioning during the computations in obtaining the optimal solution.

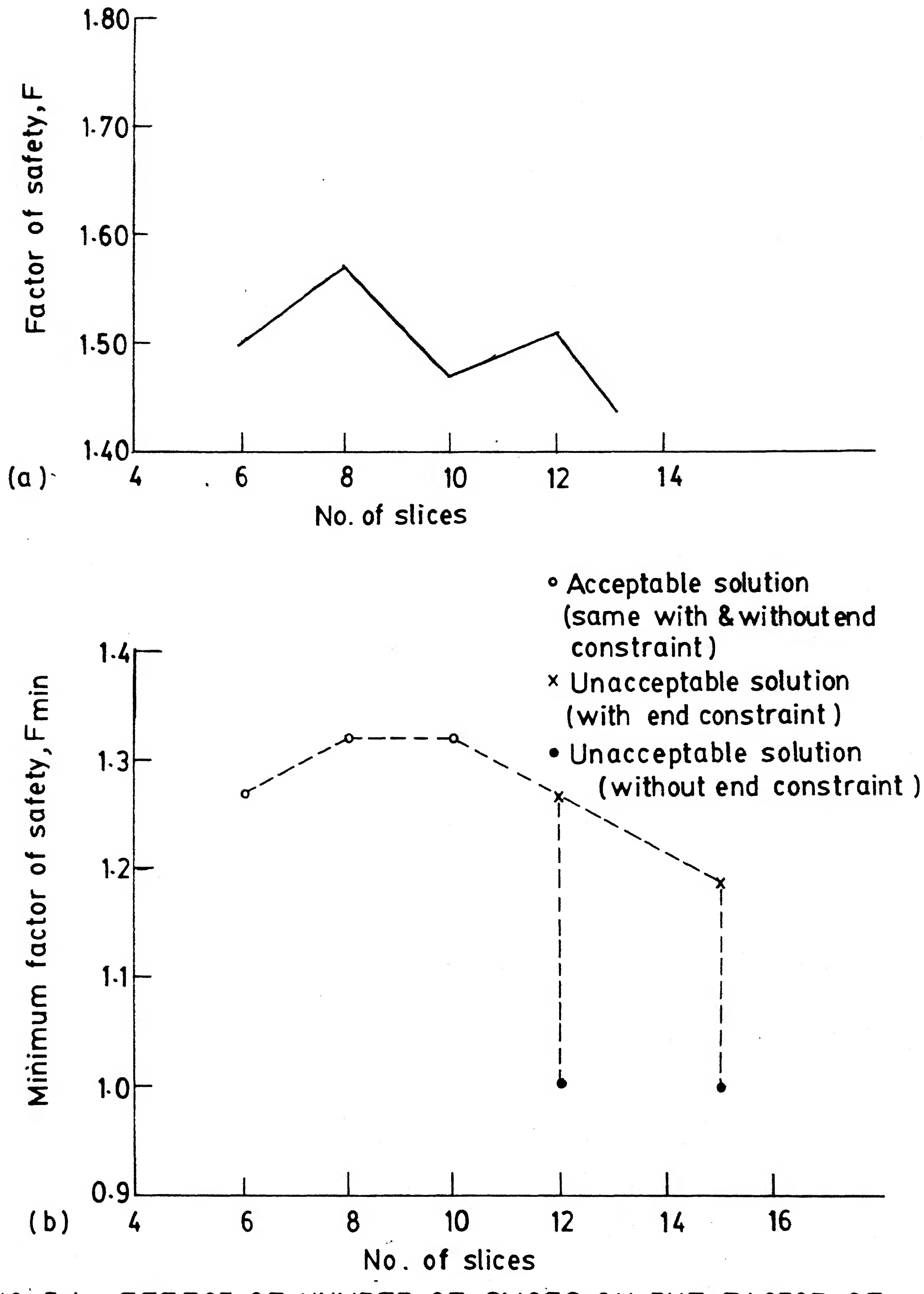


FIG. 7.4 EFFECT OF NUMBER OF SLICES ON THE FACTOR OF SAFETY OF HETEROGENEOUS SECTION
(Based on the solution of Ex. prob. 7.1 by Janbu's GPS)

TABLE 7.6

Design Vector and Constraints (Analysis by Janbu's Method)
(Example Problem 7.1)

No.of Slices = 6

No. of design variables = 7

Starting Point

F = 1.5114

Design Variables :(Not normalized)

21.375 36.ØØ 43.875 43.875 39.375 -15.ØØØ 75.ØØØ

Constraints:(all inequality) :

-1.ØØØØ -Ø.8958 -Ø.7436 -Ø.5726 -Ø.3333 -Ø.7Ø18
-Ø.9375 -Ø.8974 -Ø.5128 -Ø.619Ø -Ø.5244

Minimization by Interior penalty function method

r_o = Ø.1ØE-Ø2 f = 1.5114 ψ = 1.5288

Optimal Point

F = 1.2686

Design Variables (Not normalized) :

3Ø.48Ø 39.5Ø2 44.4Ø99 44.964Ø 43.1139 -13.4Ø23 75.8Ø34

Constraints (all inequality) :

-1.ØØØØ -Ø.8882 -Ø.7332 -Ø.5711 -Ø.38Ø3 -2.9245 -Ø.5815
-Ø.49Ø2 -Ø.2673 -1.3Ø63 -Ø.2786

No. of r-minimizations required : 4

r = Ø.1ØE-Ø5 f = 1.2686 ψ = 1.2686

Note: Out of a total of eleven constraints, first five are the upper boundary constraints, the next five are curvature constraints, and the last one is a side constraint on F. It is to be noted that in this analysis, instead of putting any constraint on the presence of hard stratum, an additional layer with very high values of c', and ϕ' have been adopted.

7.3.2.2 Analysis Using the Program SSOPT

Table 7.7 shows the results obtained for this problem using the program SSOPT based on the dynamic programming technique. The obtained minimum factor of safety is 1.182, which is the same as obtained by using the Spencer method in the proposed technique. However, as seen from the Table 7.7, the solution is not acceptable because of the following reasons:

1. The line of thrust (z) at the interslice boundaries towards the two ends of the critical slip surface is located at zero heights from the slip surface.
2. The resultant interslice force (Q) is negative at the interslice boundary near the toe.

7.3.2.3 Analysis Using the Program PCSTABL5

For the sake of comparison, the present problem has also been solved by using the PC-version of the software package STABL5 based on the random search technique. A partial computer printout of the results obtained by using the 'modified' Janbu's method is shown in Figure 7.5. The non-circular critical slip surface given by PCSTABL5 is traced from this computer output and plotted in the same figure. The same slip surface is also plotted in Figure 7.3 alongside the critical surface obtained by using the proposed technique and Janbu's GPS. It is observed from Figure 7.3 that the two surfaces are similar in shape and different in location but not by a great degree. The value of F_{min} is obtained as 1.252 which is quite close to the value of 1.268 obtained by using the proposed technique (a difference of 1.26% only).

TABLE 7.7

A Part of the SSOPT Output for the Example Problem 7.1

THE CRITICAL PATH IS

J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
YC	1	1	4	13	20	26	29	32	34	35	36	36	36	36	35	33	30	27	24	26

SLICES DATA

J	CN	L	UT	CL	U	UTX	UTY	UM	HT	N	PR	SK	Q	Z
1	0.9881	6.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19.35	1	0.00	0.00
2	0.8829	7.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	22.93	1	0.00	0.00
3	1.1382	15.01	46.41	0.00	0.00	0.00	0.00	0.00	5.33	40.29	31.06	1	25.52	15.75
4	1.1497	11.82	182.33	0.00	0.00	0.00	0.00	0.00	16.00	156.70	39.95	1	120.34	23.63
5	1.1741	10.46	295.21	0.00	0.00	0.00	0.00	0.00	25.91	248.43	48.96	1	253.51	30.22
6	0.8375	6.83	370.53	83.45	0.00	0.00	0.00	0.00	32.77	391.12	54.95	1	465.24	36.15
7	0.8829	7.62	479.54	93.04	0.00	0.00	0.00	0.00	35.81	494.83	58.61	1	702.05	40.43
8	0.9526	6.82	484.31	83.22	0.00	0.00	0.00	0.00	36.58	479.86	61.43	1	855.56	43.06
9	0.9959	6.28	470.51	76.72	0.00	0.00	0.00	0.00	35.81	460.92	62.75	1	905.01	44.51
10	0.9959	6.28	447.41	76.72	0.00	0.00	0.00	0.00	34.29	438.01	63.32	1	948.85	45.88
11	0.9881	6.10	415.03	74.43	0.00	0.00	0.00	0.00	32.00	424.04	63.13	1	885.16	46.22
12	0.9881	6.10	373.35	74.43	0.00	0.00	0.00	0.00	28.96	383.16	62.19	1	821.48	46.53
13	0.9881	6.10	331.67	74.43	0.00	0.00	0.00	0.00	25.91	341.48	61.25	1	757.80	46.82
14	0.9212	6.28	280.71	76.72	0.00	0.00	0.00	0.00	22.10	328.48	59.56	1	613.48	46.17
15	0.8149	6.82	211.16	83.22	0.00	0.00	0.00	0.00	16.76	306.09	56.36	1	411.26	44.58
16	0.6980	7.62	123.03	93.04	0.00	0.00	0.00	0.00	9.91	254.84	51.66	1	192.82	42.19
17	0.6980	7.62	46.45	93.04	0.00	0.00	0.00	0.00	3.81	146.44	46.20	1	40.21	40.22
18	0.6980	7.62	3.10	31.01	0.00	0.00	0.00	0.00	0.76	31.28	42.56	1	-0.01	0.00
19	0.9959	12.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	40.09	1	-0.01	0.00

TET = 0.886209E+01 F = 0.118292E+01

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

END OF RUN

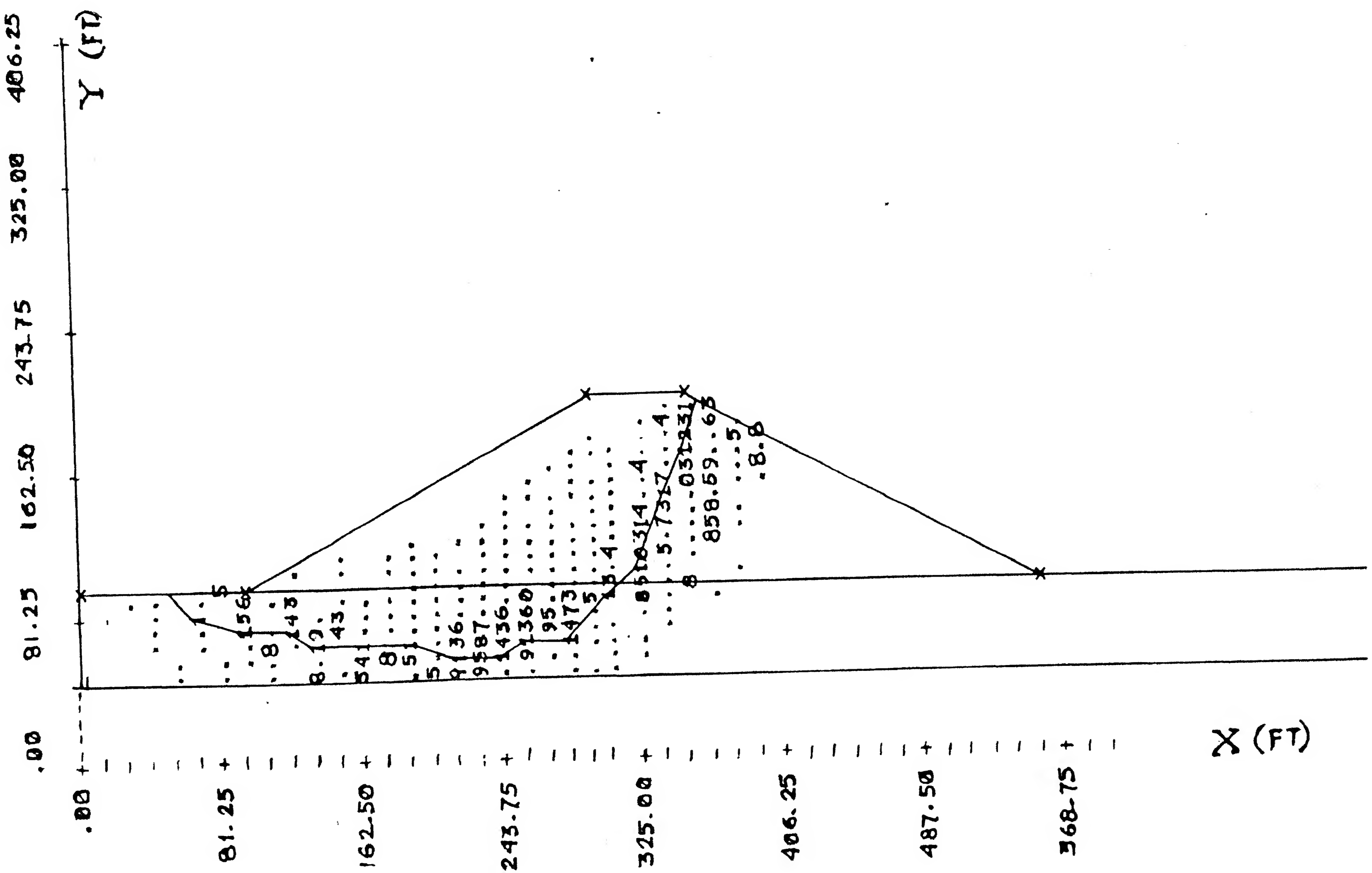


FIG. 7.5 CRITICAL SHEAR SURFACE BY PCSTABL5 FOR THE EXAMPLE PROBLEM 7.1 (Obtained from a computer output)

The internal forces and stresses not being available along with the computer output, the acceptability of the solution cannot be judged. An attempt has been made to analyse the critical surface obtained by PCSTABL5 using the developed program for Janbu's GPS. However, even a crude convergence (ε = 0.1) could not be achieved after a number of iterations. The reasons for non-convergence may be the irregular shape of the critical slip surface as well as the high value of the maximum h_m/b ratio of about 20.

7.3.3. Comparison of the Solutions Obtained by Various Techniques

Table 7.8 presents a summary of the results obtained in the present and the Wright's analyses. It is seen that the minimum factor of safety computed by the developed procedure using Janbu's and Spencer's methods are in agreement with those given by other currently available techniques. However, while in the present analysis acceptable solutions are obtained, the solutions obtained by using other techniques are not all acceptable. It is also noticeable that the value of F_{min} by Janbu's method is higher than that by Spencer's method by about 8%.

TABLE 7.8

Comparison of Different Solutions of Example Problem 7.1

Investigation	Minimization Technique	Method of Analysis	F	θ (deg.)	Remarks
A. Wright's Solution (1969)	Grid search	i) Spencer (circular)	1.199	-	
		ii) Spencer (non-circular)	1.182	-	acceptable solution
		iii) Janbu's GPS	-	-	No convergence
B. Present Solution	1. Proposed technique based on SUMT	i) Spencer (circular)	1.182	8.9862	acceptable solution
		ii) Spencer (non-circular)	1.182	8.7Ø32	acceptable solution
		iii) Janbu's GPS	1.268	-	acceptable solution
	2. Program SSOPT based on dynamic programming technique	Spencer (non-circular)	1.182	8.8621	unacceptable solution
	3. Program PCSTABL5 based on random-search technique	Modified Janbu (non-circular)	1.252	-	unacceptable solution

7.4 EXAMPLE PROBLEM 7.2

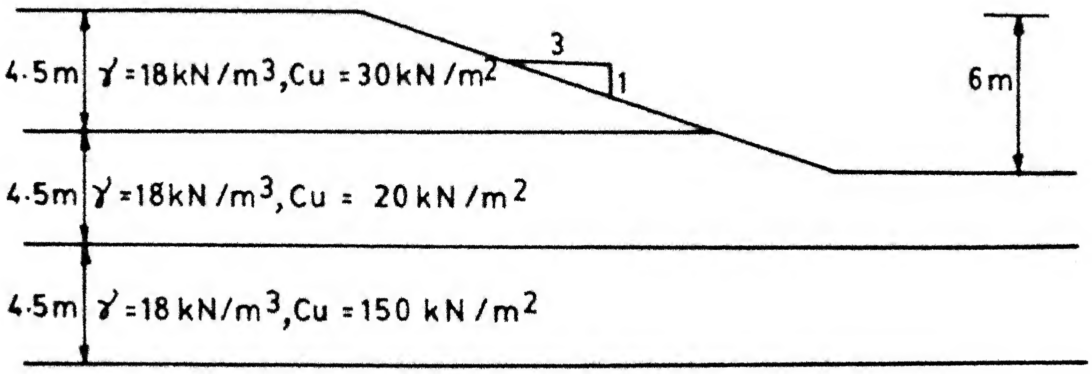


FIG. 7.6 SLOPE SECTION IN EXAMPLE PROBLEM 7.2
(B.K.Low,1989)

7.4.1 General

As stated in Section 7.1, Low (1989) has presented a semi-analytical procedure to calculate the factor of safety of embankments constructed on soft clay using the stability numbers defined for the embankment and the foundation. Design charts have also been presented. The method assumes undrained response of the soft clay foundation and is also based on a priori assumption that the failure surface is circular. The application of the method has been illustrated with the help of two example problems which are shown in Figures 7.6 and 7.8. and arked as Example problems 7.2 and 7.3 respectively. The author as also compared the results obtained by using the proposed thod with those given by the computer program STABR (Duncan and ng, 1984).

Considering the nonhomogeneous soil conditions present in the two example problems, however, a non-circular failure surface appears to be the most probable. The object of the present study is to investigate, for the said example problems, the criticality of non-circular surfaces relative to the circular ones. The proposed Direct procedure has been used for the analysis and the obtained results have been compared with those reported by Low (1989). As before, care has been taken to find an acceptable solution. Example 7.2 is taken up first.

In the Example problem 7.2 there are three horizontal clay layers, each 4.5m thick. The values of c_u for the upper, middle and lower strata are, respectively, 30, 20 and 150 kN/m^2. The unit weight is 18 kN/m^3. A cut is excavated with side slopes of 1V:3H to a depth of 6 m.

7.4.2 Results and Discussion

A starting surface ABCDEF as shown in Figure 7.7 has been intuitively chosen in the first attempt, noting that the bottom layer is a hard stratum and the shear surface will not cut across it. The Direct procedure yields a F_{min} = 1.51. However, the associated line of thrust is not satisfactory and the interslice force (Z) at the last interslice boundary is negative. In order to find an acceptable solution, water-filled tension crack of constant depth has been introduced. Several trials have been made taking the depths of tension crack as 1.5m, 2.5m, 3.5m and 4.5m. Out of these trials only the last one with 4.5m depth of tension crack yielded acceptable solution having F_{min} = 1.177 and θ = 0.1658 rad. However, for a direct determination of the

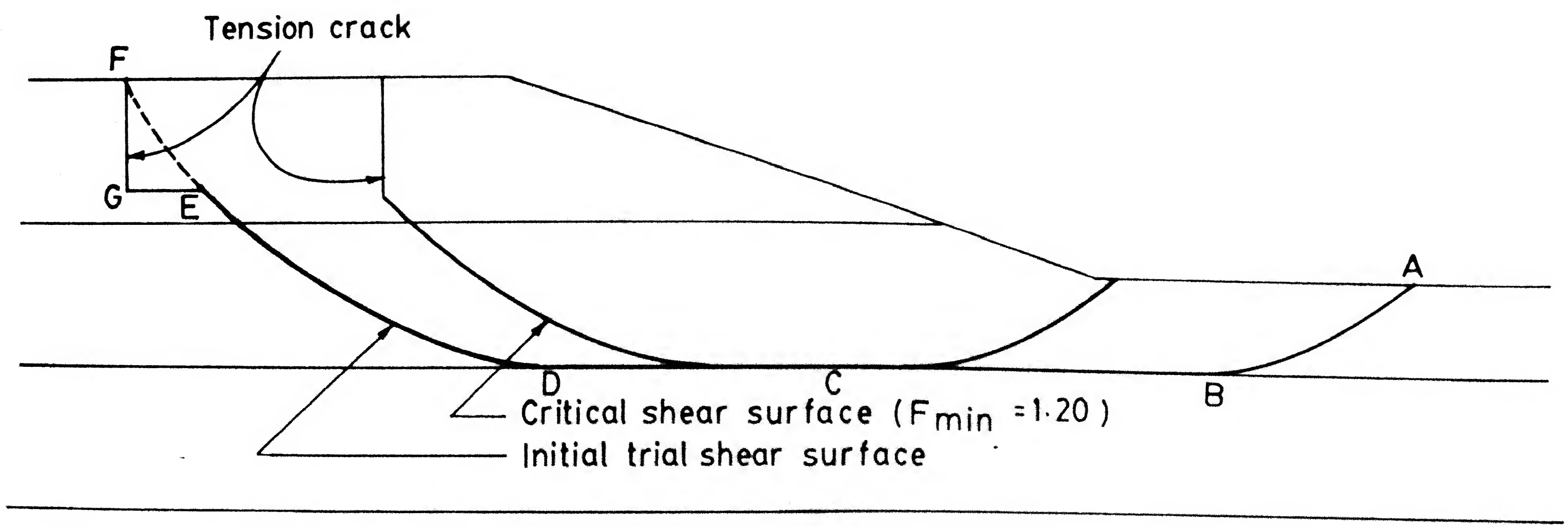


FIG. 7·7 CRITICAL NON-CIRCULAR SHEAR SURFACE BY DIRECT PROCEDURE ALONGWITH THE INITIAL TRIAL SURFACE (EX. PROB.7·2)

optimal depth of tension crack or, in other words, for a more accurate solution, z_t has been treated as a design variable and a fresh solution has been attempted. Initial value of z_t has been taken as 3.5m. The introduction of tension crack (done on the computer terminal itself) without changing the value of x_u has resulted in such an odd shaped input surface as ABCDEGF shown in Figure 7.7. The optimal value of z_t has been obtained as 3.78m and F_{min}= 1.20 and θ = 0.1560 rad. The critical surface so obtained has also been shown in Figure 7.7. The associated line of thrust, interslice force and normal and shear stress distribution along the critical surface have been presented in Table 7.9. It is seen that all these internal variables have reasonable values.

Table 7.10 presents the design vector and constraints at the starting and optimal points. It is seen that although the starting point falls in the infeasible region, the optimal point lies in the feasible region.

Comparison with Low's Solution and the Solution by STABR

As reported by Low (1989), the results obtained using the method proposed by him is 1.45 and that obtained by using the computer program STABR is 1.44 for both Ordinary Method of slices and Simplified Bishop Method. Compared with this, the acceptable solution in the present analysis giving F_{min}= 1.20 is much smaller. This means, the non-circular shear surface is more critical as anticipated. With respect to this acceptable solution, Low's solution is in error by about 21 percent and the unacceptable solution obtained in the present analysis giving

TABLE 7.9

Calculated Responses Associated with the Critical Shear Surface for the Example Problem 7.2

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma bH_t$
1	28.22	16.66		
			0.54	0.37
2	46.45	16.66		
			0.43	0.80
3	62.14	16.66		
			0.39	1.23
4	74.46	16.66		
			0.37	1.63
5	84.04	16.66		
			0.35	2.00
6	91.12	16.66		
			0.34	2.21
7	95.94	16.66		
			0.33	2.37
8	102.52	16.66		
			0.32	2.53
9	109.32	16.66		
			0.31	2.68
10	116.12	16.66		
			0.30	2.84
11	122.49	16.66		
			0.30	2.98
12	124.59	16.66		
			0.29	3.01
13	124.65	16.66		
			0.28	2.93
14	122.60	16.66		
			0.28	2.73
15	118.38	16.66		
			0.28	2.43
16	111.61	16.66		
			0.28	2.03
17	101.94	16.66		
			0.28	1.54
18	87.92	16.66		
			0.31	1.09
19	72.66	16.66		
			0.38	0.705
20	45.63	23.30		

TABLE 7.10

Design Vector and Constraints in Example Problem 7.2

No. of Slices = 20
No. of design variables = 24

Starting Point

$F = 1.2500$ $\theta = 0.1500$
Design Variables (Not normalized) :

-1.60	-2.60	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00
-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-2.80	-2.20
-1.00	0.60	2.70	28.00	-12.00	1.25	0.15	2.50

Constraints (Inequality) :

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9259	-0.8519
-0.7778	-0.7037	-0.6296	-0.5556	-0.4815	-0.4074	-0.3333	-0.3333
-0.3333	-0.3023	-0.2105	-0.6000	-0.6000	-0.4000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.2000	-0.4000	-0.6000	-0.4000	-0.5000	2.3000	-0.5945	-0.5617
-0.5731	-0.5963	-0.6255	-0.5391	-0.4814	-0.4405	-0.4102	-0.3869
-0.3685	-0.3537	-0.3415	-0.3313	-0.3503	-0.3769	-0.4145	-0.4534
-0.3328	-0.4055	-0.4388	-0.4269	-0.4037	-0.3745	-0.4609	-0.5186
-0.5595	-0.5898	-0.6131	-0.6315	-0.6463	-0.6585	-0.6687	-0.6497
-0.6231	-0.5855	-0.5466	-0.6672	-0.2600	-0.1500	-0.1718	

Constraints (Equality) : 0.1123E+01

ε_o=-0.1 δ_t=0.0001 f=1.25 ψ=1.2573 Z_n=0.2286E+03 M_n= -0.2486E+03

Optimal Point

$F = 1.2007$ $\theta = 0.1550$
Design Variables (Not normalized) :

-0.9194	-1.6549	-2.2066	-2.6009	-2.8581	-2.9887	-2.9999	-2.9999
-3.0000	-3.0000	-2.9893	-2.8687	-2.6367	-2.2884	-1.8167	-1.2041
-0.4380	0.4139	1.3171	18.8493	-3.8167	1.2006	0.1560	2.2203

Constraints (Inequality) :

-1.0000	-1.0000	-1.0000	-0.9668	-0.9211	-0.8800	-0.8411	-0.8027
-0.7633	-0.7216	-0.6796	-0.6377	-0.5957	-0.5532	-0.5039	-0.4452
-0.3725	-0.2779	-0.1465	-0.1838	-0.1839	-0.1574	-0.1370	-0.1267
-0.1193	-0.0113	-0.11E-04	-0.74E-05	-0.0107	-0.1099	-0.1114	-0.1163
-0.1235	-0.1409	-0.1535	-0.0858	-0.0513	-0.86E-06	-0.5412	-0.4313
-0.3913	-0.3682	-0.3518	-0.3385	-0.3265	-0.3165	-0.3082	-0.3012
-0.2952	-0.2896	-0.2842	-0.2791	-0.2745	-0.2718	-0.2806	-0.3136
-0.3801	-0.4588	-0.5687	-0.6087	-0.6318	-0.6482	-0.6615	-0.6735
-0.6835	-0.6918	-0.6988	-0.7048	-0.7104	-0.7158	-0.7209	-0.7255
-0.7282	-0.7194	-0.6864	-0.6199	-0.2107	-0.1550	-0.1657	

Constraints (Equality) : 0.1253E-09

No. of r-minimizations required : 6

f = 1.2007 ψ = 1.2009 Z_n = 0.1020E-02 M_n = 0.8387E-02

Note: Out of 79 inequality constraints, first 19 are boundary constraints, next 19 are curvature constraints, next 38 are constraints on line of thrust, and last 3 are the side constraints on F and θ.

F_{min} = 1.51 is in error by about 26 percent, both on the unsafe side. Apart from the magnitude of the factor of safety, the shape of the critical shear surface obtained in the present analysis is markedly different from circular.

The present example, therefore, highlights the consequences of using a priori assumption regarding shape as also of ignoring the acceptability criteria.

7.5 EXAMPLE PROBLEM 7.3

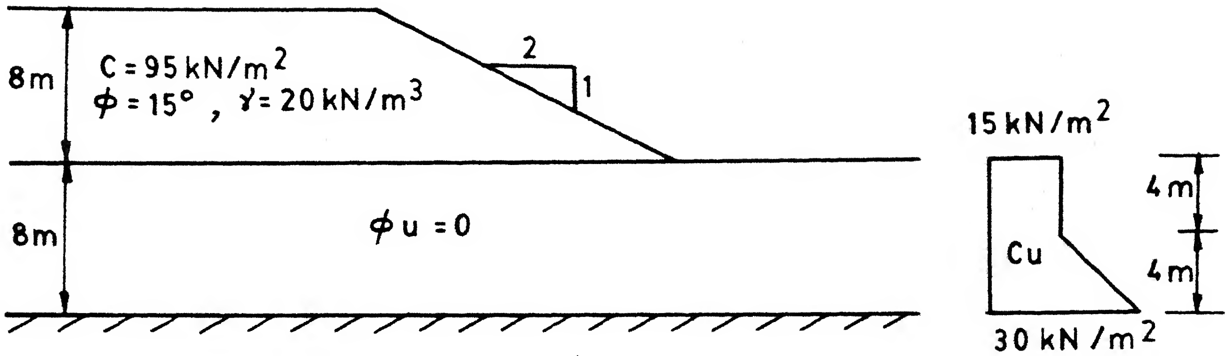


FIG. 7.8 SLOPE SECTION IN EXAMPLE PROBLEM 7.3 (B.K.Low. 1989)

7.5.1 General

As stated in Section 7.4.1, the slope shown in Figure 7.8 is the second example problem included by Low (1989) to illustrate his method of analysis of embankments on soft ground based on the assumption of circular slip surface. This example problem consists of an embankment of stiff clay, which has been constructed on soft clay. The undrained shear strength of the soft clay layer varies with depth. As in Example Problem 7.2, the results have been compared by the author with that obtained by using the computer program STABR. It has also been reported that there is some discrepancy between the solutions based on the Ordinary method of slices and the Simplified Bishop method.

In the present analysis, as in the preceding example, the proposed technique has been applied to solve the problem and the results have been compared with those reported by Low. For the sake of comparison an attempt has also been made to solve the problem using the computer program SSOPT based on the dynamic

programming technique. Further, the discrepancy in the values of factors of safety as reported by Low has been discussed.

7.5.2 Results and Discussion

Low (1989) has presented results obtained for two cases namely, $\phi = 15^{o}$ and $\phi = 0^{o}$ for the embankment material. The present analysis also deals with these two cases as such.

7.5.2.1 Case I: $\phi = 0$ for the embankment material.

(a) Analysis using Circular Slip Surface:

Before carrying out the non-circular analyses, solutions have been obtained by using both the Ordinary method of slices and the simplified Bishop method in the proposed Indirect procedure. Taking 40 slices, F_{min} has been obtained as 1.31 by both the methods. These perfectly agree with the reported value. The critical slip circle is shown in Figure 7.9. The associated normal stresses on the slice bases are found to be all positive; however these are not presented here.

(b) Analysis using Non-circular Slip Surface:

The critical slip circle obtained as discussed above has been used as the starting shear surface for the analysis with non-circular surfaces. The Direct procedure based on Spencer method has been applied. However, the procedure has been found not to converge. In order to achieve convergence the constraints on the line of thrust have been withdrawn with the result that it has now converged to a F_{min} = 1.79 but with unacceptable line of thrust and interslice forces. In an attempt to get an acceptable solution tension crack has been introduced and the depth of

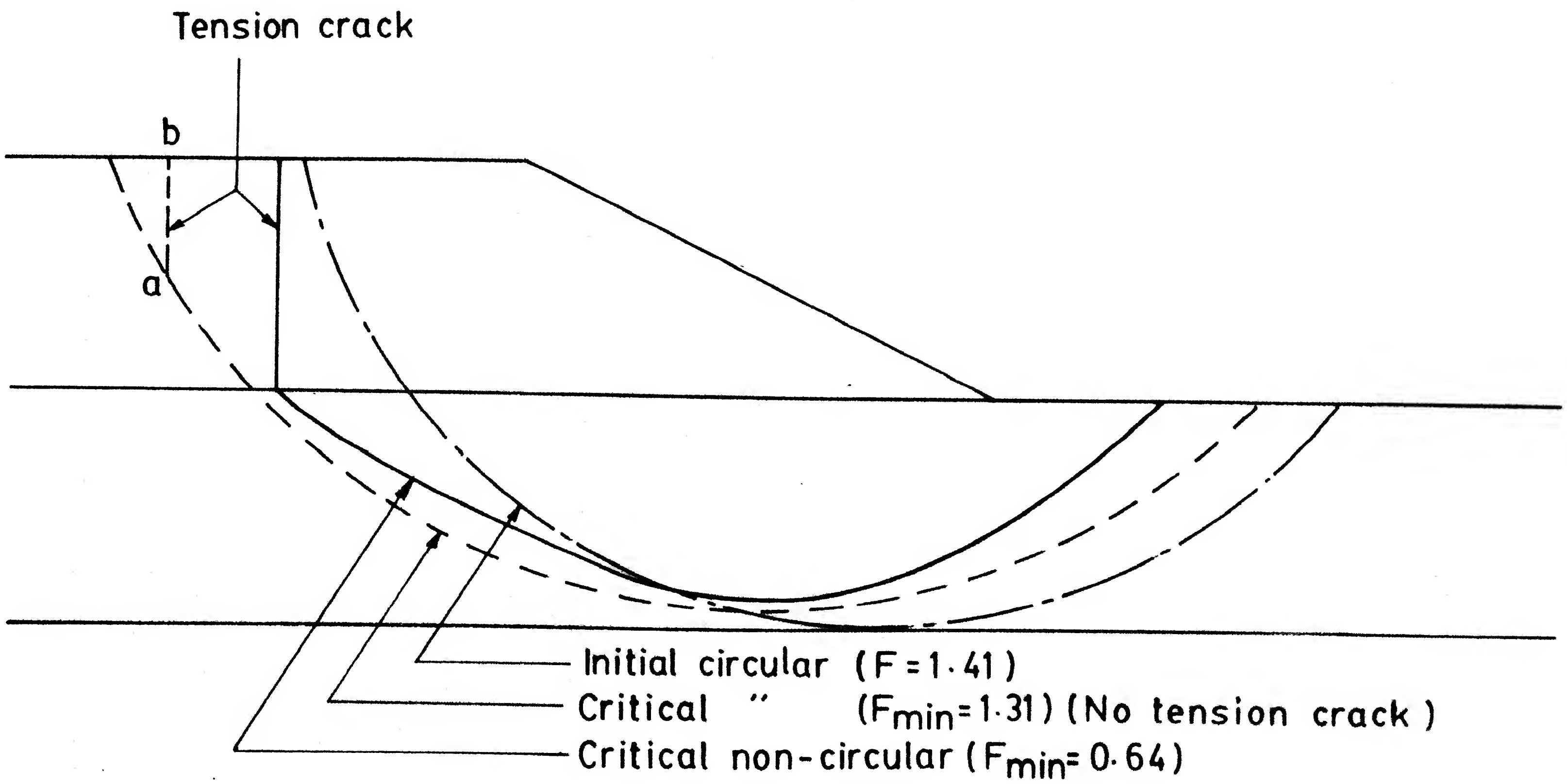


FIG. 7.9 CRITICAL CIRCULAR AND NON-CIRCULAR SHEAR SURFACES FOR THE EX. PROB. 7.3 FOR THE CASE I ($\phi = 0$)

tension crack z_t has been included in the design vector, taking the initial depth to be 4.0m. The revised upper portion of the initial surface is marked ab in Figure 7.9. However, no convergence has been possible as long as the side constraint that F cannot be less than 1.0 was imposed. Suspecting that this might be a failed slope, a lower limit of 0.5 has been tentatively used on F in place of 1.0 used earlier to get a mathematically feasible solution and at the same time save some unnecessary search. Now the solution has converged to $F_{min} = 0.64$ and the final depth of tension crack is approximately 8.0m which means the entire stiff clay layer has cracked as is the case with the Lanester embankment (Pilot et al., 1982). Further, it indicates that the slope is a failed one whereas the unacceptable solution giving $F_{min} = 1.79$ projects it to be a highly stable slope. Considering that an incipient failure condition corresponds to $F_{min} = 1.00$ (though a value of 0.64 has been obtained here), the maximum difference between the acceptable solution (say 1.0) and the unacceptable solution(1.79) in this case is an alarming 79% and on the unsafe side. The critical non-circular slip surface is also shown in Figure 7.9 and the line of thrust, the interslice forces and the stresses along this surface are shown in Table 7.11. It is seen that the line of thrust lies almost entirely within middle thirds of the heights of the interslice boundaries. The interslice forces and the stresses are all compressive and hence the solution is acceptable.

TABLE 7.11

Calculated Responses Associated with the Critical Non-Circular Shear Surface for the Example Problem 7.3 Case I(Φ = 0)

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma bH_t$
1	46.34	23.29		
			0.58	0.41
2	73.40	23.29		
			0.54	0.92
3	96.57	23.29		
			0.52	1.49
4	120.47	25.63		
			0.49	2.14
5	148.14	30.90		
			0.42	2.78
6	173.25	34.78		
			0.38	3.37
7	192.13	37.40		
			0.35	3.87
8	214.722	39.33		
			0.33	4.37
9	220.60	40.28		
			0.31	4.63
10	228.24	39.90		
			0.30	4.76
11	231.42	38.56		
			0.29	4.71
12	232.50	36.25		
			0.27	4.51
13	229.56	32.90		
			0.26	4.11
14	223.89	28.44		
			0.25	3.54
15	214.00	23.42		
			0.26	2.97
16	198.10	23.29		
			0.27	2.45
17	182.15	23.29		
			0.29	1.98
18	163.92	23.29		
			0.32	1.54
19	118.53	23.29		

7.5.2.2 Case II: $\phi = 15^{\circ}$ for the embankment material

(a) Analysis using Circular Slip Surface:

As in case I, critical slip circles have been determined using both the Ordinary method of Slices (O.M.S.) and the Simplified Bishop method. The O.M.S. gives a value of 1.36 which also agrees with that reported by Low (1989) and the Simplified Bishop method gives a value of 1.22 which is somewhat different from the corresponding reported value of 1.14. Both the critical slip circles are shown in Figure 7.10(a). However, in the solution by the Simplified Bishop method the calculated normal stresses at the bases of the last three slices towards the upper end of the slip surface (divided into 40 slices) have been found to be negative (Table 7.12). Considering the fact that the Simplified Bishop method yields a factor of safety of 1.31 for $\phi = 0$ as stated in case I, it is an apparent paradox, as pointed out by Low, that F_{min} value given by the Simplified Bishop method decreases with increasing value of ϕ for the embankment material. In an attempt to explain this discrepancy, Low (1989) states qualitatively that this might be due to the normal forces at the slice bases becoming negative. An actual calculation of the normal stress values carried out in the present analysis reveal that some such values are indeed negative and hence the solution is unacceptable. Therefore, the analysis should be carried out with the introduction of appropriate tension crack which has been done in the present analysis using non-circular or general shaped slip surface described as follows.

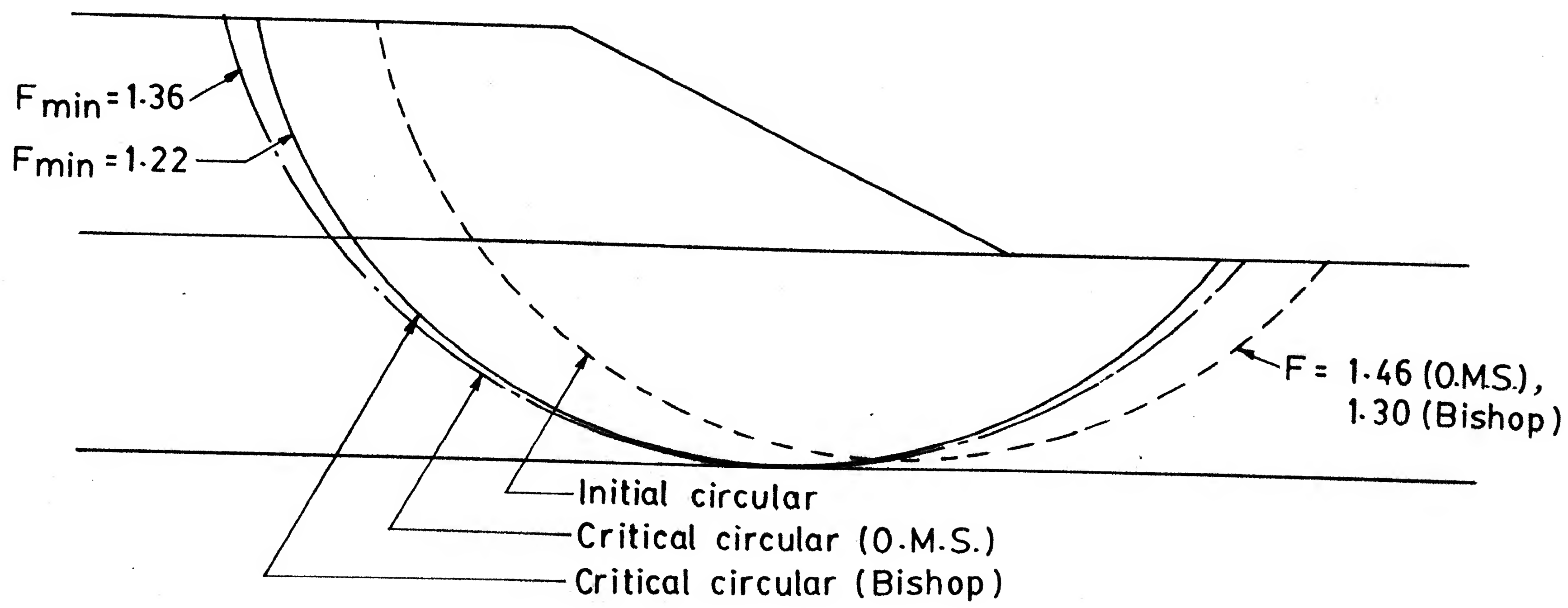


FIG. 7·10(a) CRITICAL CIRCULAR SHEAR SURFACES FOR THE EX. PROBLEM 7·3 FOR THE CASE II (ϕ=15°)

TABLE 7.12

Calculated Normal Stress Associated with the Critical Slip circle by the Simplified Bishop Method for the Example Problem 7.3 Case II ($\phi = 15^{o}$)

Slice No.	σ kPa	Slice No.	σ kPa	Slice No.	σ kPa	Slice No.	σ kPa
1	37.26	11	149.09	21	230.22	31	259.90
2	58.88	12	159.74	22	236.53	32	256.34
3	75.45	13	169.56	23	242.70	33	250.34
4	88.55	14	178.69	24	248.79	34	242.83
5	99.13	15	187.23	25	254.84	35	233.13
6	109.17	16	195.27	26	260.90	36	212.46
7	117.21	17	202.88	27	267.01	37	38.46
8	123.69	18	210.13	28	268.61	38	-25.81
9	128.90	19	217.07	29	265.96	39	-138.92
10	137.45	20	223.75	30	263.07	40	-434.59

(b) Analysis using Non-circular Slip Surface:

The critical slip circle given by the Ordinary Method of Slices as described above and shown in Figure 7.10(a) has been taken as the starting surface for the non-circular analysis using the Direct procedure. After initially facing the same kind of convergence difficulties as discussed in connection with the solution of case I i.e., for $\phi = 0$, the constraints on the line of thrust were withdrawn. The solution then converged to a F_{min} of 1.77 as against 1.79 in the $\phi = 0$ case. However, as before, the obtained line of thrust was highly unsatisfactory. To get an acceptable solution, as before, z_t has been included in the design vector with an initial value of z_t = 4.0m. The revised upper portion of the initial surface is shown as abc in Figure 7.10(b). The finally obtained convergent and mathematically feasible solution gives F_{min} as 0.63 and the depth of water-filled tension crack as approximately 8.0m. The corresponding critical slip surface is shown in Figure 7.10(b). The reason why a higher value of factor of safety has not been obtained for the $\phi = 15^{\circ}$ case is that the frictional resistance are not mobilized over the cracked portion of the embankment.

The analysis assuming that the tension crack is dry gives a higher value of F_{min} of 0.84. Table 7.13 presents the calculated responses associated with the critical shear surface corresponding to water-filled tension crack. It is seen that the line of thrust as well as other internal variables are reasonable. The design variables and the constraints at the starting and optimal points corresponding to case II ($\phi = 15^{\circ}$) and water-filled tension crack are presented in Table 7.14. The

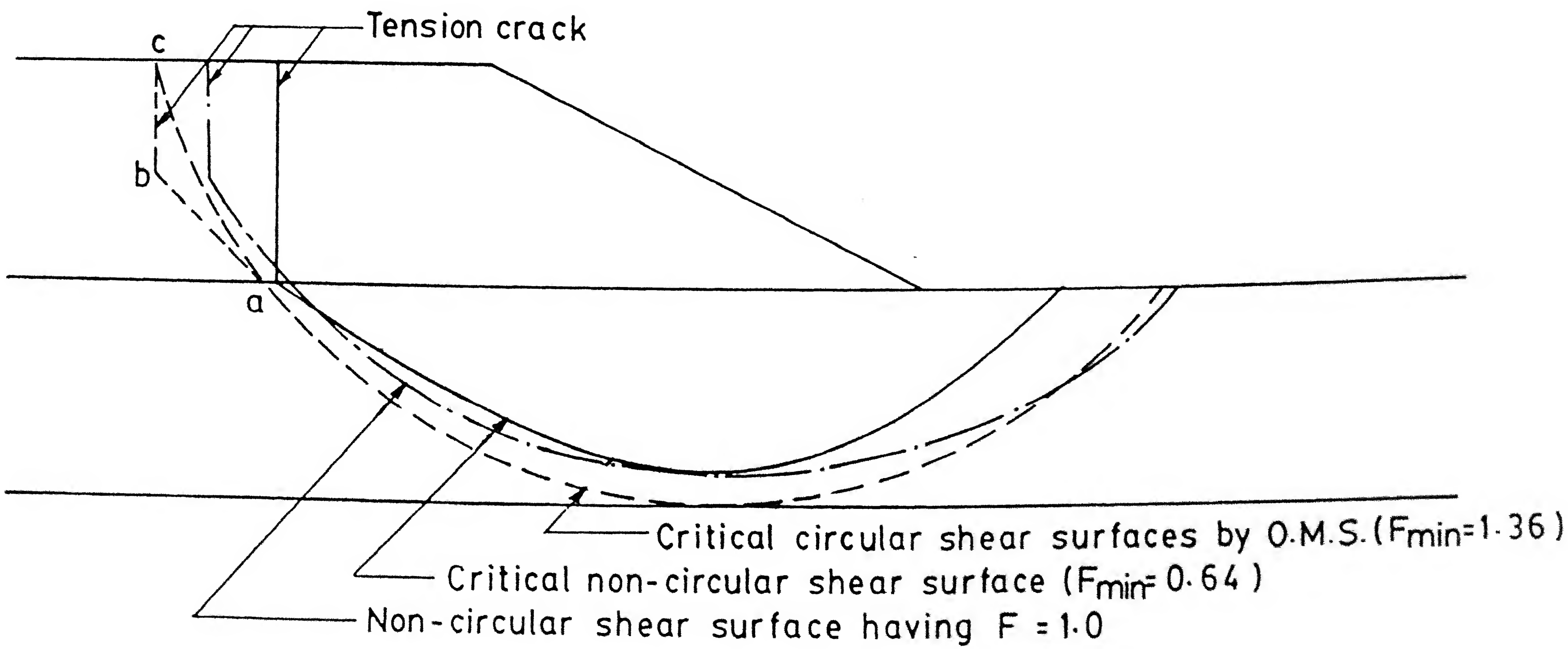


FIG. 7.10 (b) CRITICAL NON-CIRCULAR SHEAR SURFACES FOR THE EXAMPLE PROBLEM 7.3 FOR THE CASE II (ϕ = 15°)

TABLE 7.13

Calculated Responses Associated with the Critical Non-Circular Shear Surface for the Example Problem 7.3 Case II ($\Phi = 15^{\circ}$)

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma bH_t$
1	45.62	23.81		
			0.59	0.41
2	72.05	23.81		
			0.54	0.91
3	94.43	23.81		
			0.53	1.48
4	117.46	25.24		
			0.48	2.10
5	145.34	30.40		
			0.42	2.72
6	169.68	34.17		
			0.38	3.29
7	191.96	36.95		
			0.35	3.84
8	202.97	38.62		
			0.33	4.21
9	220.96	39.48		
			0.31	4.57
10	217.18	39.12		
			0.29	4.59
11	227.23	37.59		
			0.28	4.59
12	228.02	35.54		
			0.26	4.42
13	227.38	32.52		
			0.25	4.09
14	222.64	28.48		
			0.25	3.58
15	214.84	23.82		
			0.25	3.05
16	196.84	23.81		
			0.27	2.51
17	179.45	23.81		
			0.29	2.00
18	162.74	23.81		
			0.33	1.56
19	116.55	23.81		

TABLE 7.14

Design Vector and Constraints for the Example Prob.7.3 Case II(ϕ=15°)

No.of Slices = 19
No. of design variables = 23

Starting Point

F = 1.3000 θ = 0.1000
Design Variables (Not normalized) :

-1.88	-3.39	-4.61	-5.57	-6.33	-6.89	-7.27	-7.48
-7.52	-7.39	-7.09	-6.61	-5.95	-5.09	-3.99	-2.63
0.93	1.20	25.61	-12.50	1.30	0.10	4.00	

Constraints (all inequality) :

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9473	-0.8825
-0.8196	-0.7565	-0.6911	-0.6212	-0.5441	-0.4563	-0.4105	-0.3656
-0.2976	-0.1903	-0.3700	-0.2900	-0.2600	0.2000	0.2000	0.1800
-0.1700	-0.1700	-0.1700	-0.1700	-0.1800	-0.1800	-0.2000	-0.2400
-0.2600	-0.3400	-0.4300	-0.6700	-0.5535	-0.4816	-0.4625	-0.4516
-0.4326	-0.3764	-0.3374	-0.3087	-0.2865	-0.2693	-0.2665	-0.2495
-0.2575	-0.3061	-0.4602	-0.1864	1.0146	1.1640	-0.4465	-0.5184
-0.5375	-0.5484	-0.5674	-0.6236	-0.6626	-0.6913	-0.7135	-0.7307
-0.7435	-0.7505	-0.7425	-0.6939	-0.5398	-0.8644	-2.0146	-2.1640
-0.8000	-0.1000	-0.3636					

Constraints (Equality) :0.1159E+01

ε_o=-0.1 δ_t=0.0001 f=1.3000 ψ=1.30695 Z_n=-0.2406E+03 M_n= -0.2279E+04

Optimal Point

F = 0.6300 θ = 0.1519
Design Variables (Not normalized) :

-1.4087	-2.6499	-3.7414	-4.7383	-5.4765	-6.0047	6.4115	6.5655
-6.6989	-6.4449	-6.1869	-5.7546	-5.1725	4.3963	3.6060	2.7352
-1.8299	-0.9223	21.6636	-7.9699	0.6300	0.1529	-0.19E-04	

Constraints (Inequality) :

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9473	-0.8928	-0.8411
-0.7899	-0.7406	-0.6847	-0.6272	-0.5606	-0.4855	-0.3945	-0.3186
-0.2488	-0.1497	-0.1675	-0.1498	-0.0945	0.2596	-0.2084	0.1222
-0.2528	-0.0207	-0.3874	-0.0039	-0.1744	-0.1497	-0.1942	0.0140
-0.0806	-0.0344	-0.0024	-0.0146	-0.5854	-0.5423	-0.5260	-0.4843
-0.4198	-0.3784	-0.3487	-0.3263	-0.3081	-0.2912	-0.2770	-0.2638
-0.2519	-0.2436	-0.2541	-0.2686	-0.2907	-0.3256	-0.4146	0.4577
-0.4739	-0.5157	-0.5802	-0.6216	-0.6513	-0.6737	0.6920	0.7088
-0.7230	-0.7362	-0.7482	-0.7564	-0.7459	-0.7314	0.7093	0.6744
-0.1300	-0.1519	-0.3107					

Constraints (Equality) : 0.2769E-12

No. of r-minimizations required : 7
f = 0.63 ψ = 0.6302 Z_n = -0.9155E-04 M_n = 0.9079E-03

Note: Out of 75 inequality constraints, first 18 are boundary constraints, next 18 are curvature constraints, next 36 are constraints on line of thrust, and last 3 are the side constraints on F and θ. Here the lower limit of F = 0.50

previous observations that the proposed scheme effectively handles the infeasible design points as well as equality constraints are in evidence in this case also.

(c) Non-circular Shear Surface corresponding to F = 1:

An attempt has been made to find that shear surface which corresponds to a factor of safety of unity i.e., the surface corresponding to the critical equilibrium. For this purpose the objective function has been suitably chosen as,

$$f(D) = 100\ (F-1)^2$$

The multiplying factor 100 has been introduced so that the objective function value does not become too small. Except for this change in the objective function f, the Direct procedure formulated in Chapter 3 remains unchanged. As before, analysis has been carried out treating z_t as a design variable, starting from the same surface as taken earlier. As no convergence has been possible, the constraints on the line of thrust have been removed. The revised analysis has resulted into a convergent solution. The optimal shear surface so obtained is shown in Figure 7.10(b). However, as seen from Table 7.15, the associated line of thrust as well as some of the stresses and forces remain highly unsatisfactory.

Table 7.16 presents the design vector and the constraints before and after minimization corresponding to this analysis.

7.5.2.3 Solution by SSOPT

An attempt has been made to solve the present problem by the program SSOPT based on the dynamic programming technique. To represent the linear variation of its undrained strength the

TABLE 7.15

Calculated Responses Associated with the
Shear Surface Having F = 1
(Example Problem 7.3 Case II)

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma b H_t$
1	37.32	14.99		
			0.52	0.34
2	68.16	14.99		
			0.46	0.77
3	89.03	14.99		
			0.44	1.16
4	105.82	17.95		
			0.42	1.55
5	118.37	20.48		
			0.41	1.91
6	136.09	22.41		
			0.35	2.25
7	160.99	23.86		
			0.31	2.58
8	183.43	24.87		
			0.27	2.87
9	203.80	25.51		
			0.25	3.11
10	220.95	25.72		
			0.22	3.26
11	234.60	25.38		
			0.20	3.29
12	244.73	24.42		
			0.18	3.16
13	251.27	22.78		
			0.16	2.85
14	252.89	20.39		
			0.13	2.35
15	236.69	17.08		
			0.11	1.64
16	211.04	14.99		
			0.09	0.79
17	178.36	14.99		
			-0.21	-0.10
18	81.40	56.04		
			-0.30	-0.28
19	-5.14	93.62		

TABLE 7.16

Design Vector and Cosntraints in the Search for a Shear Surface Having F = 1 (Example Problem 7.3, case II)

No.of Slices = 19 Obj function $f = 100(F-1)^2$
No. of design variables = 23 Z_t is a design variable

Starting Point

f = 1.3000 θ = 0.1000
Design Variables (Not normalized) :

-1.88	-3.39	-4.61	-5.57	-6.33	-6.89	-7.27	-7.48	-7.52	-7.39
-7.09	-6.61	-5.95	-5.09	-3.99	-2.63	-0.93	1.20	25.61	-12.50
1.30	0.10	4.00							

Constraints (Inequality) :

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9473	-0.8825
-0.8196	-0.7565	-0.6911	-0.6212	-0.5441	-0.4563	-0.4105	-0.3656
-0.2976	-0.1903	-0.1968	-0.0855	-0.0564	-0.0359	-0.3159	-0.0261
-0.0234	-0.0227	-0.0226	-0.0230	-0.0254	-0.0272	-0.0336	-0.0472
-0.0652	-0.1293	-0.4624	-0.5583	-0.4000	-0.1000	-0.3636	

Constraints (Equality) : 0.1159E+01

ε_o = -0.1 δ_t=0.0001
f = 9.0000 ψ = 9.00435 Z_n = -0.2406E+03 M_n = -0.2279E+04

Optimal Point

F = 1.0000 θ = 0.0436
Design Variables (Not normalized) :

-1.0265	-1.0011	-0.9558	-0.9275	-0.9096	-0.8998	-0.8974	-0.9012
-0.9127	-0.9272	-0.9425	-0.9592	-0.9764	-0.9951	-1.0142	-1.0372
-1.1626	0.9058	0.9888	-0.7918	0.7693	0.4462		

Constraints (Inequality) :

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9556	-0.8926	-0.8320
-0.7715	-0.7094	-0.6441	-0.5749	-0.4999	-0.4185	-0.3924	-0.3552
-0.2979	-0.1943	-0.2413	-0.1331	-0.0573	-0.0326	-0.0259	-0.0189
-0.0166	-0.0140	-0.0159	-0.0231	-0.0259	-0.0296	-0.0369	-0.0540
-0.0743	-0.1201	-0.4824	-0.2168	-0.1006	-0.0436	-0.4190	

Constraints (Equality) : 0.5531E-12

No. of r-minimizations required : 4

f = 0.0000 ψ = 0.2009E-03 Z_n = 0.6104E-04 M_n = -0.2861E-03

Note: Out of 39 inequality constraints, first 18 are boundary constraints, next 18 are curvature constraints, and last 3 are the side constraints on F and θ. Lower limit of F = 0.9

lower half of the soft clay foundation layer has been divided into four sub-layers each of 1m. thickness. However, the analysis has failed to converge to any solution. The partial output concludes with the message:

"BISHOP TYPE ITERATIONS DO NOT CONVERGE"

7.5.3 Concluding Remarks

The solution to this example problem has revealed that a bias regarding the shape of the shear surface together with ignoring the acceptability criteria might be dangerous to the extent that an actually failed slope might be projected through analysis as a stable one.

7.6 EXAMPLE PROBLEM 7.4

THE SAINT-ALBAN TEST EMBANKMENT

7.6.1 Description

As already stated, the St.Alban embankment is one of the four test embankments which were brought to failure as part of a research program. Pilot et al.(1982) presented a detailed description of these tests along with the soil conditions at the sites embankment sites. The authors also presented the observed failure surfaces. Figure 7.11 shows the geometry and material properties of the St. Alban embankment, the sub-soil profile as reported by the authors. The same figure also shows the observed failure surface which, in this case, has been reported to be circular.

7.6.2 Analysis

Pilot et al. (1982) have reported about total as well as effective stress analysis of the St. Alban as well as the other three test embankments. These analyses are based on the assumption that the slip surface is circular and the Simplified Bishop method has been used for the purpose. Subsequently, Talesnick and Baker (1984) have performed total stress analysis using the program SSOPT which is based on the Spencer (1967) method for analysis of slip surfaces of general shape and the dynamic programming technique for carrying out the minimization.

In the present study, total stress analysis has been carried out using both the Direct and the Indirect procedures for general

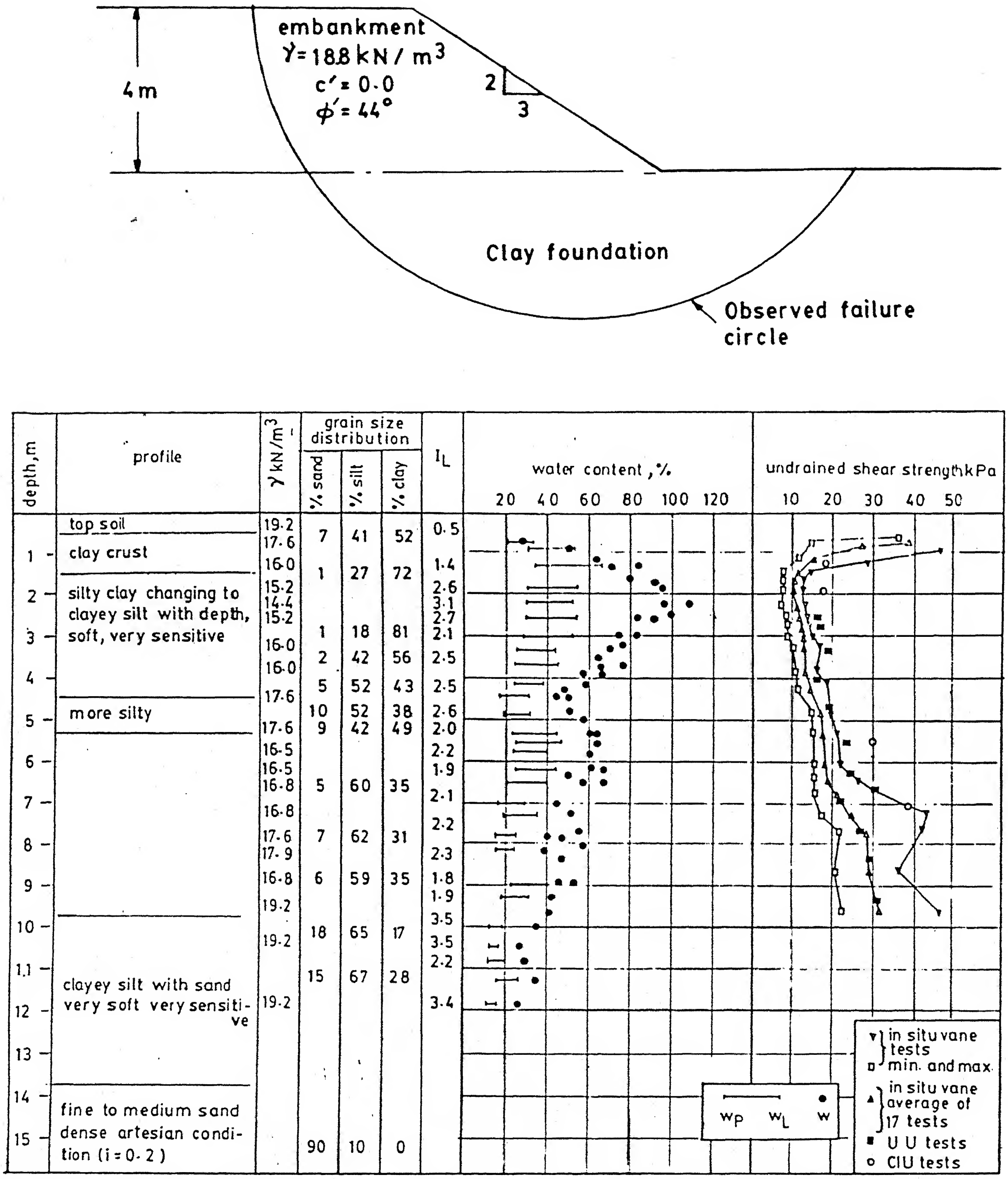


FIG. 7-11 SOIL PROFILE AND THE OBSERVED FAILURE CIRCLE FOR THE ST. ALBAN EMBANKMENT (Pilot et al., 1982)

shaped slip surfaces. The determined critical slip surfaces have been compared with the observed failure surface as well as those predicted by the previous analyses.

7.6.3 Strength profiles Used

In the present analysis, two strength profiles have been used for obtaining the results. They are, as shown in Figure 7.12,

(i) the linearly idealised average profile - the same profile was used by Talesnick and Baker (1984) in their analysis using the program SSOPT. This allows a meaningful comparison of the results obtained by using the developed procedure and the program SSOPT.

(ii) the average vane shear strength profile approximated by cubic splines. For this, the vane shear strengths have been first corrected using Mitchell's (1983) equation based on Bjerrum's (1972) recommendation, as given below :

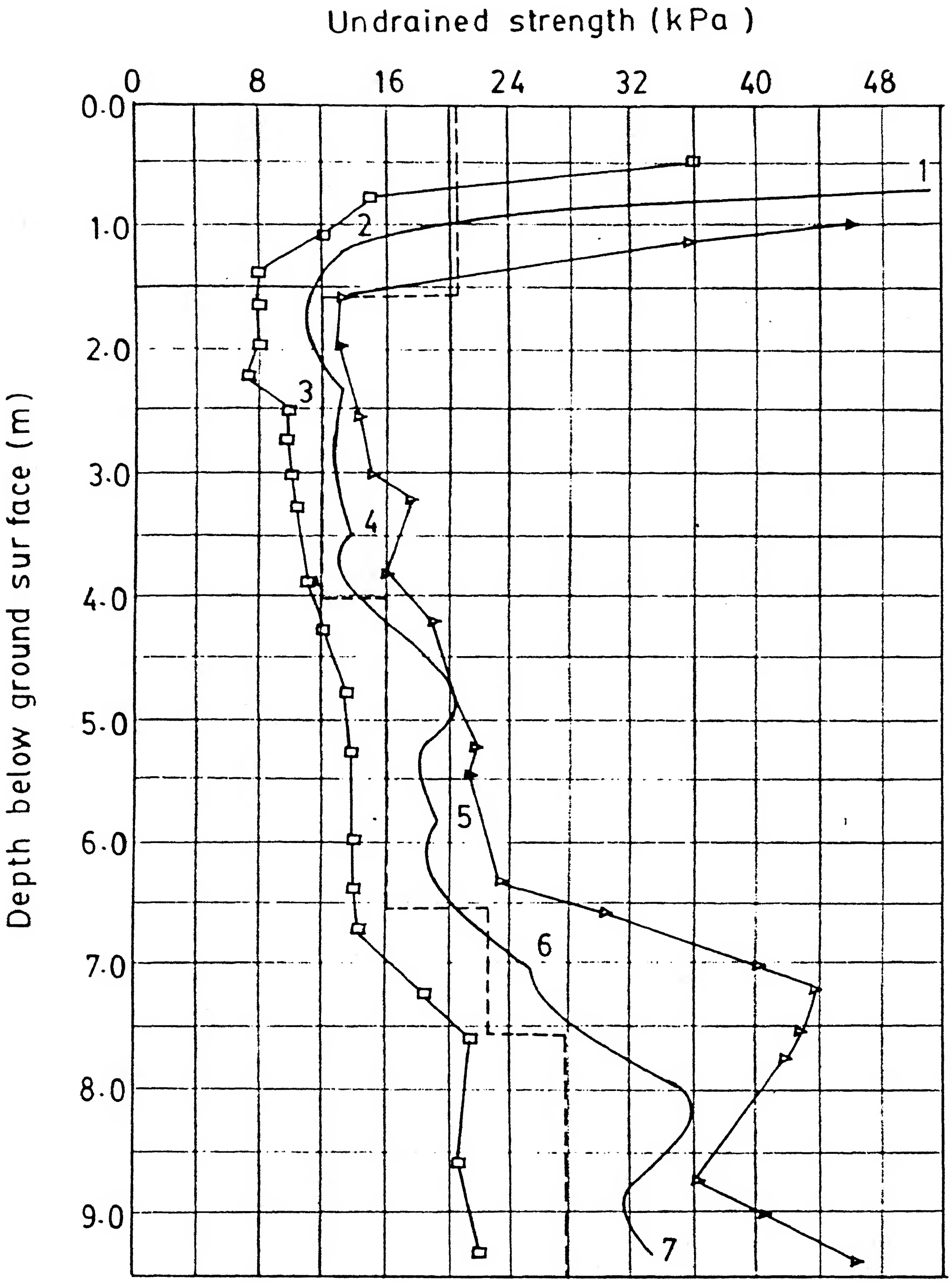
$$c_{u(corrected)} = c_{u(vane)} \left(1-0.5 \log \frac{PI}{20}\right) \qquad (7.1)$$

The corrected spline-fitted profile is obtained by joining six cubic splines marked 1-2, 2-3, 3-4, 4-5, 5-6 and 6-7 in Figure 7.12 and it is seen that it fits well the points on the corrected strength profile.

7.6.4 Results and Discussion:

7.6.4.1 Function Evaluation:

Before proceeding with the determination of the critical slip surface, the computation of the factor of safety of a given



Spline-fitted strength profile

Strength profile used by Talesnick & Baker (1984)

Minimum vane strength profile

Max. vane strength profile

FIG. 7.12 STRENGTH PROFILES CONSIDERED FOR THE SAINT ALBAN EMBANKMENT

slip surface has been checked by re-analysing the following surfaces:

1. the total stress critical slip circle reported by Pilot et al. (1982) and shown in Figure 7.13a
2. the observed failure surface (circular) shown in Figures 7.11, 7.13a and 7.13b.

For these analyses the linearised strength profile used by Talesnick and Baker (1984) has been used.

1. Re-analysis of the Critical Circular Surface reported by Pilot et al. (1982):

Solution has been attempted by using both the iterative schemes, viz., (i) the scheme suggested by Spencer and (ii) the scheme based on SUMT proposed in this thesis.

(i) *Solution using Spencer's Iterative Scheme:*

The Method of bisection has been employed to find the zeroes of Z_n and M_n. The following are the observations when several trial values of F and θ have been used. 20 slices have been considered for computations.

Trial I : F = 1.50, θ = 0.25 rad.

Result : There is no convergence. In the step 1, corresponding to θ = 0.25, F is adjusted to a value of 1.5639 till Z_n is negligible. In the step 2, corresponding to F = 1.5639, the iteration fails to adjust θ such that M_n is negligible.

Trial II : F = 1.25, θ = 0.15 rad.

Result : As before, there is no convergence. After step 1 where, corresponding to θ = 0.15, F is adjusted to a value of 1.3262 till Z_n is negligible, the iteration fails to find a θ which makes M_n negligible.

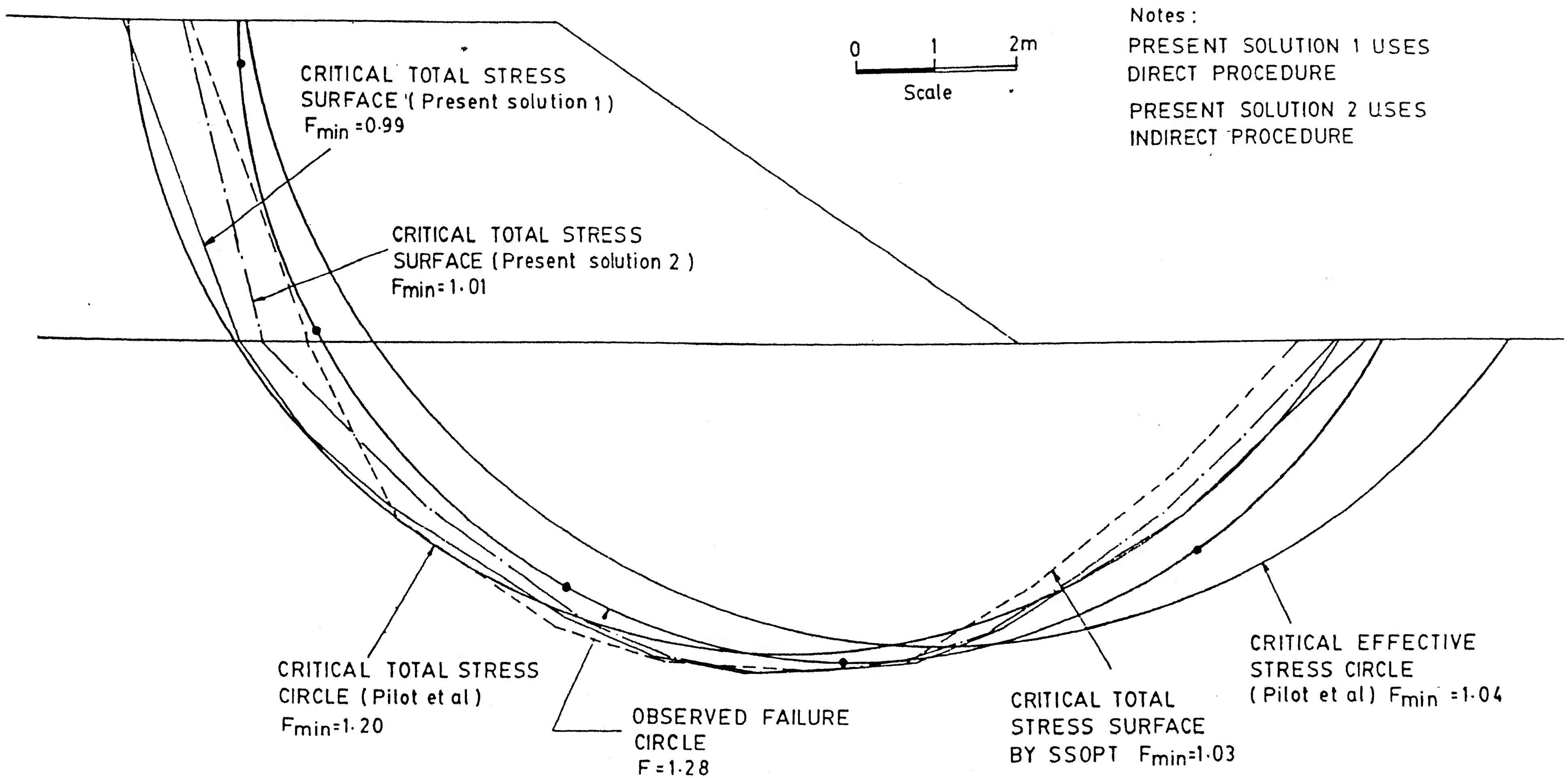


FIG. 7. 13 a . A COMPARISON OF OBSERVED AND CALCULATED SLIP SURFACES FOR THE SAINT ALBAN EMBANKMENT (Present analysis uses the same strength profile as used in the analysis by SSOPT.)

Trial III: F = 1.2Ø, θ = Ø.Ø25

Result : Finally, with these trial values for F and θ the iterations have converged to the following solution:

F = 1.1657, θ = Ø.Ø531 rad.

(ii) *Solution using the proposed scheme based on SUMT*

Result : Unlike the earlier attempted solution by Spencer's suggested scheme, convergence has been achieved in the very first attempt using the same trial values of F=1.5Ø and θ = Ø.25 rad. as taken in the 1st trial by the previous scheme. The values of F and θ have converged to:

F = 1.1659, θ = Ø.Ø53Ø rad. Corresponding

Z_n = Ø.6199E-Ø5 kN/m and M_n = Ø.2432E-Ø4 kN-m/m

The factor of safety reported by Pilot et al. (1982) is, however, 1.2Ø. The small difference between the present solution and Pilot et al.'s solution may be attributed to the possible error in scaling the slip surface as well as the subsoil strength profile from the above mentioned reference and also to the differences in the solution procedure.

2. Re-analysis of the Observed failure surface (circular)

(i) *Solution by Spencer's iterative scheme in conjunction with the Method of bisection:*

As before, several trials have been made as given below:

Trial I : F = 1.5Ø, θ = Ø.25

Result: There is no convergence. After step 1 the iteration fails to find a θ which makes M_n negligible.

Trial II : F = 1.2Ø, θ = Ø.Ø25 rad.

Result: There is no convergence as in Trial I.

Trial III: $F = 1.10$, $\theta = 0.025$ rad

Result: Again, there is no convergence. However, in this case the iteration has been found to proceed as follows:

Cycle 1:

Step 1: Corresponding to $\theta = 0.025$, F has been adjusted to 1.18 to make Z_n negligible.

Step 2: Corresponding to $F = 1.18$, θ has been adjusted to 0.066 to make M_n negligible.

Cycle 2:

Step 1: Corresponding to $\theta = 0.066$, F has been adjusted to 1.2511 to make Z_n negligible.

Step 2: Corresponding to $F = 1.2511$, in order to make M_n negligible, θ has been observed to take negative values. As a remedial measure, a control has been incorporated in the program such that if θ becomes negative, it will assume θ as zero and re-start the whole iteration process. However, this has not succeeded in this case.

(ii) *Solution using the proposed scheme:*

Results: As in the previous analysis by this method, the very first trial with $F = 1.50$, and $\theta = 0.25$ has succeeded in yielding a convergent solution having $F = 1.2091$ and $\theta = 0.0426$ rad.

Discussion:

From the two instances presented above it has been experienced that some convergence difficulties are encountered in using the iterative scheme suggested by Spencer (1973). These are discussed as follows:

Spencer's method of solution requires to find:

(i) for a given θ, that F which makes $Z_n = \emptyset$.

(ii)for a given F, that θ which makes $M_n = \emptyset$.

Each of these steps has a physical significance, and there lies the difficulty. In step (i), for instance, in order that it becomes possible to find a value of F which will satisfy force equilibrium, the value of θ must lie within some reasonable limits; this is to say that it is not possible to achieve convergence for any arbitrary value of θ. Thus, in case the initial value or, for that matter, the value of θ at any intermediate cycle is unreasonable, convergence difficulties arise in this step. Similarly, in step (ii), since it is not physically possible to satisfy moment equilibrium by iterating over θ for an unreasonable value of F, convergence difficulties arise as has been encountered in the case of the above examples. Occasionally during iteration, θ assumes negative values as observed in solving the latter example.

By several trials, one may be able to find reasonable initial guess values of F and θ such that there is no convergence difficulty in analysing that particular slip surface. This has been the case with the analysis of the Pilot's critical circle described above. The problem, however, still remains because, such difficulties may arise in any intermediate iteration in course of the determination of the critical slip surface which involves analyses of a number of trial slip surfaces. Difficulty of another kind is also encountered in the course of critical surface determination (by Indirect procedure in the case of St. Alban embankment). That is, that the pair of values (F_1, θ_1) after

convergence of step (i) and the pair of values (F_2, θ_2) after the convergence of step (ii) are found to oscillate between them. An intermediate control which has been found to work is to put

$$F = \frac{F_1 + F_2}{2} \quad \text{and} \quad \theta = \frac{\theta_1 + \theta_2}{2}$$

and restart the iterations for evaluation of the concerned trial surface.

Thus it is seen that it is imperative to use intermediate controls; however these controls do not guarantee convergence in all situations. On the other hand, with the proposed method, no such difficulty has been encountered even when started from wild guess values, as has been observed in the two examples described above. The superiority of the proposed scheme over the Spencer-suggested scheme is due to the following:

(a) In contrast to the Spencer's scheme, the proposed scheme aims in each step at finding the minimum of a hypothetical function $f = Z_n^2 + S_f.M_n^2$, and, therefore, it does not have to satisfy statics at the intermediate steps necessarily.

(b) As has already been observed in solving Example problem 6.2 the solutions obtained by the proposed scheme are not starting point dependent.

(c) The pattern search strategy involved in the Powell's method far outweighs the extra computational effort in calculating the derivatives required for evaluating the scale factor S_f in the proposed scheme. This results in higher efficiency of the proposed scheme.

7.6.4.2 Critical Surface Determination:

Results have been obtained by using both the Direct and the Indirect procedures using Spencer's method. In both the cases, in order to have a good starting point, the critical slip circle reported by Pilot et al. (1982) has been taken as the starting surface. As described in the previous section the factor of safety of this surface corresponding to the linear strength profile used by Talesnick and Baker (1984) and presented in Figure 7.12, has been obtained as 1.17. For the nonlinear spline-fitted strength profile (Figure 7.12), the factor of safety of the same surface has been obtained as 1.44 which is much higher than the expected value of unity at incipient failure condition. This is due to the high strength values near the surface in the zone of weathered, overconsolidated clay crust. La Rochelle et al. (1974) indicated that in order to obtain a safety factor of 1.0 by the Simplified Bishop method the shear strength measured in the clay crust had to be considerably reduced. Considering this, a modified profile has been used in which the top 1.5m of the spline-fitted profile has been replaced by the corresponding portion of the linear profile mentioned above. The factor of safety now obtained usingthis modified shear strength profile is 1.17 which is reasonable. The same profile has, therefore, been used in the critical surface determination. This profile is also shown in Figure 7.15.

Figure 7.13(a) shows the following slip surfaces and the associated factors of safety:

(a) observed failure circle as reported by La Rochelle et al. (1974).

(b) Critical slip circle based on total stress analysis reported by Pilot et al. (1982).

(c) Critical slip circle based on effective stress analysis reported by Pilot et al. (1982).

(d) Critical non-circular slip surface based on total stress analysis using the program SSOPT, as reported by Talesnick and Baker (1984).

(e) Several total stress critical non-circular slip surfaces obtained in the present analysis namely,

(i) Critical slip surface obtained by using the Indirect procedure and the linear strength profile used by Talesnick and Baker.

(ii) Critical slip surface obtained by using the Direct procedure and the linear strength profile used by Talesnick and Baker.

Figure 7.13(b) shows the following slip surfaces and the associated factors of safety:

(e) Surfaces described under (a) and (d) above.

(f) Total stress critical non-circular slip surfaces obtained in the present analysis by using the Direct procedure and the corrected and modified spline-fitted connected strength profiles shown in Figures 7.12 and 7.15.

The factor of safety values for various cases are presented in Table 7.17. Table 7.18 presents the line of thrust, the resultant side forces and the normal stresses at the bases of the slices corresponding to the critical slip surface obtained by using the Direct procedure and the spline-fitted (modified) strength profile after applying Bjerrum's correction. As can be seen from the table, the location of the line of thrust is

TABLE 7.17

Comparison of Factors of Safety for the St. Alban Embankment

Sl. No.	Designation of the Surface	Number of Slices Used	Strenght Profile	Factor of Safety	Remarks
1.	Critical total stress circular surface obtained by Pilot et al. (1982)				
	(a) Results reported by Pilot et al. (1982)	Not reported	Not clearly reported	1.20	
	(b) Evaluation of F.O.S. for the same surface by Talesnick and Baker (1984)	Not reported	linear	1.20	
	(c) Evaluation of F.O.S. for the same surface in the present analysis				
	(i)	14	linear	1.17	modified spline-fit refers to profile (2) of Fig.7.15
	(ii)	14	Spline-fit	1.44	
	(iii)	14	modified Spline-fit	1.17	
2.	Critical effective stress circular surface obtained by Pilot et al. (1982)	Not reported	Not clearly reported	1.04	
3.	Critical total stress non-circular surface obtained by Talesnick and Baker (1984)	Not reported	linear	1.03	
4.	Critical total stress non-circular surface obtained in the present analysis				
	(a) direct formulation				
	(i)	14	linear	0.99	
	(ii)	14	modified Spline-fit	1.03	
	(b) Indirect formulation				
	... ...	14	linear	1.00	
5.	Observed failure surface				
	(a) Evaluation of F.O.S. by Talesnick and Baker (1984) using SSOPT ...	Not reported	linear	1.28	
	(b) Evaluation of F.O.S. in the present study				
	(i)	20	linear	1.21	
	(ii)	20	Spline-fit	1.37	
	(iii)	20	modified Spline-fit	1.23	

TABLE 7.18

Calculated Responses Associated with Critical Shear Surface Obtained by the Direct Procedure and Modified Spline-fitted Strength Profile for the St. Alban Embankment

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma bH_t$
1	34.54	19.90		
			0.54	0.83
2	41.55	15.53		
			0.54	1.62
3	51.95	12.62		
			0.52	2.34
4	65.27	12.62		
			0.46	3.20
5	81.72	13.34		
			0.39	3.93
6	92.38	13.40		
			0.34	4.30
7	101.80	13.59		
			0.30	4.43
8	106.61	13.00		
			0.27	4.20
9	106.77	13.40		
			0.25	3.57
10	109.39	12.62		
			0.24	2.87
11	102.82	12.62		
			0.24	2.16
12	82.33	11.36		
			0.30	1.06
13	35.78	9.07		
			0.49	0.22
14	6.93	6.50		

reasonable and that the interslice forces and normal stresses at slice bases are all positive as they should be.

Typical design vector and constraints at the starting and the optimum points are presented in Table 7.19 corresponding to the analysis by Direct procedure using the modified spline-fitted shear strength profile.

Figure 7.14 shows the path followed by the optimization scheme in the determination of the critical surface for the case mentioned above. It is seen that the minimization of the penalty function ψ has led to the minimization of f, the objective function and hence the desired convergence has been achieved.

The following observations may be made from the Figure 7.13(a,b) and Tables 7.17, 7.18 and 7.19.

1. The proposed technique, though not restricted to a circular mode of failure, yields critical slip surfaces which may be considered to be approximately circular. The obtained surfaces compare well with those reported by various investigators who used different techniques.

2. Total stress analysis using the proposed Direct procedure and the modified spline-fitted strength profile after applying corrections gives a factor of safety of 1.03 which is identical with that reported by Talesnick and Baker (1984) for total stress analysis using the program SSOPT and is essentially the same (1.04)as that reported by Pilot et al.(1982), based on an effective stress analysis adopting the Simplified Bishop method. However, it should be noted that the obtained critical slip surfaces in these cases are not identical even though they yield

TABLE 7.19

Typical Design Vector and Constraints in the Analysis of the St. Alban Embankments

No.of Slices = 14
No. of design variables = 17

Starting Point

$F = 1.2000 \quad \theta = 0.2000$
Design Vector (Not normalized) :

-1.44	-2.4247	-3.1231	-3.6051	-3.906	-4.0439	-4.0262	-3.8521	
-3.5119	-2.9844	-2.2287	-1.1615	0.4307	10.65	-5.90	1.2	0.025

Constraints (Inequality) :

-1.0000	-1.0000	-1.0000	-1.0000	-0.9990	-0.8987	-0.8027	-0.7052
-0.6003	-0.4809	-0.4348	-0.3774	-0.2647	-0.4553	-0.2863	-0.2164
-0.1811	-0.1630	-0.1556	-0.1564	-0.1661	-0.1873	-0.2282	-0.3115
-0.5250	-1.9771	-0.5103	-0.5056	-0.4776	-0.4521	-0.3639	-0.3036
-0.2576	-0.2202	-0.1877	-0.1736	-0.1529	-0.1618	0.0640	-0.4897
-0.4943	-0.5224	-0.5479	-0.6361	-0.6964	-0.7424	-0.7798	-0.8123
-0.8263	-0.8471	-0.8382	-1.0640	-0.2000	-0.0150		

Constraints (Equality) : 0.0665E+00
$\varepsilon_o = -0.1 \quad \delta_t = 0.001$

$f = 1.2000 \quad \psi = 1.2220 \quad Z_n = -0.2464E+00 \quad M_n = 0.0882E+00$

Optimal Point

$F = 1.0350 \quad \theta = 0.0351$

Design Vector (normalized) :

-0.1693	-0.3308	-0.4434	-0.5560	-0.6289	-0.6502	-0.6439	-0.601155
-0.5174	-0.4292	-0.3341	-0.1660	0.1660	1.5711	-0.6849	0.517552
0.2724							

Constraints (Inequality) :

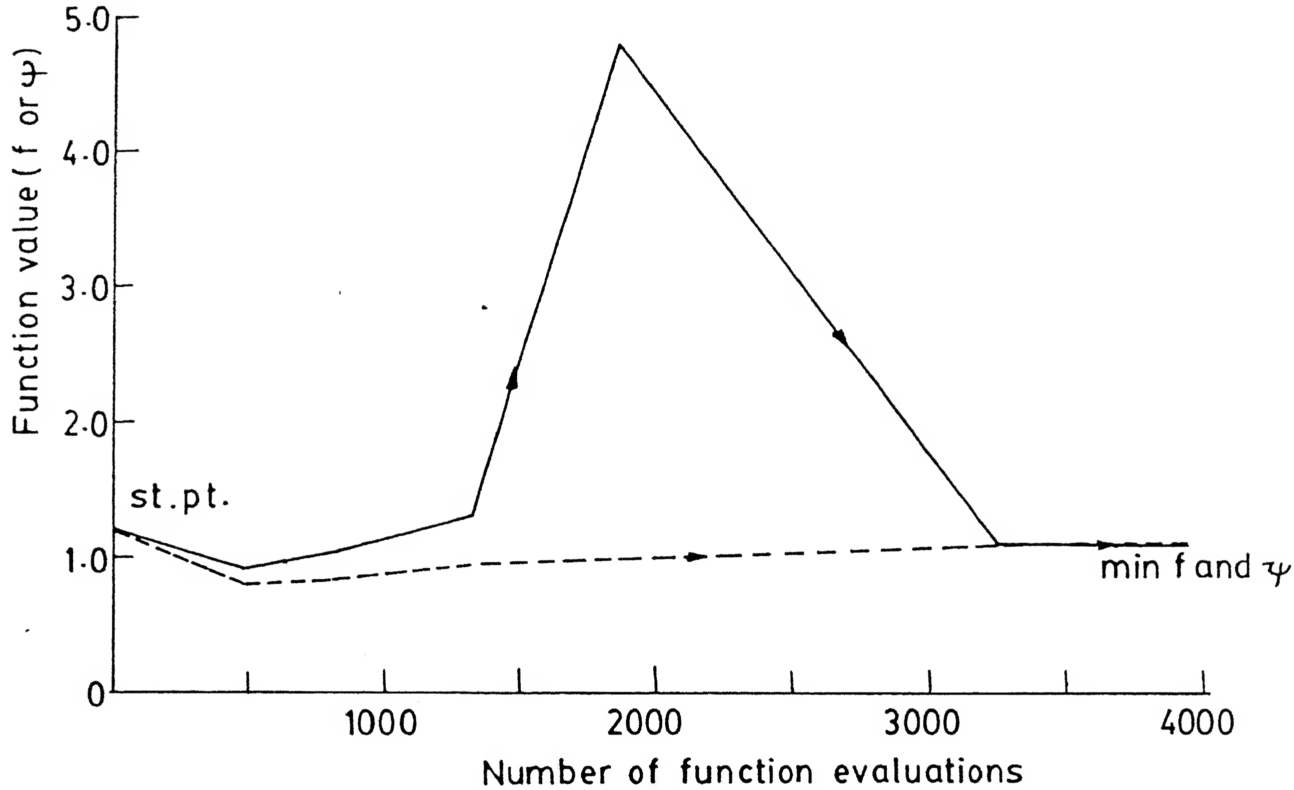
-1.0000	-1.0000	-1.0000	-1.0000	-0.9320	-0.8571	-0.7746	-0.6941
-0.6062	-0.4948	-0.3994	-0.3316	-0.2025	-0.0466	-0.2933	-0.17E-03
-0.2384	-0.3094	-0.1657	-0.2186	-0.2458	-0.0272	-0.0410	-0.4380
-0.9834	-1.0121	-0.5382	-0.5406	-0.5191	-0.4585	-0.3860	-0.3392
-0.3028	-0.2719	-0.2430	-0.2276	-0.2360	-0.2929	-0.4073	-0.4618
-0.4594	-0.4809	-0.5415	-0.6140	-0.6608	-0.6972	-0.7281	-0.7570
-0.7724	-0.7640	-0.7071	-0.5927	-0.0351	-0.0701		

Constraints (Equality) : 0.1815E-07

No. of r-minimizations required : 6

$f = 1.0350 \quad \psi = 1.0370 \quad Z_n = 0.8715E-02 \quad M_n = 0.4886E-02$

Note: Out of the 54 inequality constraints, first 13 are boundary constraints, next 13 are curvature constraints, next 26 are constraints on line of thrust, and last 2 are the side constraints on F and θ.



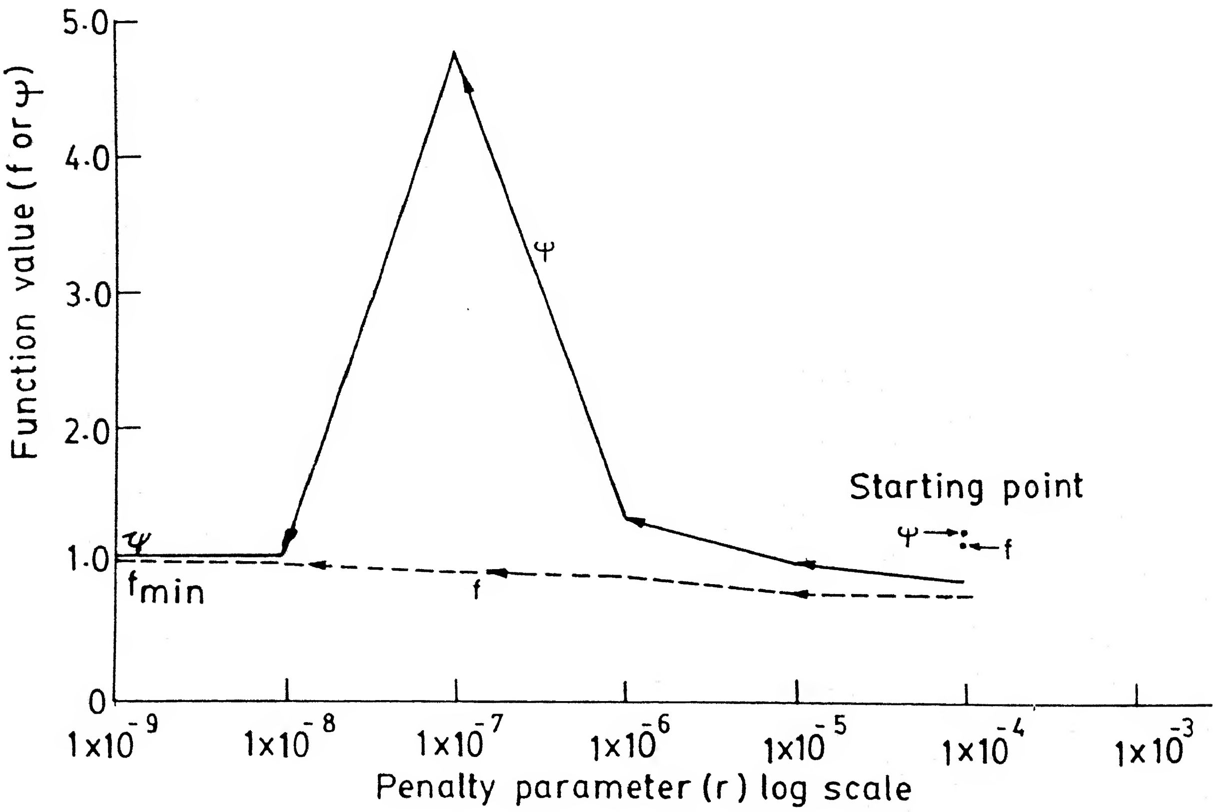


FIG.7·14 PROGRESS OF MINIMIZATION FOR THE EX.PROB. 7·4
(Direct procedure and Spline - fitted strength profile)

almost identical factor of safety values. Using Direct procedure and linear strength profile as adopted by Talesnick and Baker. the factor of safety obtained in present analysis is 0.99 signifying that the slope is a failed one and that the proposed method of analysis has good predictability. The factor of safety value obtained by using the Indirect procedure and the same profile is 1.0. As such, it is established that both the proposed procedures namely the Direct and the Indirect procedures of slope stability analysis in conjunction with Spencer's method (1973) for factor of safety computations and SUMT for minimization provides excellent results.

3. The developed methodology has the provision to compute the factor of safety for a given slip surface. This provision has been utilized to compute the factor of safety for the total stress critical circle reported by Pilot et al.(1982). The obtained value is 1.17 corresponding to both the concerned shear strength profiles as against 1.20 reported by Pilot et al. (1982) and Talesnick and Baker (1984).

It is noteworthy that the factor of safety of a correctly monitored failure surface cannot be greater than unity if the geomechanical parameters used in the analysis are accurate. The obtained higher factor of safety may be attributed to inaccuracies in the geomechanical parameters assuming that the failure surface has been determined accurately.

4. The positions of the line of thrust and the non-negativity of the normal stresses acting on the bases of the slices and the

interslice forces, as seen from Table 7.18, all these indicate that the solution is an acceptable one.

5. Results obtained by using the developed computer program based on SUMT agree well with those obtained by using the program SSOPT based on the dynamic programming technique.

7.6.4.3 Sensitivity of F to the Shear Strength Profile

As already presented, the value of the safety factor computed for the observed failure surface both in the present analysis and in the reported study by Talesnick and Baker (1984) are 1.21 and 1.28 respectively when using the same idealised linear strength profile as shown in Figure 7.12. These values are far from the value of unity which signifies incipient failure condition. It has been indicated (La Rochelle et al., 1974) that reduced values of the shear strengths measured in the clay crust should be considered in order to obtain a value close to unity. The following modifications to the spline-fitted profile obtained after Bjerrum's correction, shown in Figure 7.15, have been tried in an attempt to match the factor of safety by Spencer's method with 1.0. For the modified profiles 1,2,3 and 4 shown in Figure 7.15 the obtained factor of safety values are 1.37, 1.23, 1.14 and 1.06 respectively. Such sharp variation indicates that the factor of safety is quite sensitive to the shear strength profile used in the analysis.

It is seen from the above that the modified profile 4 gives a factor of safety (1.06) which is quite close to unity. A fresh analysis has been carried out to determine the critical slip surface using this modified strength profile. The obtained

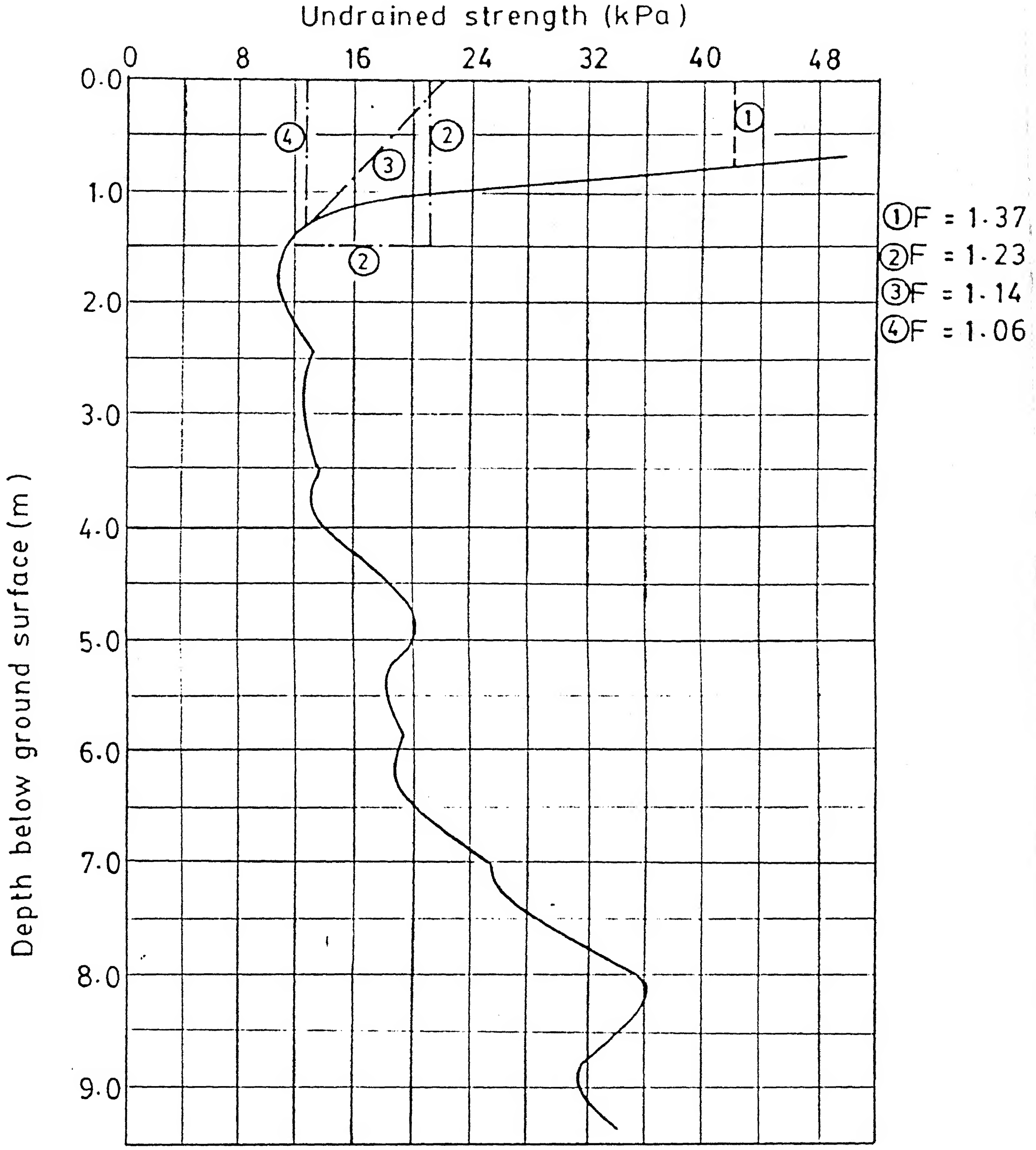


FIG. 7.15 STUDY OF SENSITIVITY OF FACTOR OF SAFETY WITH THE SUB-SOIL STRENGTH PROFILE

surface corresponds to a factor of safety of 1.Ø (the lower limit for F set in the program) and it shows a better prediction with the observed failure surface at the two outcrops, compared to the previous analysis. This surface is also plotted in Figure 7.13(b).

7.7 EXAMPLE PROBLEM 7.5

THE LANESTER TEST EMBANKMENT

7.7.1 Description

Like the St. Alban Embankment described in the preceding section, the Lanester Embankment is also a test embankment which was brought to failure as part of a research program. The embankment material is a compacted sandy clayey gravel. It has been reported that the slide was preceded by lateral displacements that caused the formation of vertical cracks in the embankment. Pilot et al. (1982) presented the observed failure surface which, in this case also, has been reported to be circular. The authors also presented a detailed description of the subsoil conditions at the embankment site. Figure 7.16 shows the geometry and the material properties of the Lanester Embankment, the sub-soil profile and also the observed failure circle.

7.7.2 Analysis

As in the case of St. Alban Embankment, the previous analyses of the Lanester Embankment consist of total and effective stress analyses based on the Simplified Bishop method as reported by Pilot et al. (1982) and a total stress analysis using the program SSOPT as reported by Talesnick and Baker (1984).

In the present study, total stress analysis has been carried out using the Direct Spencer procedure for general shaped slip surfaces. As before, the predicted critical surface has been

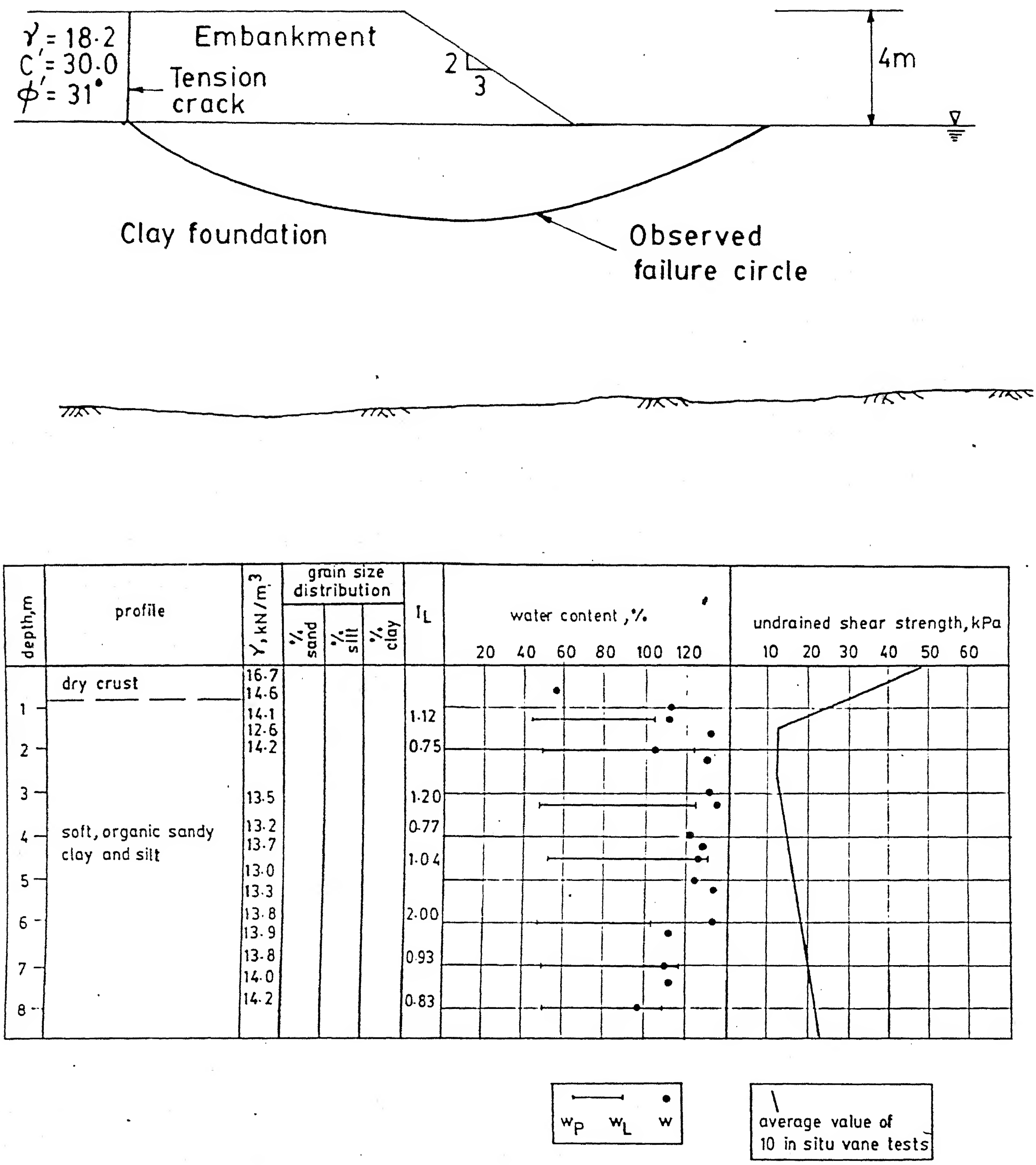


FIG. 7·16 SOIL PROFILE AND THE OBSERVED FAILURE CIRCLE FOR THE LANESTER EMBANKMENT (Pilot et al. 1982)

compared with the observed failure surface as well as with those predicted from the previous analyses.

7.7.3 Strength profiles used

The undrained shear strength profiles for the clay foundation, which have been used in the present analysis of the Lanester Embankment, are the following:

(i) the idealised profile consisting of linear segments as used by Talesnick and Baker (1984) in their analysis using the program SSOPT. This is marked as profile 2 in Figure 7.17.

(ii) the actual average vane shear strength profile reported by Pilot et al. (1982) and approximated by three straight lines. This is marked as profile 1 in Figure 7.17.

(iii) the profile formed after applying Bjerrum's correction to the profile (ii) above. This is marked as profile 3 in Figure 7.17.

7.7.4 Results and Discussion

In the search for critical non-circular surface by the Direct procedure, the critical circular slip surface reported by Pilot et al. (1982) has been taken as the starting surface, to take advantage of a good starting point. Because of the formation of the tension crack over the full-height of the embankment, all shear surfaces terminate at the bottom of these cracks. Factor of safety has been computed for the starting surface. The factor of safety varies somewhat depending upon whether the tension crack is assumed to be dry or filled with water and the number of slices used in the computation.

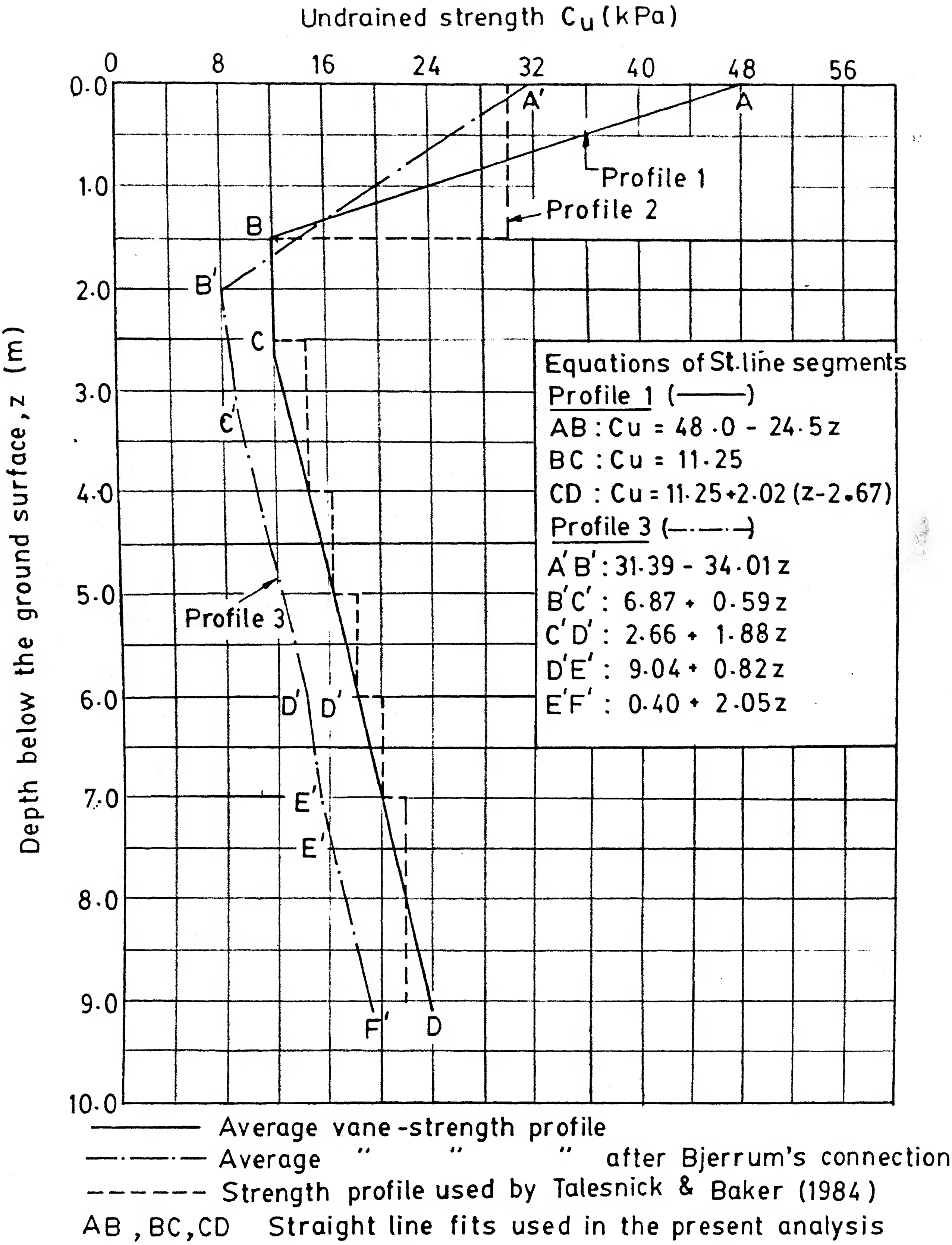


FIG. 7.17 STRENGTH PROFILES CONSIDERED FOR THE LANESTER EMBANKMENT

Figure 7.18 shows the following slip surfaces and the associated factors of safety:

(a) Observed failure circle as reported by La Rochelle et al (1974).

(b) Critical circle based on total stress analysis, reported by Pilot et al. (1982).

(c) Critical circle based on effective stress analysis, reported by Pilot et al. (1982).

(d) Critical total stress non-circular slip surface using the program SSOPT, as reported by Talesnick and Baker (1984).

(e) Several total stress critical non-circular slip surfaces obtained in the present analysis, namely,

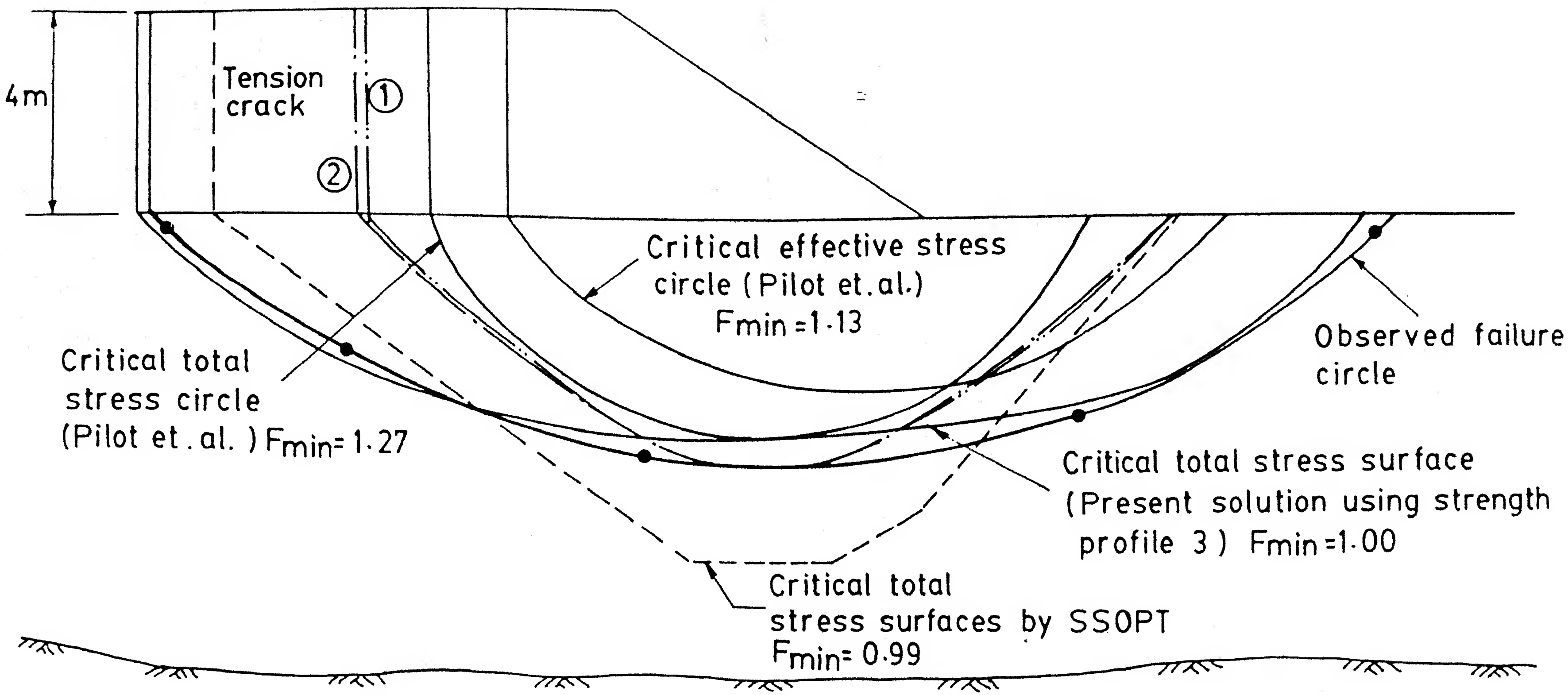
(i) Critical slip surface obtained by using the strength profile (profile 2 in Figure 7.17) used by Talesnick and Baker (1984).

(ii) Critical slip surface obtained by using the linearly idealised actual average strength profile (profile 1 in Figure 7.17).

(iii) Critical slip surface obtained by using the corrected profile (profile 3 in Figure 7.17).

The factor of safety values for various cases are separately presented in Table 7.20. The following observations may be made from Figure 7.18 and Table 7.20:

1. The F_{min} values reported by Pilot et al. (1982) using total and effective stress analysis based on Simplified Bishop method are 1.27 and 1.13 respectively which are much higher than unity. Using the strength profile used by Talesnick and Baker (1984) in



Note :-

① Criticle total stress surface by present analysis using strength profile 1 (Fig. 7·17)
② Criticle " " " " " " " " " 2 (Fig. 7·17)

① F_{min} = 1·05 ② F_{min} = 1·09

FIG. 7·18 A COMPARISON OF OBSERVED AND CALCULATED SLIP SURFACES FOR THE LANESTER EMBANKMENT

TABLE 7.20

Comparison of Factors of Safety for the Lanester Embankment

Sl. No.	Designation of the Surface	Number of Slices used	Strength Profile	Ten-Crack Assumed Dry/Filled with Water	Factor of Safety	Remarks
1.	Critical total stress circular surface obtained by Pilot et al. (1982)					Strength profile 1 and 2 refer to Figure ST 7
	(a) Results reported by Pilot et al. (1982)	Not reported	1	Not reported	1.27	
	(b) Evaluation of F.O.S. for the same surface by Talesnick and Baker (1984)	Not reported	2	Not reported	1.19	
	(c) Evasluation of F.O.S. for the same surface in the present analysis					
	(i)	14	2	Filled	1.28	
	(ii)	21	2	Filled	1.21	
	(iii)	21	2	Dry	1.32	
	(iv)	14	1	Filled	1.22	
	(v)	21	1	Filled	1.19	
2.	Critical effective stress circular surface obtained by Pilot et al. (1982)	Not reported	1	Not reported	1.13	
3.	Critical total stress non-circular surface obtained by Talesnick and Baker (1984)	Not reported	2	Not reported	0.99	
4.	Critical total stress non-circular surface obtained in the present analysis-ing					The surface correspond-ing case (v) shows the best agreement with the observed failure surface
	(a) direct formulation					
	(i)	14	2	Filled	1.09	
	(ii)	21	2	Filled	1.10	
	(iii)	14	1	Filled	1.05	
	(iv)	21	1	Filled	1.12	
	(v)	14	3	Filled	1.00	
	(vi)	14	3	Dry	1.00	
5.	Observed failure surface					
	(a) Evaluation of F.O.S. by Talesnick and Baker (1984)... ...	Not reported	2	Not reported	1.30	
	(b) Evaluation of F.O.S. in the present analysis					
	(i)	21	1	Filled	1.24	
	(ii)	21	3	Filled	0.97	
	(iii)	21	3	Dry	1.10	

their analysis by the program SSOPT, results obtained by using the proposed technique show a F_{min} value of 1.09 in contrast to the value of 0.99 reported by the authors. This discrepancy could partly be due to the error in scaling the strength profile from the above reference. Using strength profile marked 1 in Figure 7.17 i.e., the average vane shear strength profile, the proposed technique gives a F_{min} of 1.05.

2. Examination of the obtained critical slip surface reveals that it resembles the well known failure mechanism for a footing on a half-space consisting of active, radial and passive zones, where the active and passive zones are bounded by straight lines. The obtained critical surfaces are similar in shape to the surface reported by Talesnick and Baker; however, they are shallower than the latter surface. It should be noted that the "observed" failure surface does not resemble such a mechanism of bearing capacity failure likely to occur in a fully cracked embankment such as the Lanester Embankment. The failure surface predicted in the present analysis is close to the observed failure surface only over a small length in the central portion but is much closer to the total stress critical surface reported by Pilot et al. Pilot et al.'s effective stress critical circle is quite far off from both the observed and the present failure surfaces.

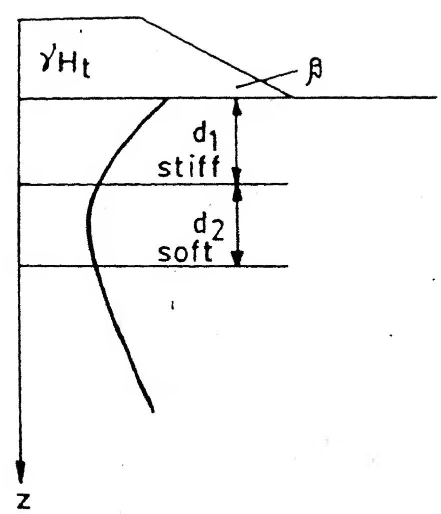
3. It has been pointed out by Talesnick and Baker (1984) that their predicted failure surface has little resemblance to the "observed" one and that it is not clear to them how the observed failure surface whose shape does not appear to be reasonable was

determined. In addition, the factor of safety of the observed failure circle has been obtained by them as 1.3 which is very high. The factor of safety computed in the present analysis using the strength profile 1 is 1.24 which is also very high.

4. However, when the strength profile 3 (shown in Figure 7.17, which has been obtained after applying Bjerrum's correction to the average vane strength profile 2) is used, not only that the predicted failure surface gives a factor of safety of 1.0, it is in remarkably close agreement with the observed failure surface especially at the two outcrops. In the central portion, however, the predicted surface is somewhat shallower than the observed one. For this analysis, the tension crack in the embankment has been assumed to be water-filled. Analysis carried out assuming the tension crack as dry also gives a surface with a F_{min} value of 1.0; however, in this case the agreement with the observed failure surface is not so close and hence it has not been presented in Figure 7.18. Considering the fact that it is most probable that the outcrops of the failure surface at the ground surface has been determined with sufficient accuracy, the prediction in the former case i.e., applying Bjerrum's correction and assuming water pressure in the tension crack appears to be very good. The above indicates that such correction may make very significant difference in the results.

Conventional Method:

Stability of embankments on soft grounds has traditionally been estimated by treating it as a bearing capacity problem. Figure 7.19 shows the effect of stratigraphy on the bearing



Key

	Reference	PI	S_t
×	Wilkes (1972)	30	—
△	Pilot (1972) (four cases)	Variable	8
o	Ladd (1972)	15	10
▽	Dascal et al (1972)	40	10
■	Eide and Holmberg (1972)	-	-
+	Raymond (1972)	40	-
▲	Lo and Stermac (1965)	21	Varved

o LI = 1·8
cot β = 4

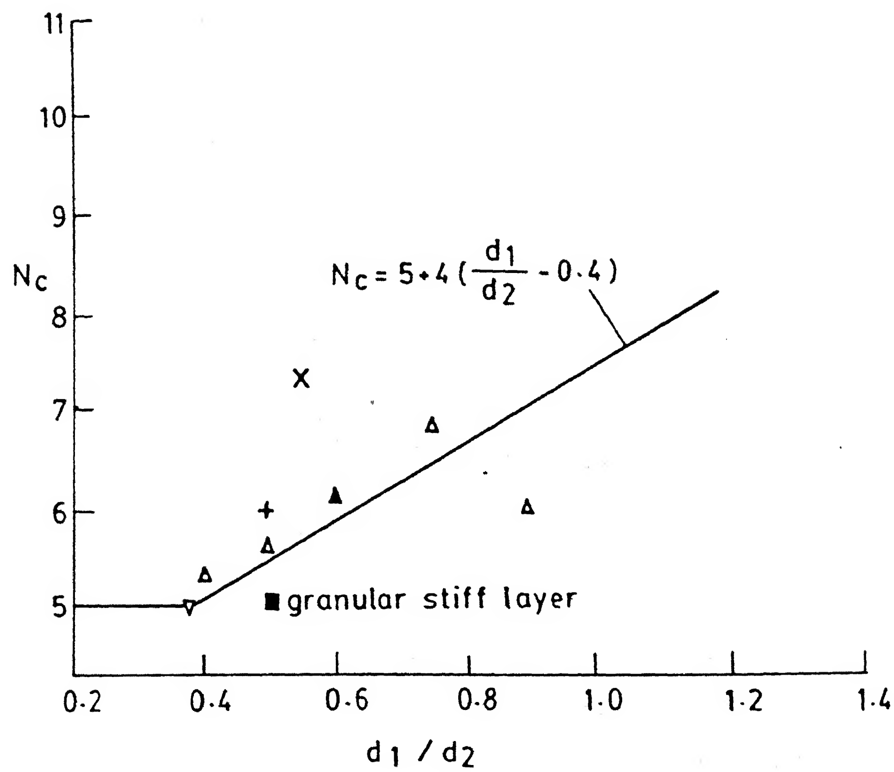


FIG. 7·19 STRATIGRAPHIC EFFECTS ON THE BEARING CAPACITY FACTOR N_c (Reproduced from Mitchell, 1982)

capacity factor, N_c, (Mitchell, 1982). The values of d_1 and d_2 defined in Figure 7.19 for the geotechnical profile of the Lanester embankment is estimated as 1.5m and 1.2m respectively and the corresponding N_c value obtained from figure 7.19 is Ø.84. The average undrained shear strength, c_u, of the foundation material is 11.25 kN/m^2. Then, the factor of safety of the embankment is estimated from the expression: $F = \frac{c_u N_c}{\gamma H_t}$ as 1.18 which is marginally higher than the solution obtained by using more refined analysis. However, when Bjerrum's corrections are applied, the obtained factor of safety following this approach is Ø.84, indicating that the embankment is a failed one which it is actually. This also demonstrates the importance of the correction to the vane shear strength as suggested by Bjerrum.

Table 7.21 present the locations of the line of thrust and the magnitudes of the interslice forces associated with the critical slip surface obtained in the present analysis using the corrected vane shear strength profile. As can be seen, the location of the line of thrust is reasonable in the sense that it is almost within the middle-third of the heights of the interslice boundaries. It is also seen that the normal stress and the interslice forces are all positive and, therefore, admissible.

Table 7.22 presents the design vectors and constraints at the starting and optimal points. The observations regarding the effectiveness of the proposed numerical scheme in handling equality constraints and in achieving proper convergence, made in earlier cases, hold for this case also.

TABLE 7.21

Calculated Responses Associated with the Critical Shear Surface Obtained by the Direct procedure using the Corrected Vane strength Profile for the Lanester Embankment

Slice No.	σ kPa	τ kPa	L/H	$Z/\gamma bH_t$
1	39.11	20.70		
			0.53	1.15
2	39.13	8.25		
			0.56	1.74
3	46.32	8.69		
			0.55	2.14
4	51.05	9.26		
			0.54	2.50
5	54.25	9.79		
			0.54	2.81
6	63.66	10.16		
			0.43	3.11
7	86.52	10.40		
			0.34	3.39
8	108.29	10.48		
			0.29	3.58
9	124.09	10.30		
			0.27	3.51
10	122.00	9.82		
			0.27	3.27
11	115.25	9.07		
			0.28	2.84
12	103.73	8.50		
			0.30	2.09
13	87.85	10.15		
			0.37	1.23
14	58.91	23.97		

TABLE 7.22

Design Vector and Constraints in the Analysis of the Lanester Embankment

No.of Slices = 14
No. of design variables = 17

Starting Point

$F = 1.2500 \qquad \theta = 0.1000$

Design Variables (Not normalized) :

-1.5219	-2.4834	-3.1577	-3.6343	-3.9543	-4.1394	-4.2000	-4.1394	
-3.9543	-3.6343	-3.1578	-2.4836	-1.5221	9.2807	-3.6807	1.25	0.10

Inequality Constraints :

-1.0000	-1.0000	-1.0000	-0.9979	-0.9148	-0.8355	-0.7561	-0.6730
-0.5823	-0.4782	-0.4412	-0.3830	-0.2756	-0.5604	-0.2872	-0.1977
-0.1566	-0.1349	-0.1245	-0.1212	-0.1245	-0.1349	-0.1565	-0.1977
-0.2873	-0.5606	-0.5305	-0.5792	-0.5714	-0.5611	-0.4739	-0.4149
-0.3728	-0.3424	-0.3220	-0.3143	-0.3575	-0.4776	-1.1745	-0.4695
-0.4208	-0.4286	-0.4389	-0.5261	-0.5851	-0.6272	-0.6576	-0.6780
-0.6857	-0.6425	-0.5223	0.1745	-0.2501	-0.1000	-0.5109	

Equality Constraints : 0.3541E+01

$\varepsilon_o = -0.1 \qquad \delta_t = 0.001$

$f = 1.2500 \quad \psi = 1.3073 \quad Z_n = -0.8953E+02 \quad M_n = -0.4065E+03$

Optimal Point

$F = 1.000 \qquad \theta = 0.0672$

Design Variables (Not normalized) :

-1.8307	-2.8582	-3.3487	-3.6820	-3.9117	-4.0755	-4.1663	-4.1637
-3.9706	-3.6517	-3.1793	-2.3677	-1.2709	14.4159	-9.3387	
0.0682							

Inequality Constraints :

-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-0.9263	-0.7780	-0.6254
-0.4944	-0.1793	-0.4557	-0.4168	-0.3140	-0.8031	-0.5371	-0.1572
-0.1036	-0.0659	-0.0729	-0.0934	-0.1906	-0.1258	-0.1535	-0.3391
-0.2852	-0.1740	-0.5317	-0.5585	-0.5507	-0.5433	-0.5377	-0.4292
-0.3446	-0.2888	-0.2697	-0.2741	-0.2822	-0.3040	-0.3712	-0.4683
-0.4415	-0.4493	-0.4567	-0.4623	-0.5708	-0.6554	-0.7112	-0.7303
-0.7259	-0.7178	-0.6960	-0.6288	-0.32E-03	-0.0672	-0.5426	

Equality Constraint : 0.5644E-10

No. of r-minimizations required : 4

$f = 1.0000 \quad \psi = 1.0026 \quad Z_n = 0.6628E-03 \quad M_n = 0.2205E-02$

Note: Out of the 55 inequality constraints, first 13 are boundary constraints, next 13 are curvature constraints, next 26 are constraints on line of thrust, and last 3 are the side constraints on F and θ respectively.

7.8 CONCLUSIONS

On the basis of the studies of the example problems belonging to stable and failed embankments on clay foundations conducted in the chapter the following conclusions are drawn:

1. The SUMT-based technique developed and tested for slopes in homogeneous soils, zoned dams and embankments has proved to be equally effective and efficient in the case of embankments on clay foundations having highly irregular sub-soil undrained shear strength profile.

2. The proposed technique has been successfully applied to locate the critical slip surface using both the Direct and the Indirect procedures coupled with methods suitable for circular as well as general shear surfaces.

3. The proposed equation solving technique has successfully analysed a given shear surface and converged to unique set of values of F and θ when different starting points have been used. For the same cases, the original scheme due to Spencer has either not converged or displayed strong starting point dependence.

4. The values of the minimum factor of safety yielded by the proposed procedures are somewhat less than or equal to the reported values obtained by using other techniques and/or methods; however the corresponding critical slip surfaces are appreciably different; this indicates two things :-

(i) the critical shear surfaces are much more sensitive to the methods used to determine them as compared to the corresponding minimum factors of safety.

(ii) the existence of a critical zone.

5. It has been demonstrated that if in the analysis the acceptability criteria are ignored and at the same time a priori assumptions are made regarding the shape of the shear surface without proper consideration to the physics of the problem, the consequence could be disastrous to the extent that a failed slope might be projected as a sufficiently safe and stable slope. In such an extreme situation, the difference between the acceptable and unacceptable solutions could be a very alarming 80%; however, in other cases this difference has been observed to be below 30%; on both occasions the unacceptable solutions are on the unsafe side.

6. In the case of well-documented failed slopes it has been observed that a total stress analysis can also predict the status of stability as accurately as an effective stress analysis if

(i) the analysis is carried out without any a priori assumption regarding the geometry of the failure surface and

(ii) the in-situ vane shear strength profile for the sub-soil is corrected.

7. The predicted failure surfaces are found to be quite sensitive to the sub-soil undrained shear strength profile. The predicted failure surface has been found to be in closer agreement with the observed failure surface when the strength profile used is subjected to Bjerrum's correction.

8. The developed methodology has proved to be a versatile tool for slope stability analysis. Its versatility lies in the fact

that apart from finding the critical shear surface it can be used to find the shear surface which corresponds to a known factor of safety by suitably modifying the objective function. The latter formulation may be utilized to advantage in the back-analysis of slopes.

CHAPTER 8

THE SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE IN THE BACK ANALYSIS OF SLOPE FAILURES

8.1 INTRODUCTION

Studies of failures carried out in the case of natural and man-made slopes generally involve assessment of reasonable or appropriate values for the geomechanical parameters for the soil at the instant of critical equilibrium (F=1) with a view to recommending further design or remedial works. The geomechanical parameters are the shear strength parameters (c', ϕ'), the pore pressure coefficient (r_u) or the set of co-ordinates defining the piezometric surface and the soil unit weight (γ) for all the concerned strata. In the case of man-made slopes such an assessment also helps to verify the geomechanical parameters previously adopted in the design.

The difficulty, however, lies in the fact that very often the shear strength parameters determined in the laboratory cannot explain the actual slide and as such attention has been focussed on the prediction of the geomechanical parameters on a theoretical basis. Towards this end, an inverse formulation is usually adopted in which the failure surface is treated as known while the concerned geomechanical parameters are treated as the unknowns. The factor of safety is set equal to 1.0. corresponding to the incipient failure or the critical equilibrium state. The failure surface is normally delineated from the results of field inspection of surface cracks and signs of slope movements, or

from information on the critical shear surface normally determined during the design stage, or by surveying the actual failed slope. In some situations, especially those involving poorly defined failure mechanisms, monitoring instruments such as inclinometers and shear strips are installed in slopes before complete failure occurrence to follow the failure process and to define the failure surface (Nguyen, 1984b). Since in this inverse formulation the unknowns normally present in the routine slope stability analyses are regarded as known and vice-versa, the procedure is popularly referred to in the literature as back analysis.

In the past, efforts were generally limited to the back calculation of only two geomechanical parameters namely, the shear strength parameters c′ and ϕ'. But even then previous methods do not provide unique solutions. As Sauer and Fredlund (1988) has commented, "it is not possible to back analyse a landslide and obtain a unique combination of effective cohesion and effective angle of internal friction". Recently, Nguyen (1984a,b) applied nonlinear programming approaches namely, the Simplex reflection technique and the Secant method to the back calculation of slope failures; reportedly such techniques also do not provide unique solutions. There has been some attempts towards finding a unique solution. Saito (1980) has recommended a numerical-graphical procedure for determination of c′ and ϕ'. However, his procedure is based on the Ordinary method of slices applicable to circular failure surface only and it is only valid for homogeneous slopes.

Yamagami et al. (1987) have also presented a procedure which is also based on nonlinear programming techniques for the back analysis of strength parameters, formulating the problem as a constrained optimization problem with mixed equality and inequality constraints. In this procedure, any arbitrary set of values for c′ and ϕ′ are chosen to start with and the corresponding critical shear surface is determined. Now, unless the chosen c′ and ϕ′ are true, the critical shear surface so determined will not match with the known failure surface. In other words, those values of c′ and ϕ′ which result in a critical shear surface coinciding with the failure surface are the true or actual values. The whole idea is that in order to ensure a correct solution the back analysed parameters should not only result in a factor of safety of 1.0 but also a critical slip surface which perfectly matches with the actual failure surface. This is a development over the early practice of arbitrarily varying c′ and ϕ′ in order to match the given value of factor of safety of 1.0, completely disregarding the fact that several combinations of c′ and ϕ′ are possible all of which will yield the same factor of safety of 1.0 but each combination will result in a different critical slip surface some of which may be widely different from the actual failure surface.

To pick the right combination, the early practice was to apply engineering judgement. However, the best way to find the most accurate set of values is to match the obtained critical slip surface with the actual failure surface. Dhawan (1986) has also stressed on this point. Based on several studies conducted using the program SUMSTAB (Satyam Babu, 1986), the author has

observed that the critical slip surface of a slope is most sensitive to c′ and relatively less sensitive to ϕ', r_u etc. He, therefore, has argued that the conventional procedure of back analysis by arbitrarily varying c′ and ϕ' to match F = 1.0, without matching the surface, is wrong.

In the nonlinear programming problem formulation by Yamagami et al.(1987), the objective function has been chosen as the sum of the squares of the difference between the corresponding ordinates of the given failure surface and the critical shear surface resulting from an arbitrary choice of c′ and ϕ'. The method has been claimed to be a versatile one. However, the procedure has been illustrated with the help of the total stress analysis of a simple homogeneous slope using the Ordinary method of slices. If, by their procedure a nonhomogeneous slope is to be back analysed using a sophisticated limit equilibrium method for general slip surfaces, the formulation and hence the computation is likely to be much more involved. Even for homogeneous slopes, lot of computations will be required by this method as it involves search for several trial critical shear surfaces during the optimization process. This is certainly not unavoidable considering that the shear surface is known and hence there is no need to iterate over a number of trial (critical) shear surfaces.

Thus, it is obvious that there is still need for an effective and efficient procedure for back analysis. In this chapter, an attempt has been made to develop an algorithm based on the SUMT to carry out a correct back analysis. The methods proposed are presented in the following sections.

8.2 PRINCIPLES OF THE PROPOSED METHODS OF BACK ANALYSIS

Two different methods of back analysis have been proposed herein and their effectiveness and limitations have been discussed on the basis of a few illustrative examples. Method I is based on those limit equilibrium methods of analysis which require solution of two nonlinear equations for the evaluation of the factor of safety, e.g., the Spencer's method (Spencer, 1973). Method II, unlike Method I, is based on all those limit equilibrium methods which requires solution of only one equation, e.g., the Janbu's GPS (Janbu, 1957, 1973), the Simplified Bishop method (Bishop, 1955) etc. Formulations and solution techniques of the Methods I and II are presented in the following sections.

8.3 DESCRIPTION OF METHOD-I

8.3.1 Problem Formulation :

In Chapter 3, the problem of finding F and θ associated with a given shear surface by a complete equilibrium method such as the Spencer's method has been formulated as one of nonlinear optimization as follows:

$$\text{Minimize } f(D) = \frac{Z_n^2 + S_f\, M_n^2}{(\gamma\ b\ H_t)^2}$$

Subject to some side constraints on the two design variables F and θ, the symbols having their usual meanings. For the solution of this optimization problem, as already stated, SUMT employing Powell's method of unconstrained minimization has been successfully used.

In the above mentioned forward or conventional analysis, the geometry of the slope and the shear surface as well as the geomechanical parameters, c', ϕ', r_u are known and hence Z_n and M_n become functions of F and θ only. Now, if F and θ associated with a given surface are known, e.g., F = 1 for an actual failure surface, Z_n and M_n become functions only of the geomechanical parameters if the geometry is kept constant. Following the same principle, therefore, the back analysis problem can also be formulated as one of nonlinear optimization as follows:

$$\text{Find } \mathbf{D} = \left[c', \phi', r_u, \ldots\right]^T$$

such that

$$f(\mathbf{D}) = Z_n^2 + S_f M_n^2 \longrightarrow \text{Min.}$$

subject to the constraints :

$$c'_{min} \le c' \le c'_{max}$$

$$\phi'_{min} \le \phi' \le \phi'_{max}$$

$$r_{u_{min}} \le r_u \le r_{u_{max}}$$

where the suffix min. and max. refer to the anticipated minimum and maximum values of the unknown parameters respectively, which may be obtainable from shear test results, visual examination and experience. These side constraints are useful in guarding against the design variables assuming too unreasonable values apart from saving a lot of unnecessary computations by restricting the search zone.

out earlier using the given values for the geomechanical parameters). With respect to the given slip surfaces, the slopes are stable i.e., they have values of the safety factors greater than unity.

Since the procedure is exactly the same for both failed and stable slopes, it does not make any difference whether the slopes in the Example problems are failed slopes or not. In both the cases the slip surface and the corresponding factor of safety are known. But to validate the proposed numerical method it has been preferred to find the factor of safety corresponding to a given slip surface and geomechanical parameters and then apply the proposed method of back analysis to check whether it is possible to get back the same values of geomechanical parameters for that slip surface and the previously obtained F and θ from forward or conventional analysis.

8.5

EXAMPLE PROBLEM 8.1

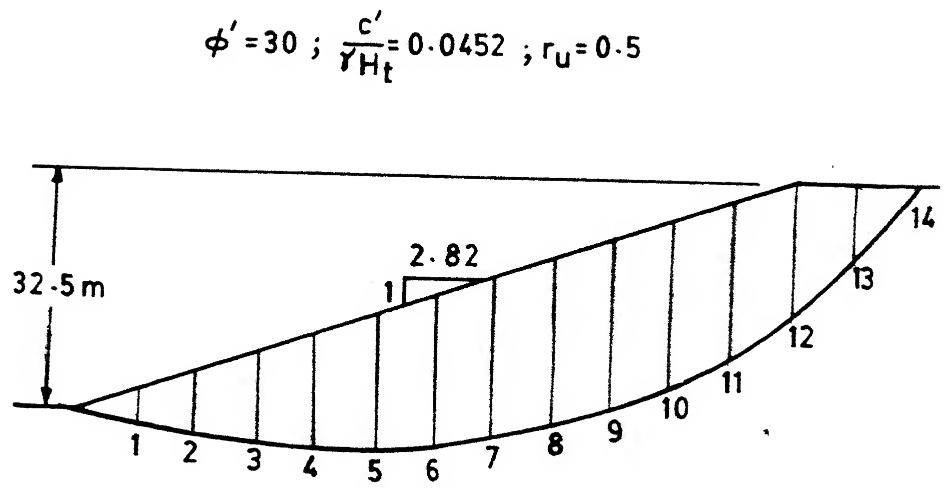


FIG. 8.1 SLOPE SECTION IN EX. PROBLEM 8.1 (Spencer, 1973)

8.5.1 General

The slope section in the Example problem 8.1 is shown in Figure 8.1 and has been taken from the illustrative examples in Spencer's (1973) paper. The soil and pore pressure properties are shown in the Figure 8.1 itself. Assuming γ = 20 kN/m^3, c' for H_t = 32.5m is calculated as 29.38 kPa.

The non-circular shear surface shown in the figure was analysed by Spencer (1973) using the method developed by him. The results of the analysis, corresponding to the assumption of parallel interslice forces i.e., k=1 throughout, as reported by the author, are as follows:

$$F = 1.42$$

$$\theta = 0.25 \text{ rad.}$$

where the symbols have their usual meanings. Using the

proposed equation-solver for Spencer's method, formulated in Chapter 3, the above values have been obtained as

$$F = 1.42297$$

$$\theta = 0.250076 \text{ rad.}$$

These results have already been reported in Chapter 4; however, for ready reference they have also been presented here.

8.5.2 RESULTS AND DISCUSSION

8.5.2.1 General

Design variables have been normalized as follows unless otherwise mentioned

Variable	c'	ϕ'	r_u	θ
	kPa	deg.	-	rad.
Normalized form	$\frac{c'}{0.1\gamma H_t}$	$\frac{\pi\phi'}{180}$	r_u	$\frac{\theta}{\beta/2}$

where H_t and β are the height and inclination of the slope respectively.

The side constraints used are as follows unless otherwise mentioned :

$$0.0 \leq c' \leq 100.0$$

$$\phi' \geq 0.0$$

$$0.0 \leq r_u \leq 0.55$$

$$0.0 \leq \theta \leq \beta$$

8.5.2.2 Two-Parameter Back-analysis Case I: c', ϕ'

To start with, a 2-parameter back analysis has been attempted. The two parameters considered are the effective cohesion c' and the effective angle of shearing resistance ϕ'. It is assumed that the pore pressure coefficient r_u and the unit weight γ are both known quantities for this particular analysis. The knowledge of the shape and location of the shear surface (Figure 8.1) as well as the associated F and θ values are, as discussed in the formulation, the essential pre-requisites for the back analysis. The design variables in this case are c' and ϕ' and they have been normalized for use in the optimization subroutines.

The results of the back analysis together with the actual values of c' and ϕ' (which have been used in the forward analysis earlier to find F and θ) and their initial values used in the minimization scheme are presented in Table 8.1. It is seen that the back analysed values of the parameters are in excellent agreement with their actual values. The function values Z_n and M_n before and after minimization are also presented.

Figure 8.2 shows the path followed by the objective function f and the composite function ψ with both the number of function evaluations and the penalty parameter r. The orders of magnitude of the objective function f and the composite function ψ are such that for all practical purposes both of them are zero. Such closeness in values of f and ψ signifies that there has been no ill-conditioning during the progress of minimization.

TABLE 8.1

Results of a Two parameter (c', ϕ') Back Analysis by Method I of the Slope Section in Example Problem 8.1

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c' (kPa)	10.0	29.382	29.38	0.0007%
ϕ' (deg.)	15.0	30.001	30.00	0.0003%
Z_n (kN/m)	-0.6633E+04	0.2798E-01	-	-
M_n (kN-m/m)	-0.6497E+05	-0.9516E-01	-	-

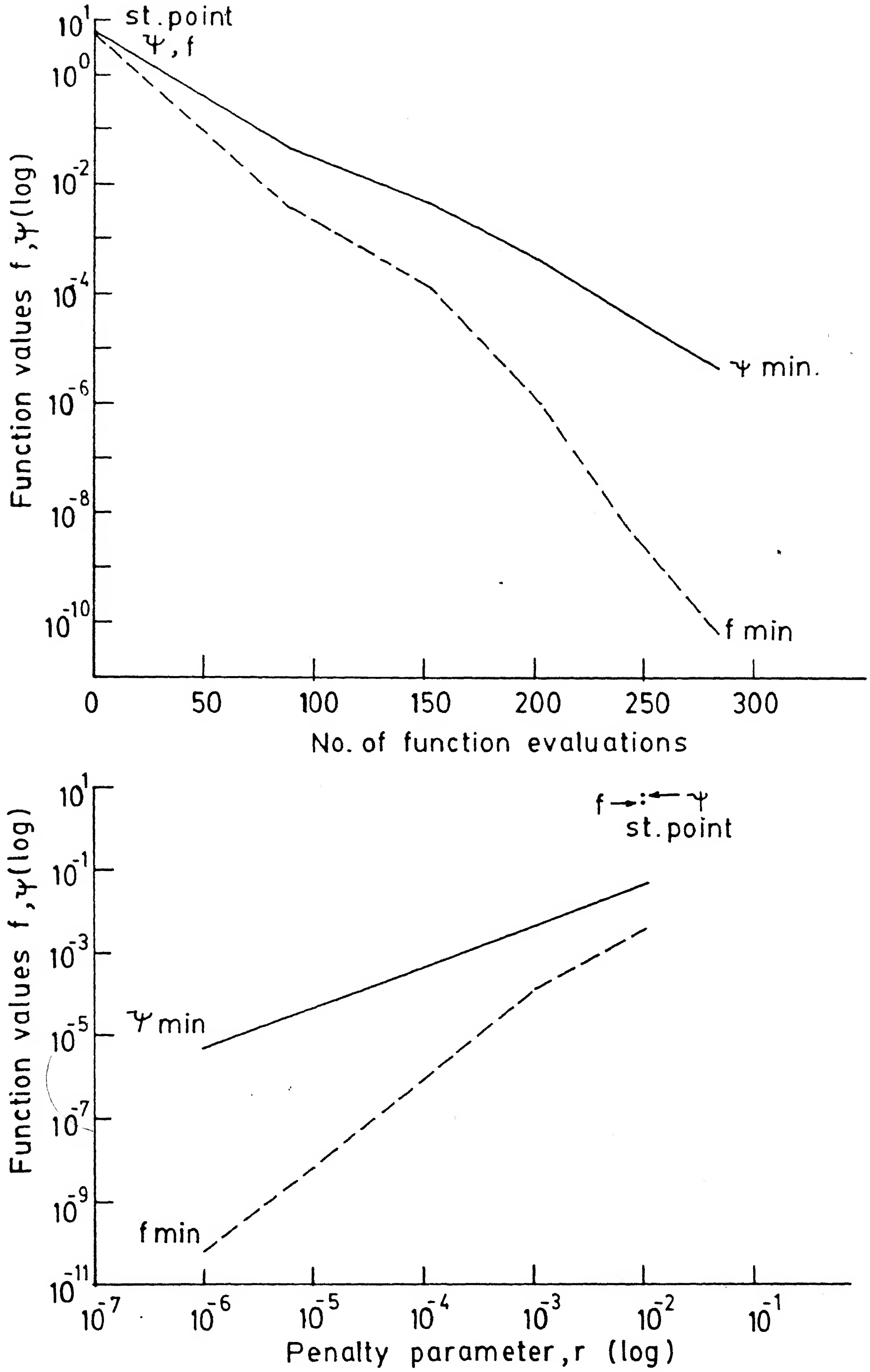


FIG. 8.2 PROGRESS OF MINIMIZATION IN THE BACK ANALYSIS OF (c', ϕ') BY METHOD I FOR EX. PROBLEM 8.1

8.5.2.3 Three-parameter Back analysis Case I : c', ϕ', θ

The 2-parameter back analysis explained above requires that the interslice force inclination θ be known in order to use the sophisticated and complete equilibrium method due to Spencer (1973) along with the shear surface (shape and location) and its factor of safety F. Now, in the case of back analysis of a slide, the value of F at failure is set to 1.Ø but θ is not known beforehand for the observed failure surface.

It becomes necessary, therefore, to treat θ also as an unknown in addition to the geomechanical parameters to be back-calculated. When only 2-parameters namely c' and ϕ' are to be found, inclusion of θ makes it a 3-parameter (c' , ϕ' , θ) back analysis. In such an attempt, the known quantities, as before, are r_u, γ and F, apart from the shear surface. The design variables in this case are c' , ϕ' and θ and are normalized as discussed earlier.

The results of this back analysis have been presented in Table 8.2 in the same manner as Table 8.1. It is observed from the table that the back analysed values of the parameters do not perfectly match with their actual values; however, they are reasonably close. The maximum error is in the prediction of c' and is equal to about 13 percent. It should be noted that the error in determining the appropriate strength parameters c' and ϕ' may often be much higher than the obtained discrepancy.

TABLE 8.2

Results of a Three parameter (c', ϕ', θ) Back Analysis by Method I of the Slope Section in Example Problem 8.1

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c' (kPa)	10.0	25.672	29.38	-12.62%
ϕ' (deg.)	15.0	31.150	30.00	+3.83%
θ (rad)	0.15	0.253194	0.250076	-1.25%
Z_n (kN/m)	-0.6506E+04	0.6981E-02	-	-
M_n (kN-m/m)	-0.6736E+05	-0.9925E-01	-	-

8.5.2.4 Two-parameter Back-analysis Case II: c', θ

It has been observed from the results of the 3-parameter back analysis presented in the previous sub section that from a practical point of view the back analysed values are in reasonably good agreement with the actual known values of the geomechanical parameters c' and ϕ' and the additional unknown θ. However, from the point of view of a mathematical analysis, it would be desirable to have a more perfect agreement between the two sets of values, as indeed have been obtained in the case of the 2-parameter back analysis presented in the earlier section. Considering the success of the 2-parameter back analysis, it appears that the increase in the number of parameters from 2 to 3 has contributed to the error in prediction observed in the latter case. It may be noted again that although the number of geomechanical parameters involved in the back analysis are only two, namely, c' and ϕ', the sophisticated method of analysis (Spencer's method) has itself introduced an extra unknown in the form of θ, thus making the total number of parameters equal to three.

Based on experience with the laboratory determination of the shear strength parameters it may generally be taken to be true that compared to c', the uncertainties and inaccuracies involved in the determination of ϕ' are much less. If this is accepted ϕ' determined from the laboratory tests may be considered as reasonably correct and hence for all practical purposes ϕ' need not be back analysed. This leaves only c', and θ as the unknowns which are likely to be back analysed most accurately. The dropping of ϕ' from the list of unknowns will remove the

possibility of unrealistic or unjustified adjustment or combinations between c′ and ϕ′ themselves. This will thus help lay all the emphasis on the accurate back analysis of c′. Judging by Dhawan's findings (Dhawan, 1986) that the critical slip surface is most sensitive to c′ the above mentioned attempts to back analyse c′, and θ treating ϕ′ as a known quantity seems justified. Results of such an analysis conducted with reference to the current example problem are presented in Table 8.3. As in the previous 2-parameter back analysis involving c′ and ϕ′, the back analysed values are in excellent agreement with the actual values of the parameters c′ and θ. This exercise firmly establishes the success of the 2-parameter back analysis using the proposed method.

8.5.2.5 Three-Parameter Back-analysis Case II: c′, ϕ′, r_u

Three parameter back analysis case I involving c′, ϕ′ and θ presented in section 8.5.2.3 has met with reasonable success. To check whether the same trend of results are obtainable for other combinations of variables or not, a similar test has been conducted treating c′, ϕ′ and r_u as unknowns and F and θ as known parameters with reference to the current example problem. The results presented in Table 8.4, however, show that the prediction has not been satisfactory in this case. The maximum error, occurring in the value of ϕ′, is found to be more than 40%.

8.5.2.6 Four-parameter Back-analysis: c′, ϕ′, r_u, θ

To further investigate the trend of predictability as the number of parameters to be back calculated increases, a four

TABLE 8.3

Results of a Two parameter (c', θ) Back Analysis by Method I of the Slope Section in Example Problem 8.1

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c' (kPa)	10.0	29.38	29.38	Nil
θ (rad)	0.15	0.250076	0.250076	Nil
Z_n (kN/m)	-0.2287E+04	-0.2288E-04	-	-
M_n (kN-m/m)	-0.2345E+04	-0.1526E-02	-	-

TABLE 8.4

Results of a Three parameter (c', ϕ', r_u) Back Analysis by Method I of the Slope Section in Example Problem 8.1

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c' (kPa)	10.0	24.76	29.38	-15.72%
ϕ' (deg.)	15.0	17.65	30.00	-41.17%
r_u	0.4	0.45	0.50	-10.00%
Z_n (kN/m)	0.4483E+04	0.3719E-01	-	-
M_n (kN-m/m)	0.5689E+05	0.6219E-01	-	-

parameter back analysis has been carried out treating all of c', ϕ', r_u and θ as unknowns. The results are presented in Table 8.5. It is observed that the back analysed values of the parameters are widely different from their actual values with the maximum difference, occurring in c', reaching about 65% which is even worse than the 3-parameter case.

Thus it is seen that as the number of parameters increases, the predictability of the proposed scheme diminishes sharply.

TABLE 8.5

Results of a Four parameter (c', ϕ', r_u, θ) Back Analysis by Method I of the Slope Section in Example Problem 8.1

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c' (kPa)	10.00	48.51	29.38	+65.11%
ϕ' (deg.)	15.00	16.55	30.00	-44.83%
r_u	0.30	0.2547	0.50	-49.06%
θ (rad)	0.15	0.22994	0.250076	-8.05%
Z_n (kN/m)	-0.4839E+04	-0.2577E-01	-	-
M_n (kN-m/m)	-0.4285E+05	-0.3257E-01	-	-

8.6 EXAMPLE PROBLEM 8.2

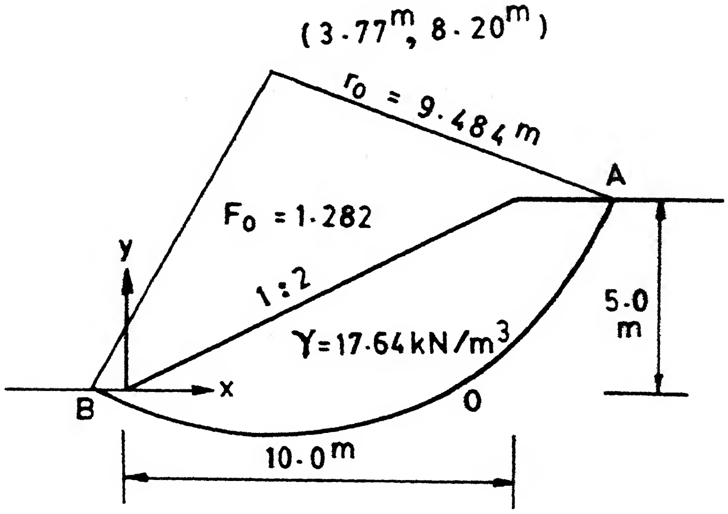


FIG. 8.3 SLOPE SECTION IN EX. PROBLEM 8.2 (Yamagami et al,1989)

8.6.1 General

Figure 8.3 shows the slope section considered in this Example problem which has been taken from Yamagami et al.'s paper (1987). The authors have used this problem to illustrate their approach of back analysis. Performing, initially, a total stress forward analysis based on the Ordinary method of slices they determined the critical circular slip surface which is also shown in Figure 8.3. The corresponding factor of safety reported by them is: F = 1.282 corresponding to the following strength parameters:

$$c_u = 9.8 \text{ kN/m}^2 \quad \text{and} \quad \phi_u = 10^\circ$$

Using 21 slices and the Ordinary method of slices the F value determined in the present study is, 1.2814. Since the proposed method is based on the Spencer's method of analysis, it

is necessary, before starting the back analysis, to compute the factor of safety of the same shear surface using the Spencer's method. Using 21 slices again the following F and θ values have been obtained:

$$F = 1.3594 \quad \theta = 0.22446 \text{ rad.}$$

The back analyses carried out with reference to this example problem are presented and discussed in the subsequent sections. The design variables have been normalized as discussed before. The side constraints imposed are as discussed earlier with the exception that the upper limit of c_u has been reduced from 100 to 50 to avoid unnecessary search.

8.6.2 RESULTS AND DISCUSSION

8.6.2.1 Two-parameter Back-analysis Case I: c_u, ϕ_u

The results of a 2-parameter back analysis to find c_u and ϕ_u (while the shear surface as well as the F and θ values are known from the forward analysis) are presented in Table 8.6. As in the case of the Example Problem 8.1, the back analysed values of c_u and ϕ_u obtained by using the proposed method are in perfect match with their actual values. It should be noted that to compare the relative efficiency of the proposed method the starting point for the optimization has been taken the same as used by Yamagami et al. The back analysed values reported by them are also presented in the same table. Actually, they have obtained results using two techniques, namely, the SUMT and the flexible tolerance method. The best results reported by them are those already presented in Table 8.6. It is seen that the errors in prediction are much less in the proposed method compared to Yamagami et

TABLE 8.6

Results of Two-Parameter (c_u, ϕ_u) Back Analysis by Method I of the Slope Section in Example Problem 8.2

Parameter OR Function	Initial value	Back Analysed Value		Actual value	Error in prediction	
		Proposed Method	Yamagami et. al's Method		Proposed Method	Yama-gami et.al's Method
c_u kPa	11.76	9.7961	10.035	9.80	-0.04%	+2.40%
ϕ_u deg	21.8014 ($\tan\phi'$ =0.40)	10.0040 ($\tan\phi'$ =0.1764)	9.5422 ($\tan\phi'$ =0.1681)	10.00 ($\tan\phi'$ =0.1763)	+0.04%	-0.51%
Z_n kN/m	0.1840E+03	-0.3015E-01	-	-	-	-
M_n kN-m/m	0.4374E+03	0.1787E-01	-	-	-	-

al.'s method. Practically speaking, the values predicted by the proposed and the Yamagami et al.'s methods and the actual values are insignificantly different from one another. However, they are significant from the point of view of reliability and desirable perfection.

8.6.2.2 Two-parameter Back analysis Case II: c_u, θ

The case II of the 2-parameter back analysis consists in finding c_u and θ treating ϕ_u, F and, of course, the shear surface, as known. The utility and justification of such an exercise have been discussed in Section 8.5.2.4 in connection with the solution of Example Problem 8.1. The results of the analysis carried out with reference to the current example problem are presented in Table 8.7. Normalization of design variables are done in the same way as in case of Example Problem 8.1. The side constraint on c_u are as in Table 8.6. The observation from this table is that a two-parameter back analysis can be performed most accurately using the proposed method, no matter whether it is c_u, ϕ_u, combination or c_u, θ. Similar observations were made earlier in case of Example Problem 8.1.

8.6.2.3 Three-parameter Back-analysis Case I: c_u, ϕ_u, θ.

The 3-parameter back analysis to find c_u, ϕ_u and θ have been performed as described and discussed in Section 8.5.2.3. The results obtained here are presented in Table 8.8. It is seen that unlike Example Problem 8.1, the back analysed values show rather poor agreement with the corresponding actual values with the maximum error exceeding 55%. The normalization of design variables and the side constraints have been done as before.

TABLE 8.7

Results of Two parameter (c_u, θ) Back Analysis by Method I of the Slope Section in Example Problem 8.2

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c_u (kPa)	11.76	9.800	9.80	Nil
θ (rad)	0.1750	0.22456	0.22446	+0.04%
Z_n (kN/m)	0.2832E+02	0.1442E-01	-	-
M_n (kN-m/m)	0.9977E+02	-0.1183E-01	-	-

TABLE 8.8

Results of a Three parameter (c_u, ϕ_u, θ) Back Analysis by Method I of the Slope Section in Example Problem 8.2

Parameter/ Function	Initial value	Back analysed value	Actual value	Error in prediction
c_u (kPa)	11.76	5.427	9.80	-44.6%
ϕ_u (deg.)	21.8014 (tan ϕ_u=0.4)	15.592	10.0	+55.92%
θ (rad)	0.175	0.2666	0.22446	+18.77%
Z_n (kN/m)	0.1681E+03	0.8525E-02	-	-
M_n (kN-m/m)	0.5079E+03	0.7376E-02	-	-

8.7 DESCRIPTION OF METHOD - II

8.7.1 Statement of the Inverse Problem:

Find, for a given slope with a given slip surface and the associated factor of safety (in the case of a failed slope the given slip surface will be the failure surface and the associated factor of safety will be 1.Ø), the geomechanical parameters which yield the same value of the factor of safety subject to the non-negativity of the parameters.

8.7.2 Analysis:

The geomechanical parameters, c', ϕ', r_u etc. to be back analysed must satisfy the following relationship:

$$F = F_o$$

where F stands for the factor of safety expression used in the analysis and F_o is the factor of safety of the given slip surface. For a failed slope, as already stated, $F_o = 1.Ø$

With this introduction, the back analysis problem can be formulated as one of nonlinear constrained optimization as follows:

$$\text{Find } \mathbf{D} = [\,c', \phi', r_u, \ldots.]^T$$

such that

$$f(\mathbf{D}) = (F-F_o)^2 \longrightarrow \text{Min.}$$

subject to the constraints:

$$c'_{min} \le c' \le c'_{max}$$

$$\phi'_{min} \le \phi' \le \phi'_{max}$$

$$r_{u\ min} \le r_u \le r_{u\ max}$$

As discussed while formulating Method I, the shear surface being "locked", the number of design variables are greatly reduced and thus a lot of computation is saved.

The advantage of the Method II over Method I is that it does not require the value of θ, the interslice force angle for its solution. However, it should be pointed out that the Method I is based on a more sophisticated analysis satisfying complete equilibrium.

8.7.3 Method of Solution

The solution technique for this method is the same as discussed in the case of Method I.

8.8 ILLUSTRATIVE EXAMPLE

8.8.1 General

Application of the Method II using Janbu's GPS will be illustrated in the following sections. The Example Problem 8.1 has been chosen again for the purpose. The value of the factor of safety, calculated using 14 slices, has been obtained as,

$$F_o = 1.4668$$

The line of thrust assumed for Janbu's analysis procedure is presented in Table 8.9(a). Such a line of thrust has been assumed in accordance with Janbu's (1973) suggestion. The design variables are normalized as discussed earlier. As far as the side constraints are concerned, no upper limit has been imposed on ϕ'. The other limits will be mentioned at the appropriate places.

TABLE 8.9(a)

Line of Thrust Assumed In the Analysis of the Example Problem 8.1 By Method II Using Janbu's GPS

Interslice boundary number	1	2	3	4	5	6	7	8	9	10	11	12	13
Line of thrust position h_t/z	0.34	0.35	0.36	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.43

TABLE 8.9(b)

Results of Two Parameter (c', ϕ') Back-Analysis of the Slope Section in Example Problem 8.1 by Method II using Janbu's GPS

Parameter/ Function	Initial Value	Back - Analysed Values				Actual Value	Error
		Test 1 $0 \le c' \le 150$	Test 2 $0 \le c' \le 100$	Test 3 $0 \le c' \le 75$	Test 4 $0 \le c' \le 60$		
c' kPa	10.0	42.65	37.64	29.64	29.38	29.38	Nil
ϕ' deg.	15.0	25.75	27.35	29.92	30.0	30.0	Nil
F	0.7321	1.46698	1.46698	1.46696	1.4668	1.4668	-
f	0.5398	0.00	0.00	0.00	0.00	-	-

8.8.2 RESULTS AND DISCUSSION

8.8.2.1 Two-Parameter Back Analysis Case I: c', ϕ'

To start with, a 2-parameter back analysis has been attempted. The two parameters considered are c' and ϕ' assuming, as before, that r_u and γ are known for this analysis in addition to the shape and location of the shear surface and its factor of safety. The results are presented in Table 8.9(b). Unlike Method I, here it is observed that no unique solution is obtainable for 2-parameter back analysis and the closeness of a solution with the correct solution depends on the narrowness of the search domain for the various design variables. The narrower the search domains the more are the chances of trapping the actual solution. As shown in Table 8.9(b), several experiments have been carried out varying the value of c'_{max}, the upper limit on c', put as a side constraint. It is seen that by gradually narrowing down the search domain for c' from an initial c'_{max} of 150 kPa, a close solution has been obtained corresponding to c'_{max} = 75 kPa and a still better solution has been obtained when c'_{max} = 60 kPa. Considering that 75 is more than 2.5 times the actual value of 29.38 it is expected that such a crude estimate may not be difficult to have from the laboratory tests. No such upper limit on ϕ' has been imposed as it has been observed that it is the c'_{max} which controls.

Figure 8.4 shows the variation of the objective function f and the composite function ψ with the progress of minimization it shows that there has been no ill-conditioning during the computations.

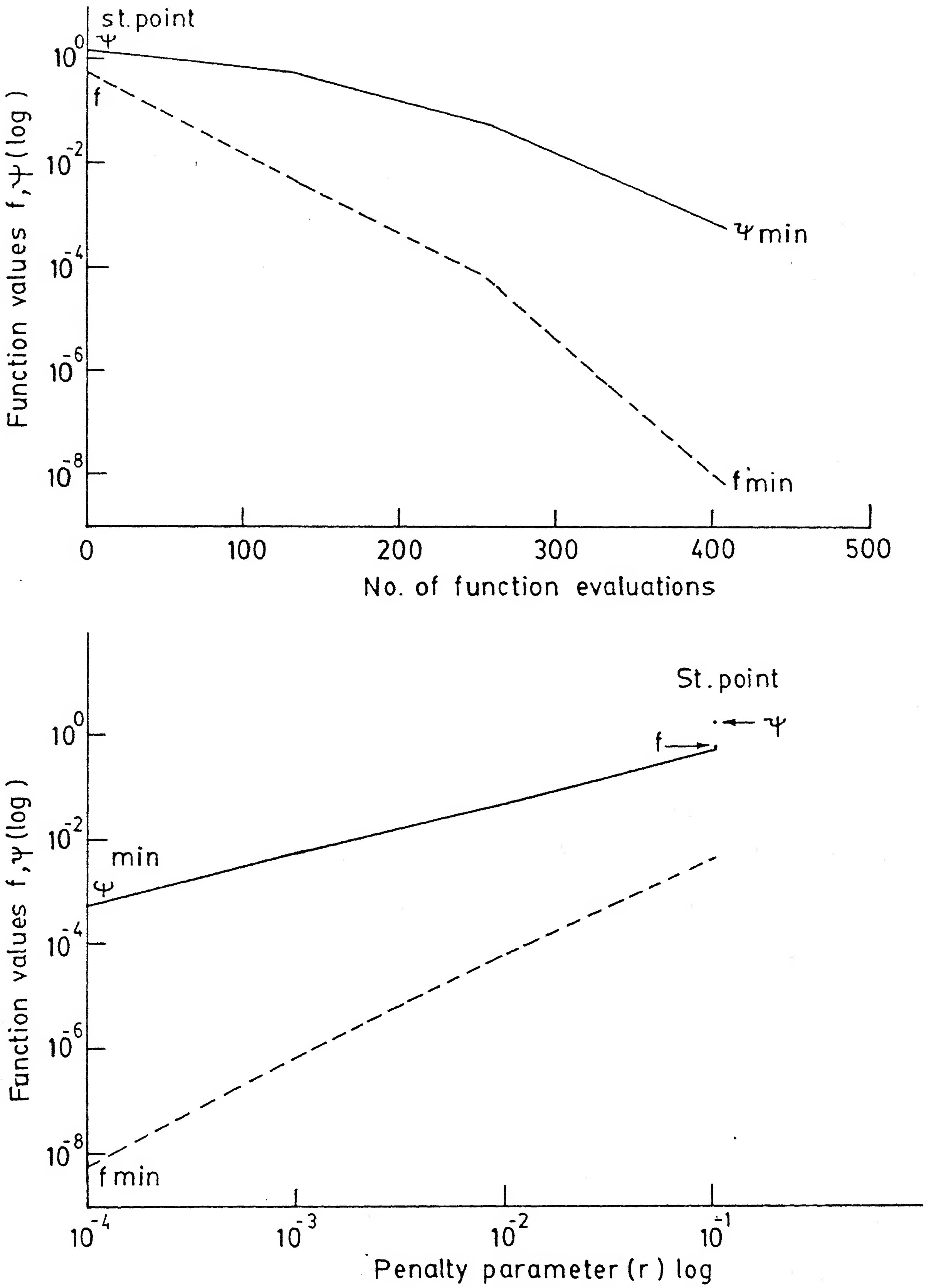


FIG. 8.4 PROGRESS OF MINIMIZATION IN THE TEST 4 OF THE BACK ANALYSIS OF (C', ϕ') BY METHOD II USING JANBU'S GPS FOR EX. PROBLEM 8.1

8.8.2.2 Two-Parameter Back-Analysis Case II c', r_u

In this case of back analysis with r_u as unknown along with c' the dependence of the solution on the bounds imposed on the variables are even more pronounced. The two experiments conducted show that for an accurate analysis the search domain has to be narrowed down by using good engineering guess and judgement about the existing soil and pore pressure conditions. From the experience gained in the previous analysis.involving c' and ϕ', the upper limit for c' has been kept at 75.0 as before. As far as the pore pressure coefficient r_u is concerned, the lower and upper limits have been placed as 0.3 and 0.55 respectively for both the experiments. The lower limit on c' was kept 0.0 in the first experiment. Observing that this has resulted in a too small value of c', it has been raised to 20.0 in the second experiment. Though not accurate, a reasonably close result has been obtained in the second case as presented in Table 8.10. The maximum error is in c' and is equal to 8.85%.

8.8.2.3 Three-Parameter Back Analysis (c', ϕ', r_u)

The observations made earlier also hold in the case of the 3-parameter back analysis to find c', ϕ' and r_u. The variables have to be given narrow ranges of variation to obtain an accurate solution. The results, presented in Table 8.11, show that a somewhat reasonable solution has been obtained in Test 2 in which case a lower limit of 0.3 on r_u has been imposed in addition to the upper limits on c' and r_u. A lower limit of 0.0 has been used for c'. The maximum error is in the prediction of r_u and is equal to 14.62%.

TABLE 8.10

Results of Two Parameter (c', r_u) Back-Analysis of the Slope Section in Example Problem 8.1 by Method II using Janbu's GPS

Parameter/ Function	Initial Value		Back Analysed Values		Actual Value	Error in prediction
	Test 1	Test 2	Test 1	Test 2		
c' kPa	10.00	20.00	18.4750	26.7800	29.38	-8.85%
r_u	0.35	0.35	0.4385	0.4851	0.5	-2.98%
F	1.6054	1.7010	1.4669	1.4669	1.4668	-
f	0.0192	0.0548	0.00	0.00	-	-

N.B. For Test 1 : $0 \le c' \le 75$; $0.3 \le r_u \le 0.55$

Test 2 : $20 \le c' \le 75$; $0.3 \le r_u \le 0.55$

TABLE 8.11

Results of Three Parameter (c', ϕ' r_u) Back-Analysis of the SlopeSection in Example Problem 8.1 by Method II using Janbu's GPS

Parameter/ Function	Initial Value		Back Analysed Values		Actual Value	Error in prediction
	Test 1	Test 2	Test 1	Test 2		
c' kPa	10.00	10.00	28.425	29.20	29.38	-0.61%
ϕ'	15.00	15.00	21.80	26.45	30.00	-11.83%
r_u	0.30	0.35	0.295	0.4269	0.50	-14.62%
F	0.9804	0.9178	1.4669	1.4669	1.4668	-
f	0.2367	0.3015	0.00	0.00	-	-

N.B. For Test 1 : $0 \le c' \le 75$; $\phi' \ge 0$; $0 \le r_u \le 0.55$

Test 2 : $0 \le c' \le 75$; $\phi' \ge 0$; $0.3 \le r_u \le 0.55$

8.9 COMPARISON OF METHOD I AND METHOD II:

The essential difference between the two methods is that unique solutions are obtained for 2-parameter back analysis using Method I whereas Method II gives solutions which are dependent on the starting point and/or the side constraints imposed on the variables. This can be explained by the fact that Method I, in effect, is based on the solution of two equations, namely, the force equilibrium and the moment equilibrium expressed as:

$$Z_n = 0 \quad \text{and} \quad M_n = 0$$

Due to the availability of two equations, two unknowns can be uniquely solved for. This has been observed previously in the case of forward analysis to find F and θ when the soil and pore pressure conditions were known. Now the uniqueness of solution has been observed in the back analysis where (F,θ) and (c',ϕ') interchange themselves, as it were.

In contrast, Method II is based on the solution of only one equation namely, $F = F_o$. Therefore, it cannot uniquely solve for more than one unknown unless supported by other equations in the form of side constraints and hence it is dependent on the side constraints imposed in a particular case. The knowledge of θ, however, is not necessary in this method and it is more versatile in the sense that any limit equilibrium method requiring solution of only one factor of safety equation may be used.

When more than two parameters are to be back analysed, however, both methods provide solutions which are not unique. This can also be explained from what has been stated above.

8.10 CONCLUSIONS

From the studies contained in this chapter the following conclusions may be drawn:

1. The proposed schemes of back analysis are better than the numerical procedure proposed by Yamagami and Ueta (1987) in the sense that one need not search for the critical surface and match it with the observed failure surface to obtain the design parameters.
2. The method I presents unique values of c' and ϕ' or c' and θ; however, in method II, the solution is dependent on the prescribed domain of c'. But when pore pressure parameter is treated as unknown, none of the methods present unique solution.
3. Considering the fact that ϕ' can be determined in the laboratory with reasonable accuracy considering c' and θ to be unknowns, c' can be predicted with confidence using method I.
4. The limited studies conducted to back predict more than two geomechanical parameters for homogeneous slopes indicate that it is unlikely that a solution with any degree of confidence can be arrived at either for a homogeneous or a nonhomogeneous slope. Since in the present study keeping the shear surface unchanged several combinations of geomechanical parameters are obtainable for the same factor of safety, it can be concluded that even matching the shear surface does not guarantee the uniqueness of the solution.

5. From the limited study on the subject it may be opined that from the field and laboratory investigations of the slide zone, meaningful upper and lower bounds on the geomechanical parameters should be found out so that these limits can be utilized as side constraints in the back analysis of these parameters. This would ensure that the obtained parameters are of physical significance pertaining to the problem.

CHAPTER 9

SUMMARY, GENERAL OBSERVATIONS AND CONCLUDING REMARKS

9.1 SUMMARY

In this thesis the problem of stability of earth slopes has been formulated as a mathematical programming problem and solved by adopting the Sequential Unconstrained Minimization Technique (SUMT) in conjunction with the limit equilibrium methods of slices. The analyses are essentially based on Spencer method (1973) and Janbu's generalized procedure of slices (1973) for general slip surfaces; however, in a few cases the Simplified Bishop Method (Bishop, 1955) has also been used. A complete stability analysis consists of two parts:

Part I. Evaluation of the factor of safety associated with a given slip surface.

Part II Determination of the critical slip surface.

For Part I involving Spencer method , a numerical method for solving the equations of stability has been proposed towards developing an efficient and powerful equation solver.

For Part II, initially a formulation has been attempted in which the factor of safety is minimized with respect to co-ordinates of discrete points on the shear surface subject to some behavioural and side constraints which are of inequality type. This formulation is referred to as the Indirect formulation.

For Part II involving Spencer method, as the use of Indirect formulation is time consuming, attention has been focussed on more efficient schemes. Towards this end, a new formulation has been attempted in which the need for iterations to satisfy the equilibrium conditions for each trial slip surface is eliminated and the critical slip surface is found out directly. This has been realized by putting the equilibrium conditions as equality constraints and treating both the factor of safety and the constant slope of interslice forces as design variables in addition to the slip surface co-ordinates. This new formulation has been referred to as the Direct formulation in which the factor of safety forms the objective function as well as a design variable.

In finding the critical slip surface, care has been taken to see that the obtained solution satisfies some acceptability criteria in addition to the basic requirement that the final surface corresponds to the minimum factor of safety.

In addition to the above, the inverse problem of finding the geomechanical parameters when the actual failure surface is known has also been formulated as a mathematical programming problem and the solution has been attempted by applying the SUMT. The solution procedure based on SUMT utilizes the Extended Penalty Function Method (Kavlie, 1971) as it accepts infeasible starting points and it can handle equality constraints. In the case of Indirect formulation and availability of feasible starting point, the Interior Penalty Function method may also be used. Powell's method and Quadratic fit have been selected for multidimensional

unconstrained search and one dimensional search respectively as they do not require gradient evaluations.

To test the working of the gradient based methods such as the Davidon-Fletcher-Powell method (DFP), a study was initiated to the analysis of a homogeneous slope using the Direct formulation. As explicit gradients are not available, finite difference technique using central difference was used. However, it was observed that the values of the partial derivatives by the finite difference formula are very sensitive to the grid-size chosen. Further it was much more time consuming compared to the Powell's method even for homogeneous slopes. Due to the above reasons the study was not pursued in applying the gradient methods to the nonhomogeneous slopes. However, more studies are required to make any conclusive comment on the usefulness or otherwise of these methods in analysing nonhomogeneous slopes.

The application of the proposed numerical scheme of stability analysis has been illustrated by solving a variety of problems categorised as under:

1. Homogeneous slopes considering both linear and nonlinear strength envelopes.
2. Slopes in soils with nonhomogeneous and anisotropic strength variation.
3. Zoned dams and embankments including those founded on thin shear zones. A special formulation has been presented for the analysis of the same.
4. Embankments on soft ground.
5. Back analysis of slope failures.

To establish the relative efficiency of the proposed procedure, the results obtained in studying the above problems have been compared with those obtained by using other currently available techniques such as, variational technique, dynamic programming, grid search, random search, simplified search (Celestino and Duncan, 1981), simplex reflection technique etc.

9.2 GENERAL OBSERVATIONS AND CONCLUDING REMARKS

On the basis of numerical studies the following concluding remarks are made:

1. The proposed numerical method for solving the stability equation in Spencer method has been found to be very efficient and powerful. The method works even in situations where the original solution scheme of Spencer fails to converge.

2. Using the proposed equation solver, unique solution has been obtained when different initial guess values for the factor of safety and the interslice force angle were used. This finding is in contrast to the existence of multiplicity of solutions reported earlier (Chugh, 1982).

3. The proposed method of determination of critical shear surface is more versatile and flexible than the other existing methods in the following respects:

i) it has the provision for introduction of tension cracks dry or water-filled and either of constant or variable depth.

ii) Constraints can be added or dropped at will without affecting the basic structure of the developed computer program.

4. Almost in all the problems studied herein, the minimum factor of safety obtained in the present analysis have been observed to be equal or lower than those given by some of the other existing softwares.

5. The acceptability constraints, $Ø < L_i < H_i$ as used in the present analysis using Spencer's method are generally observed to be adequate in the sense that when these constraints are satisfied at all the interslice boundaries, is general the line of thrust for effective stress satisfies the criteria

$$Ø.25H_i \leq L'_i \leq Ø.65H_i$$

laid down by Hamel (1968). Wherever the above criteria are satisfied or nearly satisfied the resultant effective normal forces on the side and base of each slice are generally observed to be compressive. Moreover, in the case of homogeneous slopes, the factor of safety against shear failure along the vertical interslice boundaries are also observed to be greater than the average factor of safety against shear failure along the shear surface and hence no separate constraint needs to be applied to guard against such violation of the Mohr-Coulomb failure criteria.

However, situations arise where the above constraints are found hard to satisfy particularly in locations near the crest. In some cases this also happens that the constraints $Ø < L_i < H_i$ are satisfied yet the line of thrust for effective stress is not satisfactory. In such situations introduction of tension crack

generally resulted in acceptable line of thrust; in a few cases, in addition to tension crack other assumptions regarding slopes of interslice forces had to be tried in order to obtain acceptable solutions. This corroborates Spencer's finding (Spencer, 1968, 1973).

Although it seems more logical to put the acceptability constraints straight away on the line of thrust for effective stress in the form

$$0.33H_i \leq L'_i \leq 0.67H_i,$$

it has been observed that imposition of constraints in this form prove to be too stringent for the smooth progress of the optimization.

6. The studies have revealed that if the acceptability criteria are not taken care of the analysis may lead to a large error 30% in the value of the factor of safety on the unsafe side. The error could be disastrous if the neglect of the acceptability criteria is combined with a priori assumption regarding the shape of the failure surface.

7. The Direct minimization procedure involving the Spencer method has been observed to be as effective as the Indirect procedure. However, it has reduced the computational time to one forth to one-sixth of that required by the Indirect procedure.

8. The Extended penalty function technique can effectively handle the equality constraints imposed in the Direct formulation, in addition to accepting infeasible starting points.

Fairly good results have been obtained by choosing the initial value of ε, the tolerance, and δ_t, a factor representing the transition between the two types of penalty terms as -0.1 and 0.001 respectively. The initial value of the penalty term r chosen such that the value of the composite function ψ is 1.1-1.2 times the value of the objective function has generally produced better results.

9. Powell's method of multidimensional search along with Quadratic fit of unidirectional search have been found to give good results.

10. Design variables normalized by dividing by their initial values together with a step length of 0.1 have been found to give best results in case of Direct formulation. When Indirect formulation is used, normalization does not usually make appreciable difference in the final results. However, in the case of high dam or embankment where there is large difference in the values of the shear surface co-ordinates normalization improves the result.

Regarding the convergence criteria to be used in Powell's method, it has been observed that in the search for critical shear surface the use of the convergence criterion

$$|D_{i+1} - D_i| \leq \varepsilon$$

takes much longer computation time yielding only marginal change in the optimal solution as compared to the solution obtained by

12. The special formulation proposed herein for analysis of dam or embankments founded on thin shear zones has proved to be ver effective and enables one to avoid the difficulties in thi regard reported earlier (Saytam Babu, 1986). Moreover, this ha reduced the number of design variables and thus reduced th c.p.u. time to almost 1/3rd of that required in using the genera formulation.

13. In the case of failed slopes, the failure surfaces predicte by the present analysis match the observed failure surfaces mor closely than the other predictions reported in the literature.

14. Although Janbu's generalized procedure of slices does no strictly satisfy moment equilibrium equation the values of th minimum factor of safety given by this method differ by not mor than ±8% of those obtained by using the Spencer method whic satisfies all conditions of equilibrium. However, the critica slip surfaces given by the two methods are generally different.

15. Regarding the inclusion of the end constraints in the searc schemes for critical surface the following observations have bee generally made. In the analysis by Spencer method no difficult arises in achieving proper convergence as a result o non-inclusion of such constraints. As such these constraint have not been imposed to allow more freedom to the shear surface keeping in mind that the numerical scheme should not be undul burdened with too many constraints.

In contrast, the analysis by Janbu's method, in most cases, meet with convergence difficulties when appropriate en

constraints are not included. The reason for this anomaly in behaviour probably lies in the fact that the line of thrust is pre-assigned in the Janbu's method whereas it is obtained as part of the solution in the Spencer method.

16. As regards the number of slices to be used, unlike in Spencer method, convergence difficulties arise in using Janbu's method to analyse heterogeneous slopes in particular when the number of slices is large so as to make the ratio of maximum mean-height to width of slice exceed about 6. Janbu (1980) has recommended a maximum value of 3 for this ratio. Whenever the number of slices have been restricted accordingly, no such difficulties have been encountered.

It has been observed, with regard to both the methods, that there are cases which normally yield unacceptable solutions but on reducing the number of slices lead to acceptable solutions. However, taking too few number of slices may lead to suppression of the true stress conditions (true in the context of the method concerned). This confirms earlier findings (Spencer, 1968).

17. It has been demonstrated that the nonlinear strength envelope can be incorporated in the standard slope stability computations using Spencer method in both Indirect and Direct formulations. Using the Direct formulation in particular, it has been shown quantitatively that nonlinear failure envelope gives shallower critical slip surface with a lower factor of safety as compared to the conventional approach using constant values of c' and ϕ'

18. For slopes in soils having nonhomogeneous and anisotropic strength variation, the minimum factor of safety obtained in the present analysis based on non-circular slip surfaces are observed to be quite close to the values for the corresponding cases as reported by Lo (1965). The obtained general critical slip surfaces are fairly close to the critical slip circles as per Lo's solution.

19. It is the shape and location of the critical shear surface rather than the value of the minimum factor of safety which is more dependent on the minimization procedure adopted. In many cases it has been observed that although the F_{min} values obtained by using various techniques are quite close, corresponding critical shear surfaces are quite apart from one another. This also indicates the presence of a critical zone rather than a sharply defined critical slip surface.

20. The proposed procedure of back analysis based on SUMT in conjunction with the Spencer method can accurately back calculate two parameters, namely, the cohesion (c') and the angle of shearing resistance (ϕ'). This is an improvement over the earlier works in the field in respect of the following:

i) Most of the earlier works are good for back calculation of only one parameter.

ii) The method suggested by Yamagami et. al. has been demonstrated to be capable of back analysing 2-parameters namely the total stress shear strength parameters c_u and ϕ_u (assumption of total stress analysis eliminates the average pore pressure

parameter, r_u). However, the method requires a lot of computational effort as it involves in its iteration procedure the determination of a number of (trial) critical slip surfaces. The present formulation, on the other hand, does not require any surface optimization as such and as a result it is very quick and simple.

iii) Unlike the earlier ones, the present approach is based on a more accurate method of factor of safety computations which satisfies all equations of equilibrium.

A limitation of the method, however, is that the slope of the interslice forces 'θ'(in Spencer method) is to be either known or assumed. However, in those cases where ϕ' obtained from the geotechnical exploration can be accepted as reliable the method can accurately calculate c' and θ.

Another limitation of the method, which is common to the earlier methods also, is the difficulty concerning the back analysis of the average pore water pressure parameter r_u. It has been observed that whenever r_u forms one of the design variables in combination with either c' and ϕ', the method shows extreme starting point dependence. Thus, unless the initial guess for r_u is very good, the back analysed values of the parameters are likely to be inaccurate.

9.3 SCOPE OF FURTHER STUDIES

The solution technique established in the present thesis may be utilized in studying the following problems:

1. Application of the principle of the Direct formulation to the Morgenstern and Price method which is supposed to be the most accurate of the limit equilibrium methods. Generally, the use of the Morgenstern and Price method is limited in view of the large computational time required in the large number of iterations involved in its method of solution. The efficiency of the Direct formulation in conjunction with this method with respect to computational effort should be studied.

2. The limit equilibrium method proposed by Sarma (1979) uses the internal strength of material for the solution of the problem. However, as Sarma (1979) points out, the method is not really suitable for the analysis of sections where large number of probable slip surfaces have to be analyzed i.e., where the critical shear surface is to be determined. This is because of the large number of iterations involved in finding the set of inclinations of the interslice boundaries. The principle of the proposed Direct procedure may be applied to solve this problem efficiently for a general heterogeneous slope section.

3. Determination of three dimensional critical shear surface:

A limit equilibrium approach of analysis similar to the method of columns, by Chen and Chameau (1982) may be adopted. However, instead of making any a priori assumption regarding the

geometry of the shear surface, some co-ordinates defining its shape and location may be taken as design variables. Upward concavity may be maintained by imposing constraints in both longitudinal and transverse directions. Other appropriate constraints may also be imposed.

4. Determination of the three dimensional critical surface assumed to be consisting of five planes - two on the sides, two at the ends and one at the bottom.

Following a three dimensional wedge analysis the factor of safety may be expressed as a function of the direction cosines of the normals to the five planes constituting the shear surface together with the co-ordinates of the four corners of the central plane at the bottom of the sliding mass. These four co-ordinates and five direction cosines may, thus, be considered as the design variables. When there exists a shear zone in the foundation, the direction cosine of the normal to the central plane is, however, known.

Appropriate physical constraints consistent with the geometry of the slope and geological features of the site need to be imposed on the design vectors.

5. A detailed study on the back analysis of geomechanical parameters.

6. Obtaining optimal lower bound solutions of slope stability problems wherein the minimum factor of safety is found out subjected to the constraints that stresses at all points within the slope are statically admissible and do not violate the appropriate yield conditions.

APPENDIX A

DETAILED DERIVATIONS OF WORKING EXPRESSIONS IN THE EXTENDED VERSIONS OF THE METHODS BY JANBU AND SPENCER

This appendix contains the theoretical derivations of the working formulae and equations which are referred to in Chapter 2 and used in Chapter 3. These belong to the extended versions of the Spencer's method (Spencer, 1973) and Janbu's GPS (Janbu, 1957, 1973). The original versions of these methods are extended, as stated in Chapter 2, in order to incorporate external forces and moments acting on a slice in the case of Spencer's method and only moment in the case of Janbu's method. Such forces are already considered in the original version of the Janbu's method; the extension of this method is only in respect of including external moment. Derivations of the partial derivatives of the external balancing force and moment in the Spencer's method and are used in Chapter 3 are also presented here.

A-1. Derivation of Equation (2.25) in Spencer's Method

Referring to Figure 2.2(b) and resolving normal to the base of the slice,

$$(W+P_e)\cos\alpha - b\sec\alpha(\sigma'+u)-Q_e\sin\alpha - Z_R\ \sin(\alpha-\delta_R)+Z_L\sin(\alpha-\delta_L)=0$$

Hence,

$$\sigma' b\sec\alpha=(W+P_e)\cos\alpha-Z_R\sin(\alpha-\delta_R)+Z_L\sin(\alpha-\delta_L)-ub\sec\alpha-Q_e\sin\alpha \quad \text{(A-1)}$$

Resolving parallel to the base of the slice,

$$(W + P_e)\sin\alpha - \tan\phi'_m\ \sigma' b\sec\alpha - c'_m b\sec\alpha + Q_e\cos\alpha + Z_R\cos(\alpha - \delta_R) - Z_L\cos(\alpha - \delta_L) = 0 \tag{A-2}$$

Substituting for $\sigma' b\sec\alpha$ from equation (A-1),

$$(W+P_e)\sin\alpha - \tan\phi'_m\ [(W + P_e)\cos\alpha - Z_R\sin(\alpha - \delta_R) + Z_L\sin(\alpha - \delta_L) - ub\sec\alpha - Q_e\sin\alpha] - c'_m b\sec\alpha + Q_e\cos\alpha + Z_R\cos(\alpha - \delta_R) - Z_L\cos(\alpha - \delta_L) = 0 \tag{A-3}$$

Transposing,

$$Z_R = \frac{1}{\cos(\alpha - \delta_R)[1 + \tan\phi'_m\ \tan(\alpha - \delta_R)]}\Big\{c'_m\ b\sec\alpha - (W + P_e)\sin\alpha + \tan\phi'_m\ [(W + P_e)\cos\alpha + Z_L\sin(\alpha - \delta_L) - ub\sec\alpha - Q_e\sin\alpha] + Z_L\cos(\alpha - \delta_L)\Big\}$$

$$Z_R = \frac{1}{\cos(\alpha - \delta_R)[1 + \tan\phi'_m\ \tan(\alpha - \delta_R)]}\Big\{c'_m\ b\sec\alpha - (W + P_e)\sin\alpha + Q_e\cos\alpha + \tan\phi'_m[(W + P_e)\cos\alpha - ub\sec\alpha - Q_e\sin\alpha] + Z_L\cos(\alpha - \delta_L)[1 + \tan\phi'_m\ \tan(\alpha - \delta_L)]\Big\} \tag{A-4}$$

$$Z_R = \frac{c'_m b' - W'\sin\alpha - Q_e\cos\alpha + \tan\phi'_m\ (W'\cos\alpha - Q_e\sin\alpha - ub')}{\cos(\alpha - \delta_R)[1 + \tan\phi'_m\ \tan(\alpha - \delta_R)} + Z_L\left\{\frac{\cos(\alpha - \delta_L)[1 + \tan\phi'_m\ \tan(\alpha - \delta_L)]}{\cos(\alpha - \delta_R)[1 + \tan\phi'_m\ \tan(\alpha - \delta_R)]}\right\} \tag{A-5}$$

where $c'_m = c'/F$

$\tan\phi'_m = \tan\phi'/F$

$b' = b\sec\alpha$

$W' = W + P_e$

A-2 Derivation of Equations (2.26) and (2.27)

Referring to Figure A-1 and neglecting the curvature of the slip surface between adjacent inter-slice boundaries (it has been assumed already that the slice widths are small), in slice 1, taking moments of all the forces about O_1 the mid point of the base of the slice 1,

$$Z_1\cos\delta_1(Y_1O_1) + M_{e_1} + Q_{e_1} h_1 = \emptyset$$

Therefore,

$$Y_1O_1 = -\frac{Q_{e_1} h_1 + M_{e_1}}{Z_1\cos\delta_1} \qquad \text{(A-4)}$$

In slice 2,

$$X_2O_2 = Y_1O_1 + \frac{1}{2}\,[\tan\delta_1(b_1 + b_2) - b_1\tan\alpha_1 - b_2\tan\alpha_2]$$

(The signs of α_1 and α_2 are negative in accordance with the sign convention) Rearranging

$$X_2O_2 = Y_1O_1 + \frac{1}{2}[b_1(\tan\delta_1 - \tan\alpha_1) + b_2(\tan\delta_1 - \tan\alpha_2)] \qquad \text{(A-5)}$$

Taking moments about O_2 of all forces

$$Z_2\cos\delta_2(Y_2O_2) - Z_1\cos\delta_1(X_2O_2) + M_{e_2} + Q_{e_2} h_2 = \emptyset$$

Therefore

$$Y_2O_2 = \frac{Z_1\cos\delta_1(X_2O_2) - Q_{e_2} h_2 - M_{e_2}}{Z_2\cos\delta_2}$$

and substituting X_2O_2 in equation (A-5) and rearranging

$$Y_2O_2 = \frac{Z_1\cos\delta_1}{2Z_2\cos\delta_2}\left[b_1(\tan\delta_1 - \tan\alpha_1) + b_2(\tan\delta_1 - \tan\alpha_2)\right] - \left[\frac{M_{e_1} + M_{e_2} + Q_{e_1} h_1 + Q_{e_2} h_2}{Z_2\cos\delta_2}\right] \qquad \text{(A-6)}$$

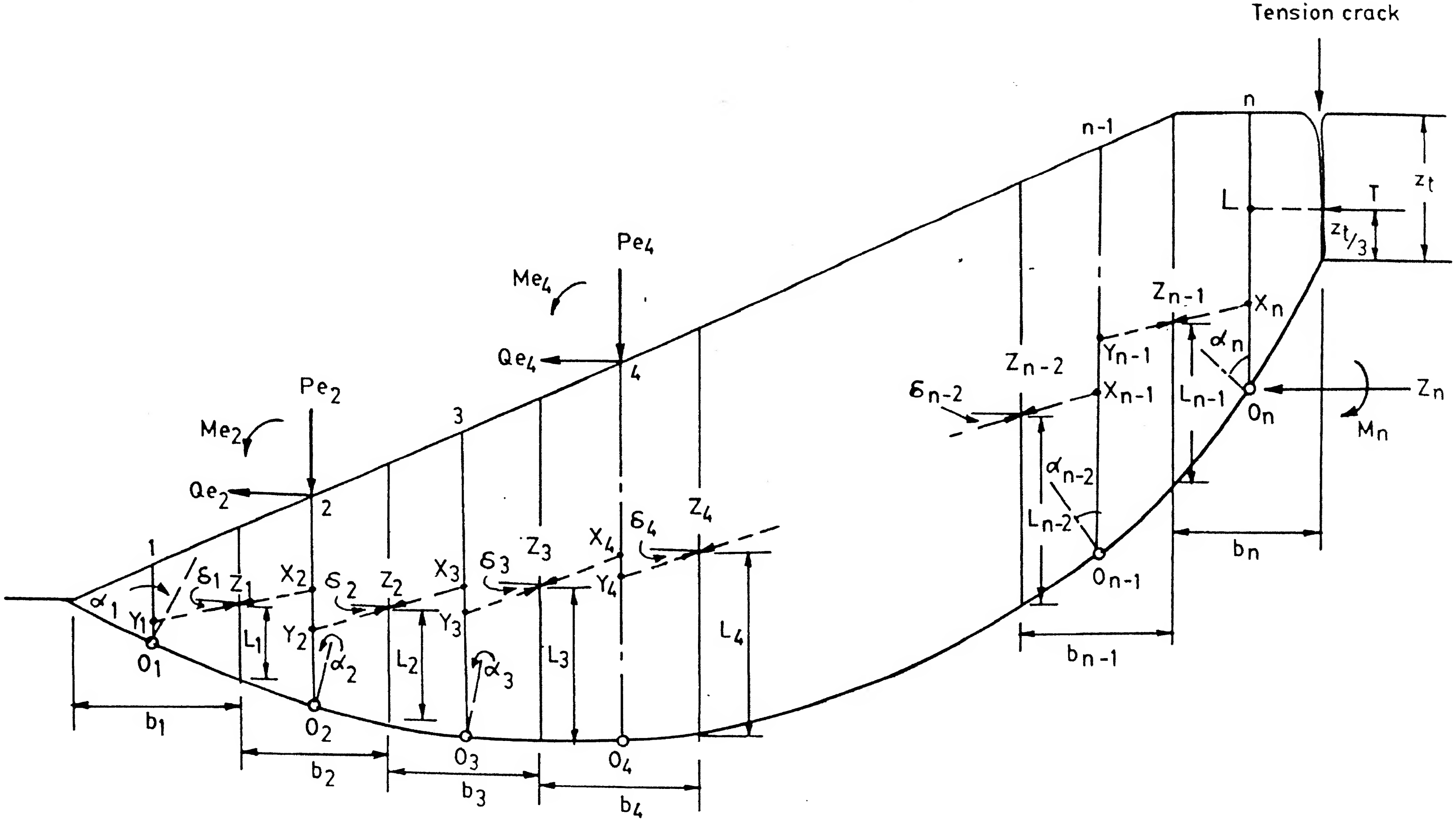


FIG. A-1 SLIDING MASS ALONG WITH THE DETAILS OF INTERSLICE AND BOUNDARY FORCES

In slice 3,

$$X_3O_3 = Y_2O_2 + \frac{1}{2}\left[\tan \delta_2(b_2 + b_3) - b_2\tan\alpha_2 - b_3\tan\alpha_3\right]$$

and substituting for Y_2O_2 in equation (A-6) and rearranging

$$X_3O_3 = \frac{1}{2Z_2\cos\delta_2}\left[Z_1\sin\delta_1(b_1 + b_2) + Z_2\sin\delta_2(b_2 + b_3) - Z_1\cos\delta_1(b_1\tan\alpha_1 + b_2\tan\alpha_2) - Z_2\cos\delta_2(b_2\tan\alpha_2 + b_3\tan\alpha_3)\right] - \left[\frac{M_{e_1} + M_{e_2} + Q_{e_1}h_1 + Q_{e_2}h_2}{Z_2\cos\delta_2}\right] \qquad \text{(A-7)}$$

Taking moment about O_3 of all forces

$$(Y_3O_3)\,Z_3\cos\delta_3 - (X_3O_3)Z_2\cos\delta_2 + M_{e_3} + Q_{e_3}h_3 = \emptyset$$

and $$Y_3O_3 = \frac{(X_3O_3)\,Z_2\cos\delta_2}{Z_3\cos\delta_3} - \frac{Q_{e_3}h_3 + M_{e_3}}{Z_3\cos\delta_3}$$

substituting X_3O_3 in (A-7) and rearranging

$$Y_3O_3 = \frac{1}{Z_3\cos\delta_3}\left\{\frac{1}{2}Z_1\left[\sin\delta_1(b_1+b_2) - \cos\delta_1(b_1\tan\alpha_1+b_2\tan\alpha_2)\right] + \frac{1}{2}Z_2\left[\sin\delta_2(b_2 + b_3) - \cos\delta_2(b_2\tan\alpha_2 + b_3\tan\alpha_3)\right] - (M_{e_1} + M_{e_2} + M_{e_3} + Q_{e_1}h_1 + Q_{e_2}h_2 + Q_{e_3}h_3)\right\} \qquad \text{(A-8)}$$

This applies to slice 3. Similarly, in slice I, the corresponding dimension

$$Y_IO_I = \frac{1}{Z_I\cos\delta_I}\left[\sum_{i=1}^{I-1}\left\{\frac{1}{2}Z_i\left[\left(b_i + b_{i+1}\right)\sin\delta_i - \left(b_i\tan\alpha_i + b_{i+1}\tan\alpha_{i+1}\right)\cos\delta_i\right]\right\} - \sum_{i=1}^{I}\left(M_{e_i} + Q_{e_i}h_i\right)\right]$$

Since $Z_o = \emptyset$, this expression can be written alternatively,

$$Y_I O_I = \frac{1}{Z_I \cos\delta_I} \sum_{i=1}^{I} \left\{ \tfrac{1}{2} Z_{i-1} \left[\sin\delta_{i-1} \left(b_{i-1} + b_i \right) \right. \right.$$

$$\left. \left. - \cos\delta_{i-1} \left(b_{i-1} \tan\alpha_{i-1} + b_i \tan\alpha_i \right) \right] - \left(Q_{e_i} h_i + M_{e_i} \right) \right\}$$

$$Y_I O_I = \frac{1}{Z_I \cos\delta_I} \sum_{i=1}^{I} \left\{ [J] - \left[Q_{e_i} h_i + M_{e_i} \right] \right\} \qquad \text{(A-9)}$$

where

$$J = \tfrac{1}{2} Z_{i-1} \left[\sin\delta_{i-1} \left(b_{i-1} + b_i \right) - \cos\delta_{i-1} \left(b_{i-1} \tan\alpha_{i-1} + b_i \tan\alpha_i \right) \right] \qquad \text{(A-10)}$$

Now $L_I = Y_I O_I + \frac{1}{2} b_I (\tan\delta_I - \tan\alpha_I)$

Substituting for $Y_I O_I$ in (A-9)

$$L_I = \tfrac{1}{2} b_I (\tan\delta_I - \tan\alpha_I) + \frac{1}{Z_I \cos\delta_I} \sum_{i=1}^{I} \left\{ [J] - \left[Q_{e_i} h_i + M_{e_i} \right] \right\} \qquad \text{(A-11)}$$

In the last (n^{th}) slice, taking moments about O_N of all forces

$$T(LO_n) - Z_{n-1} \cos\delta_{n-1} (X_n O_n) - M_n + M_{e_n} + Q_{e_n} h_n = 0 \qquad \text{(A-12)}$$

but $X_n O_n = L_{n-1} + \frac{1}{2} b_n (\tan\delta_{n-1} - \tan\alpha_n)$ (A-13)

Substituting for L_{n-1} from equation (A-11)

$$X_n O_n = \tfrac{1}{2} b_{n-1} (\tan\delta_{n-1} - \tan\alpha_{n-1}) + \frac{1}{Z_{n-1} \cos\delta_{n-1}} \sum_{i=1}^{n-1} \left\{ [J] \right.$$

$$\left. - \left[Q_{e_i} h_i + M_{e_i} \right] \right\} + \tfrac{1}{2} b_n (\tan\delta_{n-1} - \tan\alpha_n) \qquad \text{(A-14)}$$

From equation (A-10)

$$\sum_{i=1}^{n-1} [J] = \sum_{i=1}^{n} [J] - \tfrac{1}{2} Z_{n-1} \left[\sin\delta_{n-1} \left(b_n + b_{n-1} \right) - \cos\delta_{n-1} \left(b_n \tan\alpha_n + b_{n-1} \tan\alpha_{n-1} \right) \right.$$

Therefore,

$$\frac{1}{Z_{n-1}\cos\delta_{n-1}} \sum_{i=1}^{n-1} [J] = \frac{1}{Z_{n-1}\cos\delta_{n-1}} \sum_{i=1}^{n} [J] - \frac{1}{2}\Big[\tan\delta_{n-1}\big(b_n + b_{n-1}\big) - \big(b_n \tan\alpha_n + b_{n-1}\tan\alpha_{n-1}\big)\Big]$$

Substituting in equation (A-14)

$$X_n O_n = \frac{1}{2} b_{n-1}(\tan\delta_{n-1} - \tan\alpha_{n-1}) + \frac{1}{2} b_n (\tan\delta_{n-1} - \tan\alpha_n) + \frac{1}{Z_{n-1}\cos\delta_{n-1}} \sum_{i=1}^{n} [J] - \frac{1}{2}\Big[\tan\delta_{n-1}\big(b_n + b_{n-1}\big) - \big(b_n \tan\alpha_n + b_{n-1}\tan\alpha_{n-1}\big)\Big] - \frac{1}{Z_{n-1}\cos\delta_{n-1}} \sum_{i=1}^{n-1} \Big[Q_{e_i} h_i + M_{e_i}\Big]$$

Hence

$$X_n O_n = \frac{1}{Z_{n-1}\cos\delta_{n-1}} \left\{ \sum_{i=1}^{n} \Big[J\Big] - \sum_{i=1}^{n-1} \Big(Q_{e_i} h_i + M_{e_i}\Big) \right\}$$

Substituting in equation (A-12)

$$T(LO_n) - \sum_{i=1}^{n} \Big[J\Big] + \sum_{i=1}^{n-1} \Big(Q_{e_i} h_i + M_{e_i}\Big) - M_n + M_{e_n} + Q_{e_n} h_n = \emptyset$$

or $$T(LO_n) - \sum_{i=1}^{n} \Big[J\Big] + \sum_{i=1}^{n} \Big(Q_{e_i} h_i + M_{e_i}\Big) - M_n = \emptyset$$

and $$T(LO_n) = \frac{1}{2}\gamma_w z_t^2 \left(\frac{z_t}{3} + \frac{b_n}{2}\tan\alpha_n\right)$$

i.e., $$M_n = \gamma_w \frac{z_t^2}{2} \left[\frac{z_t}{3} + \frac{b_n}{2}\tan\alpha_n \right] - \sum_{i=1}^{n} \Big[J\Big] + \sum_{i=1}^{n} \Big(Q_{e_i} h_i + M_{e_i}\Big) \qquad \text{(A-15)}$$

A-3 Expressions for Z_n and its Derivatives

Equation (A-1) may be re-written as,

$$Z_i = \frac{1}{r_i}\left[p_i + q_i\ Z_{i-1}\right] \tag{A-16}$$

where

$$p_i = p_{1_i} + p_{2_i} \tan\phi'_{m_i} \tag{A-17}$$

and $$p_{1_i} = c'_{m_i} b_i \sec\alpha_i - W_i \sin\alpha_i - p_{e_i} \sin\alpha_i - Q_{e_i} \cos\alpha_i \tag{A-18}$$

$$p_{2_i} = W_i \cos\alpha_i - u_i b_i \sec\alpha_i + p_{e_i} \cos\alpha_i - Q_{e_i} \sin\alpha_i \tag{A-19}$$

$$c'_{m_i} = \frac{c'_i}{F} \tag{A-20}$$

$$\tan\phi'_{m_i} = \frac{\tan\phi'_i}{F} \tag{A-21}$$

$$\tan\delta_i = k_i \tan\theta_i \tag{A-22}$$

$$q_i = \cos(\alpha_i - \delta_{i-1})[1 + \tan\phi'_{m_i}\ \tan(\alpha_i - \delta_{i-1})] \tag{A-23}$$

$$r_i = \cos(\alpha_i - \delta_i)[1 + \tan\phi'_{m_i}\ \tan(\alpha_i - \delta_i)] \tag{A-24}$$

For the last slice, substituting i=n and considering presence of water pressure in the tension crack, from the above recurrence relation one obtains,

$$Z_n + T = \frac{1}{r_n}\ [p_n + q_n\ Z_{n-1}]$$

Therefore, $$Z_n = \frac{1}{r_n}\ [p_n + q_n\ Z_{n-1}] - \frac{1}{2}\gamma_w\ z_t^2 \tag{A-25}$$

Differentiation of expression (A-16) with respect to F and θ gives:

$$r_i \frac{\partial Z_i}{\partial F} = -\left(\frac{\partial r_i}{\partial F}\right) Z_i + \frac{\partial q_i}{\partial F} Z_{i-1} + q_i \frac{\partial Z_{i-1}}{\partial F} + \frac{\partial p_i}{\partial F} \tag{A-26}$$

$$r_i \frac{\partial Z_i}{\partial \theta} = -\left(\frac{\partial r_i}{\partial \theta}\right) Z_i + \frac{\partial q_i}{\partial \theta} Z_{i-1} + q_i \frac{\partial Z_{i-1}}{\partial \theta} \tag{A-27}$$

The derivatives of p, q and r can easily be obtained from equations (A-17) through (A-24) as:

$$\frac{\partial p_i}{\partial F} = \frac{\partial p_{1_i}}{\partial F} - p_{2_i}\frac{\tan\phi'_i}{F^2} \qquad \text{(A-28)}$$

$$\frac{\partial q_i}{\partial F} = \frac{-\tan\phi'_i \sin(\alpha_i - \delta_{i-1})}{F^2} \qquad \text{(A-29)}$$

$$\frac{\partial r_i}{\partial F} = \frac{-\tan\phi'_i \sin(\alpha_i - \delta_i)}{F^2} \qquad \text{(A-30)}$$

Further,

$$\frac{\partial q_i}{\partial \theta} = \left[\sin(\alpha_i - \delta_{i-1}) - \tan\phi'_{m_i} \cos(\alpha_i - \delta_{i-1})\right]\frac{\partial \delta_{i-1}}{\partial \theta} \qquad \text{(A-31)}$$

$$\frac{\partial r_i}{\partial \theta} = \left[\sin(\alpha_i - \delta_i) - \tan\phi'_{m_i} \cos(\alpha_i - \delta_i)\right]\frac{\partial \delta_i}{\partial \theta} \qquad \text{(A-32)}$$

where

$$\frac{\partial p_{1_i}}{\partial F} = \frac{-c'_i b_i \sec\alpha_i}{F^2} \qquad \text{(A-33)}$$

Again, referring to equation 2.29

$$\frac{\partial \delta_i}{\partial \theta} = k_i \frac{1 + \tan^2\theta}{1 + \tan^2\delta_i} \qquad \text{(A-34)}$$

Starting with

$$Z_o = 0 = \frac{\partial Z_o}{\partial F} = \frac{\partial Z_o}{\partial \theta} \qquad \text{(A-35)}$$

Equations (A-25), (A-26) and (A-27) may be used to obtain

Z_n, $\frac{\partial Z_n}{\partial F}$ and $\frac{\partial Z_n}{\partial \theta}$, noting that,

$$\frac{\partial Z_n}{\partial F} = \left.\frac{\partial Z_i}{\partial F}\right|_{i=n} \qquad \text{(A-36)}$$

$$\frac{\partial Z_n}{\partial \theta} = \left.\frac{\partial Z_i}{\partial \theta}\right|_{i=n} \qquad \text{(A-37)}$$

A-4 Expression for Derivatives of M_n:

Differentiating expression (A-15) with respect to F and θ,

$$\frac{\partial M_n}{\partial F} = -\sum_{i=1}^{n}\left[\frac{\partial J}{\partial F}\right] \qquad \text{(A-38)}$$

$$\frac{\partial M_n}{\partial \theta} = -\sum_{i=1}^{n}\left[\frac{\partial J}{\partial \theta}\right] \qquad \text{(A-39)}$$

$\frac{\partial J}{\partial F}$ and $\frac{\partial J}{\partial \theta}$ can be obtained by differentiating expression (A-10) with respect to F and θ respectively as :

$$\frac{\partial J}{\partial F} = \frac{1}{2}\frac{\partial Z_{i-1}}{\partial F}\left[\sin\delta_{i-1}\left(b_i+b_{i-1}\right)-\cos\delta_{i-1}\left(b_i\tan\alpha_i+b_{i-1}\tan\alpha_{i-1}\right)\right] \qquad \text{(A-40)}$$

$$\frac{\partial J}{\partial \theta} = \frac{1}{2}\left\{\frac{\partial Z_{i-1}}{\partial \theta}\left[\sin\delta_{i-1}\left(b_i + b_{i-1}\right) - \cos\delta_{i-1}\left(b_i\tan\alpha_i + b_{i-1}\tan\alpha_{i-1}\right)\right]\right.$$

$$+ Z_{i-1}\left[\left(b_i + b_{i-1}\right)\cos\delta_{i-1}\frac{\partial \delta_{i-1}}{\partial \theta} + \left(b_i\tan\alpha_i + b_{i-1}\tan\alpha_{i-1}\right)\right.$$

$$\left.\left.\sin\delta_{i-1}\frac{\partial \delta_{i-1}}{\partial \theta}\right]\right\} \qquad \text{(A-41)}$$

where $\frac{\partial Z_{i-1}}{\partial F}$, $\frac{\partial Z_{i-1}}{\partial \theta}$ and $\frac{\partial \delta_{i-1}}{\partial \theta}$ can be obtained from equations (A-26), (A-27) and (A-34) respectively.

A-5 Derivation of Equation (2.6) in Janbu's GPS

Referring to Figure A-2, taking moments about the point of application of the total resultant ΔN, (i.e., where ΔW intersects the base) one obtains,

$$(E+\Delta E)h_m - E(\Delta Y_t + h_m) - (T+\Delta T)(\Delta x - b_m) - b_m.T - \Delta Q(z_Q + b_m \tan\alpha) - \Delta M = \emptyset \tag{A-42}$$

or $$-E\Delta Y_t + \Delta E h_m - T\Delta x - \Delta T\Delta x + \Delta T b_m - (z_Q + b_M \tan\alpha)\Delta Q - \Delta M = \emptyset \tag{A-43}$$

Rearranging and dividing by Δx,

$$T = -E\tan\alpha_t + h_m \frac{\Delta E}{\Delta x} - \frac{(\Delta x - b_m)\Delta T}{\Delta x} - (z_Q + b_m \tan\alpha)\frac{\Delta Q}{\Delta x} - \frac{\Delta M}{\Delta x} \tag{A-44}$$

Noting that, $\lim\limits_{\Delta x \to \emptyset} \frac{\Delta E}{\Delta x} = \frac{dE}{dx}$

$$\lim_{\Delta x \to \emptyset} \frac{\Delta Q}{\Delta x} = \frac{dQ}{dx}$$

$$\lim_{\Delta x \to \emptyset} \frac{\Delta M}{\Delta x} = \frac{dM}{dx}$$

$$\lim_{\Delta x \to \emptyset} \frac{\Delta T}{\Delta x} = \frac{dT}{dx}$$

Equation (A-44) can be, in the limit, written as :

$$T = -E\tan\alpha_t + h_m \frac{\Delta E}{\Delta x} - (\Delta x - b_m)\frac{dT}{dx} - (z_Q + b_m \tan\alpha)\frac{dQ}{dx} - \frac{dM}{dx} \tag{A-45}$$

However, Janbu's expression, in absence of ΔM, is as follows:

$$T = -E\tan\alpha_t + h_t \frac{dE}{dx} - z_Q \frac{dQ}{dx} \tag{A-46}$$

Equation (A-46) obviously is not the same as equation (A-45). The discrepancy and its reasons have been discussed by Madej (1971) and hence are not presented here. Keeping, however, Janbu's version unchanged, the extension due to the inclusion of ΔM takes the following form :

$$T = -E\tan\alpha_t + h_t \frac{dE}{dx} - z_Q \frac{dQ}{dx} - \frac{dM}{dx} \tag{A-47}$$

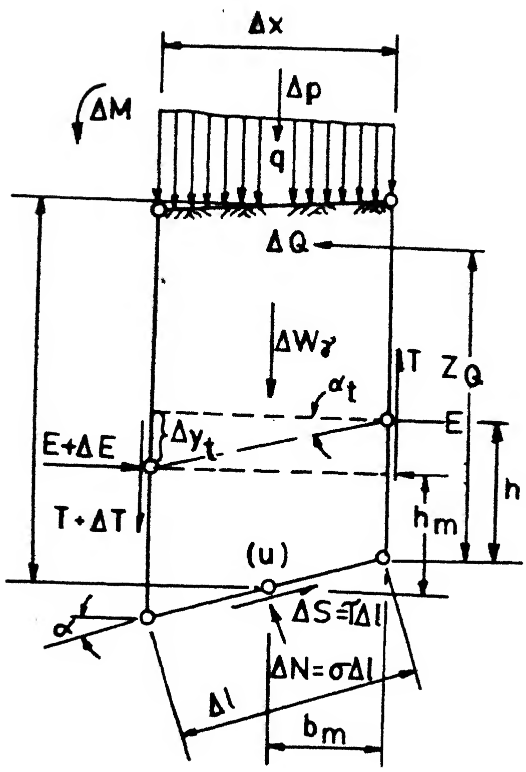


FIG. A-2 FORCES ACTING ON THE BOUNDARIES OF A SINGLE SLICE IN JANBU'S GPS

APPENDIX B

CALCULATION OF SLICE CHARACTERISTICES IN THE CASE OF A VERTICAL SECTION

It has been mentioned in chapter 6 that when the slope surface or any of the layer boundaries has a vertical section, the proposed generalized procedure for the calculation of slice details in a complex heterogeneous soil and pore pressure conditions can still be used by taking an imaginary range or slice of zero thickness i.e, introducing two vertical intersection boundaries at the same x. The detailed procedure of such calculations in the case of a vertical cut are presented in this Appendix.

B-1 Choice of the Discretization Model:

Unless the shear surface is forced through the toe, ideally the possibility of a slope failure i.e., the possibility of a shallow failure surface terminating at a point on the vertical section above the toe, needs also to be investigated. To keep this provision one of the shear surface co-ordinates constituting the design variables has to be allowed to move along the vertical section. In the Model II of discretization, as already discussed in chapter 3, the discrete points on the shear surface are allowed to vary only in the y-direction while keeping their x-co-ordinates fixed, and the same can be advantageously used here. This is because, unlike Model I, Model II ensures that the

vertical section forms one of the interslice boundaries which means one design variable will vary along the vertical wall section and thus the possibility of a slope failure can be investigated.

B-2 Steps to be Followed :

The following are the steps:

1. To include the slice of zero thickness by taking, in the program input, the number of "ranges" and hence the number of intersection boundaries as their actual number plus one. At the vertical section, there will be two intersection boundaries having the same x. At the left boundary the slope surface has zero y co-ordinate and at the right boundary it has a y-co-ordinate of H_t, the height of the wall.

2. By inspection, the ranges, R_1, R_n and R_o (Figure B-1) containing the points of intersection of the shear surface with the ground surface (A and B in Figure B-1) and the vertical section are found. The co-ordinates (x_U and x_L) of the intersection points A and B are then found out from a knowledge of the equations of the ground surface and the shear surface segments at those ranges. Details of such calculations have been already presented in chapter 3 in connection with the description of the discretization Model II.

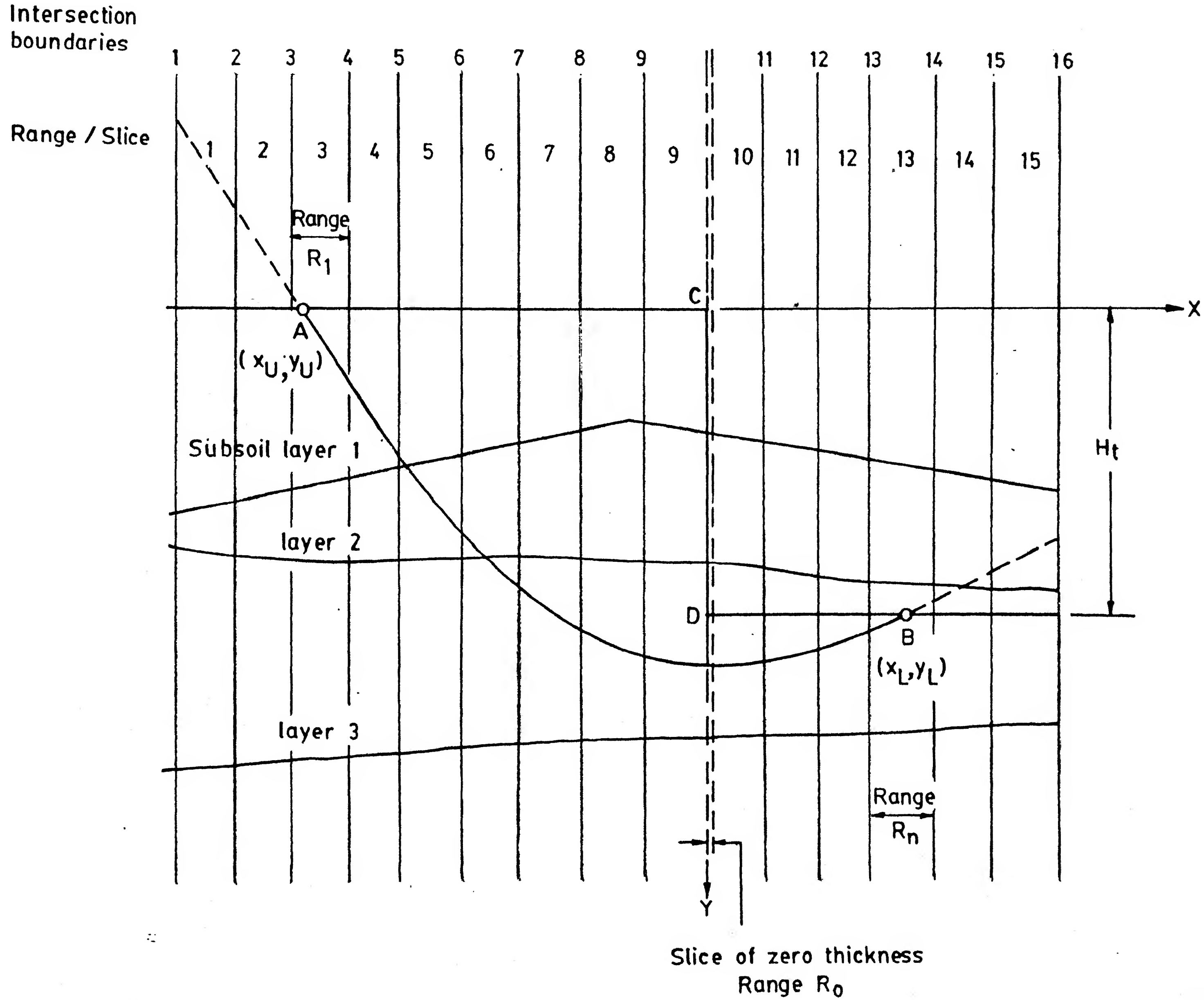


FIG. B-1 DISCRETIZATION MODEL II IN A VERTICAL SLOPE SECTION

3. The total number of slices within the sliding mass, including the one having zero-thickness, is given by,

$$n' = n_R - (n_{L_1} + n_{L_2})$$

where n_R = the total number of ranges

$n_{L_1} = R_1 - 1$

and $n_{L_2} = n_R - R_n$

The slice-details such as the weights, the pore pressures etc. are now calculated for each of the n' slices using the generalized procedure described in chapter 6 for any heterogeneous section.

4. Before the above calculated values are returned from the subroutine developed for this purpose, they are rearranged to exclude the imaginary slice. The actual number of slices for use in the expression for factor of safety is obtained as $n = n' - 1$. In Figure B-1, for instance, $n' = 12$ and hence $n = 11$. The slice-details up to the 7^{th} slice remains unchanged whereas those to the right of the 7^{th} slice are adjusted as follows:

Actual slice number <u>without zero slice</u>	Corresponding slice number <u>including zero slice</u>
8	9
9	10

and so on.

5. Although two intersection boundaries are considered present at the vertical section, only one variable (y_v) is employed to move along the intersection boundary coinciding with the vertical

section in the search for critical shear surface. If, during search, y_v becomes less than H_t, x_L is set equal to zero and it is not required to calculate the intersection point. A non-negativity constraint is imposed on this variable (as per the axis system shown in Figure B-1) so that the shear surface does not go above the ground surface. Other constraints, however, remain as discussed earlier.

It should be pointed out, however, that the above formulation has not been tested in the present study.

APPENDIX C

IDEALISATION OF THE IN-SITU UNDRAINED SHEAR STRENGTH PROFILE BY PIECEWISE POLYNOMIAL APPROXIMATION IN THE FORM OF CUBIC SPLINES

It has been mentioned in chapter 7 that the idealisation of the given irregular vane-strength variation may be performed by means of piecewise polynomial approximations in the form of cubic splines. This is discussed in some detail in this Appendix.

C-1 Spline-Fit:

A typical undrained (vane shear) strength profile is given in Figure C-1. It can be seen that the profile has slope discontinuities at point B,C,...,K. The problem of finding a function s(z) which fits the data is easily provided by cubic spline interpolation to the given strength data using N_P (=11) numbers of cubic splines which join continuously but with differing slopes at the above mentioned points (B through K), as shown in Figure C-2. The procedure for fitting the cubic splines followed in the computation performed in chapter 7 is described as follows:

Suppose, for example that the first piece AB is required to fit (N+1) points whose abscissae (z_i) and ordinates $f(z_i)$, i=1,2,...N+1, are given and let

$$a = z_1 < z_2 < \ldots\ldots < z_{N+1} = b$$

where N is the number of sub-intervals and is equal to four in this case.

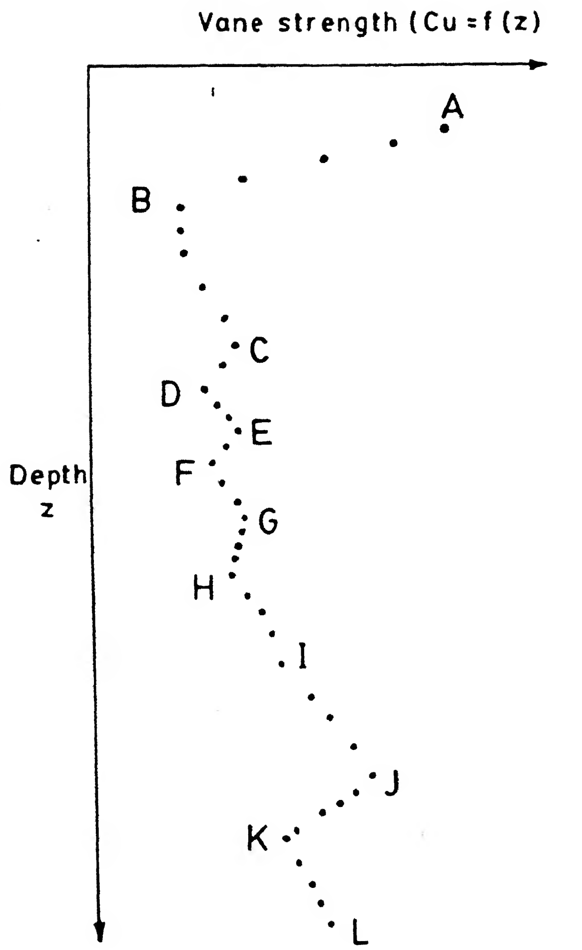


FIG. C1 IN SITU STRENGTH PROFILE

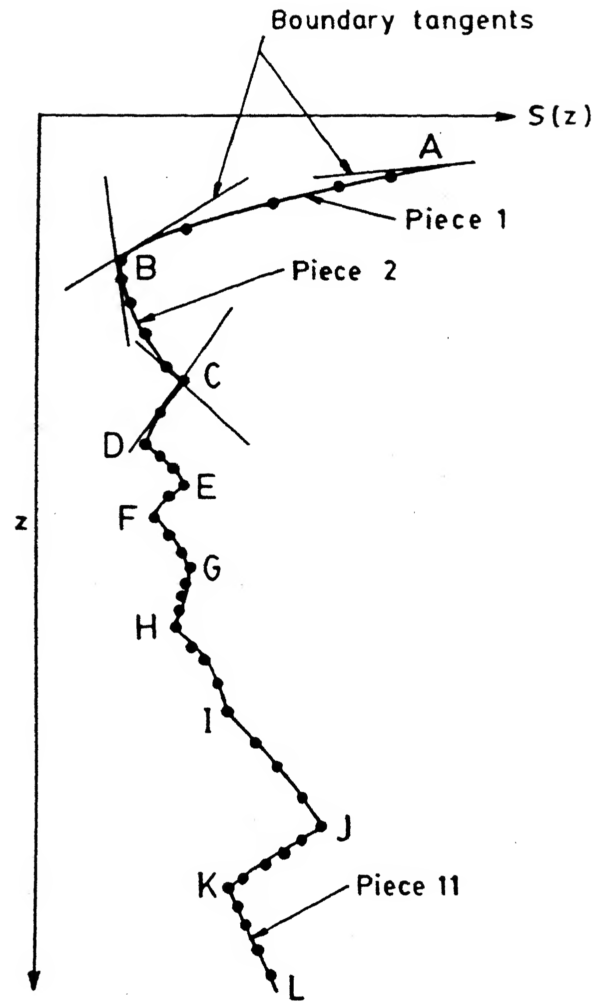


FIG. C3 SPLINE FITTED PROFILE

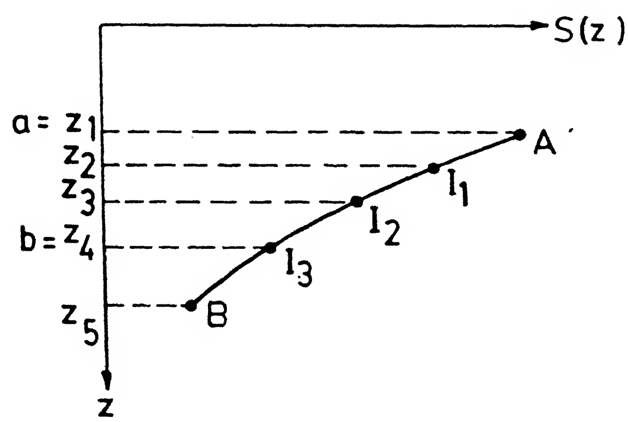


FIG. C2 CUBIC SPLINE-FIT PIECE 1

The interior interpolation points $z_2, \ldots, z_N$ are considered to be the breakpoints for the spline function $s(z)$; that is, on each sub-interval $[z_i, z_{i+1}]$, $s(z)$ is constructed as a certain cubic polynomial $P_i(z)$, $(i = 1,2,\ldots,N)$. To facilitate the use of $s(z)$ in subsequent calculations, each cubic piece $P_i(z)$ of $s(z)$ is written as:

$$P_i(z) = C_{1,i} + C_{2,i}(z-z_i) + C_{3,i}(z-z_i)^2 + C_{4,i}(z-z_i)^3 \quad ..(C\text{-}1)$$

Referring to Figure C-2, $P_i(z)$ fits the sub-piece AI_1, $P_2(z)$ fits the sub-piece I_1 I_2 etc. The problem now reduces to the determination of the coefficients $C_{j,i}$, $j=1,..,4$, $i=1,\ldots,N$ which makes it a total of 4N coefficients. The conditions available are the following:

1. The fitted spline function is continuous on [a,b]. This means:

$$\begin{aligned} P_i(z_i) &= f(z_i) \\ P_i(z_{i+1}) &= f(z_{i+1}) \qquad (i=1,2,\ldots,N) \end{aligned} \qquad (C\text{-}2)$$

(C-2) gives 2N equations.

2. The spline function is twice continuously differentiable on [a,b]. This implies:

$$P'_{i-1}(z_i) = P'_i(z_i)$$

and $$P''_{i-1}(z_i) = P''_i(z_i) \qquad (i=2,3,\ldots,N) \qquad (C\text{-}3)$$

(C-3) supplies 2(N-1) equations.

So from (C-2) and (C-3) a total of (4N-2) equations are available. Hence 2 more equations are needed for a unique solution of the 4N coefficients. These two additional conditions are obtained from the knowledge of the boundary derivatives $f'(a)$ and $f'(b)$ which are either estimated graphically or logically assumed. In the present study, they have been graphically estimated. These are also supplied as input in addition to the co-ordinates z_i and $f(z_i)$.

More details about the solution technique are available in standard text books on numerical analysis (Conte et. al., 1986).

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